ISSN 0735-2727, Radioelectronics and Communications Systems, 2011, Vol. 54, No. 7, pp. 345–353. © Allerton Press, Inc., 2011.
Original Russian Text © A.P. Trifonov, M.B. Bespalova, P.A. Trifonov, 2011, published in Izv. Vyssh. Uchebn. Zaved., Radioelektron., 2011, Vol. 54, No. 7, pp. 3–9.

Effectiveness of Ultra-Wideband Range and Velocity Estimation in Presence of Narrowband Interference

A. P. Trifonov, M. B. Bespalova, and P. A. Trifonov
Voronezh State University, Voronezh, Russia
Received in final form April 4, 2011

Abstract—Characteristics of two algorithms for estimating range and velocity are found. The influence of narrowband Gaussian interference on precision of the estimates is analyzed.

DOI: 10.3103/S0735272711070016

In [1–4] and other works the possibilities of using ultra-short (subnanosecond) pulses and their sequences in radar technologies are considered. Short-pulse signals and their sequences represent a special case of ultra-wideband signals (UWBS), using which has peculiarities and allows broadening the capabilities of radars. In [4] the characteristics of ultra-wideband range and velocity estimates are found in conditions of white Gaussian noise (WGN). However in real conditions besides WGN narrowband intentional interference, which may be interpreted as Gaussian narrowband random process [5], often affects the signal. Hence let's consider range and velocity estimation in conditions of both WGN and Gaussian narrowband interference (GNI).

Similarly to [4] the probing sequence (UWBS) will be represented by

$$\tilde{s}_N(t) = \sum_{k=0}^{N-1} s_0[t-(k-\mu)\theta-\lambda], \tag{1}$$

where function $s_0(\cdot)$ describes the shape of one pulse, θ is the period, λ is the time position of the sequence. Parameter μ denotes the point of the sequence (1), which is connected to its time position. When $\mu = 0$ the quantity λ represents the time position of the first pulse of the sequence, when $\mu = (N-1)/2$, it denotes the position of the middle, and when $\mu = N-1$ it references the last pulse in the sequence.

We assume that the probing sequence (1) is scattered by the target with range R_0 and velocity V_0, so that

$$|V_0| << c, \tag{2}$$

where c is the light velocity. Then the received signal may be represented by [4]

$$s(t, R_0, V_0) = \sum_{k=0}^{N-1} s[t-2R_0/c-(k-\mu)\theta(1+2V_0/c)]. \tag{3}$$

Function $s(\cdot)$ describes the shape of one received pulse and, in a general case, differs from $s_0(\cdot)$ in (1) [4].

Under the influence of WGN and GNI the following realization is observed on the interval $[0, T]$

$$x(t) = s(t, R_0, V_0) + n(t) + y(t). \tag{4}$$

Here $n(t)$ is the centered WGN with one-sides spectral density N_0, $y(t)$ is the centered GNI with correlation function $K_y(\tau) = \langle y(t)y(t+\tau)\rangle$ and spectral density

$$G_y(\omega) = \int_{-\infty}^{\infty} K_y(\tau)\exp(-j\omega\tau)\,d\tau. \tag{5}$$

Processes $n(t)$ and $y(t)$ are assumed to be statistically independent.

Let's assume that correlation function and spectral density of GNI are unknown. In this case to estimate range and velocity we suggest using the maximum likelihood algorithm, synthesized for the case of no GNI. Let the duty cycle of the sequence (3) be rather large, so that separate pulses do not overlap and the observation interval $[0, T]$ is greater than the duration of the whole sequence, i.e. $T > N\theta$. Then in conditions of WGN only logarithm of the likelihood ratio functional, neglecting an insignificant summand, may be expressed as [4, 6]

$$L_1(R,V) = \frac{2}{N_0} \times \sum_{k=0}^{N-1} \int_0^T x(t)s[t - 2R/c - (k-\mu)\theta(1+2V/c)]dt. \tag{6}$$

Realization of the observed data $x(t)$ (4) besides WGN $n(t)$ contains GNI $y(t)$. Hence the estimates

$$(\hat{R}_1, \hat{V}_1) = \mathrm{argsup} L_1(R,V) \tag{7}$$

are not the maximum likelihood estimates (MLE). These estimates may be called quasi-likely estimates (QLE) [7] since they coincide with MLE when $y(t) \equiv 0$, i.e. in the absence of GNI.

To determine characteristics of QLE let's represent (6) as a sum of signal and noise functions [6]

$$L_1(R,V) = S_1(R,R_0,V,V_0) + N_1(R,V). \tag{8}$$

Here the signal function is represented by

$$S_1(R,R_0,V,V_0) = \frac{2}{N_0}\sum_{k=0}^{N-1}\sum_{n=0}^{N-1}\int_0^T s\big[t - 2R_0/c - (k-\mu)\theta(1+2V_0/8c)\big]$$

$$\times s\big[t - 2R/c - (n-\mu)\theta(1+2V/c)\big]dt, \tag{9}$$

and the noise function is expressed as $N_1(R,V) = L_1(R,V) - \langle L_1(R,V)\rangle$ and is a realization of Gaussian random field. First two moments of the noise function are represented by

$$\langle N_1(R,V)\rangle = 0,$$

$$K_1(R_1,R_2,V_1,V_2) = \langle N_1(R_1,V_1)N_1(R_2,V_2)\rangle$$

$$= \sum_{k=0}^{N-1}\sum_{n=0}^{N-1}\left\{\frac{2}{N_0}\int_0^T s[t - 2R_1/c - (k-\mu)\theta(1\right.$$

$$+2V_1/c)]s\big[t - 2R_2/c - (n-\mu)\theta(1+2V_2/c)\big]dt$$

$$+\frac{4}{N_0^2}\int_0^T\int_0^T K_y(t_2 - t_1)s[t_1 - 2R_1/c - (k-\mu)\theta(1$$

$$+2V_1/c)]s[t_2-2R_2/c-(n-\mu)\theta(1+2V_2/c)]dt_1dt_2\Big\}. \tag{10}$$

Since we assume that $T > N\theta$, the whole received sequence (3) is located inside the observation interval, and the integration limits in (9) and (10) may be substituted by infinities. As a result we obtain

$$S_1(R,R_0,V,V_0)=\sum_{k=0}^{N-1}\sum_{n=0}^{N-1}S_f\{2(R-R_0)/c$$

$$+(n-k)\theta+2\theta[nV-kV_0-\mu(V-V_0)]/c\}, \tag{11}$$

$$K_1(R_1,R_2,V_1,V_2)=\sum_{k=0}^{N-1}\sum_{n=0}^{N-2}K_H\{2(R_2-R_1)/c$$

$$+(n-k)\theta+2\theta[nV_2-kV_1-\mu(V_2-V_1)]/c\}. \tag{12}$$

In (11) and (12)

$$S_f(\tau)=\frac{2}{N_0}\int_{-\infty}^{\infty}s(t)s(t-\tau)dt$$

is the signal function (the uncertainty function) [6] for a single UWBS of the sequence (3), while

$$K_H(\eta)=S_f(\eta)+\frac{4}{N_0^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}K_y(t_2-t_1+\eta)s(t_1)s(t_2)dt_1dt_2.$$

Let's denote duration of one pulse in the sequence (3) and GNI correlation time by τ_s and τ_y, respectively, so that $S_f(\pm\tau_s)\approx 0$ and $K_y(\pm\tau_y)\approx 0$. Let's limit the consideration to the central peaks of signal (11) and correlation (12) functions, assuming that besides (2) the following condition is satisfied

$$\max\{|R-R_0|,|R_1-R_2|\}\le c\theta/2. \tag{13}$$

Let the duty cycle of the received UWBS sequence (3) be rather large, so that

$$\tau_s \ll \theta,\quad \tau_y \ll \theta. \tag{14}$$

Then for the summands in (11) we obtain

$$S_f\{2(R-R_0)/c+(n-k)\theta+2\theta[nV-kV_0-\mu(V-V_0)]/c\}\approx 0$$

when $n \ne k$ and

$$S_f\{(R-R_0)/c+(n-k)\theta+2\theta[nV-kV_0$$

$$-\mu(V-V_0)]/c\}=S_f[2(R-R_0)/c+2\theta(k-\mu)(V-V_0)/c],$$

when $n = k$. Correspondingly, for the summands in (12) we may write down

$$K_H\{2(R_2-R_1)/c+(n-k)\theta+2\theta[nV_2-kV_1-\mu(V_2-V_1)]/c\}\approx 0$$

when $n \neq k$ and

$$K_H\{2(R_2 - R_1)/c + (n-k)\theta + 2\theta[nV_2 - kV_1 - \mu(V_2 - V_1)]/c\} = K_H[2(R_2 - R_1)/c + 2\theta(k-\mu)(V_2 - V_1)/c],$$

when $n = k$. As a result, when (2), (13), (14) are satisfied, functions (11) and (12) may be represented by

$$S_1(R, R_0, V, V_0) = \sum_{k=0}^{N-1} S_f[2(R - R_0)/c + 2\theta(k-\mu)(V - V_0)/c], \tag{15}$$

$$K_1(R_1, R_2, V_1, V_2) = \sum_{k=0}^{N-1} K_H[2(R_2 - R_1)/c + 2\theta(k-\mu)(V_2 - V_1)/c]. \tag{16}$$

Obviously [6], the signal function (15) reaches its maximum when $R = R_0, V = V_0$. Hence the signal-to-noise ratio (SNR) [6] may be expressed as

$$z_1^2 = S_1^2(R_0, R_0, V_0, V_0)/K_1(R_0, R_0, V_0, V_0). \tag{17}$$

Substituting the values of functions (15) and (16) into (17) yields

$$z_1^2 = z^2/\chi_1 = Nz_0^2/\chi_1. \tag{18}$$

Here $z^2 = Nz_0^2$ is the SNR at the output of maximum likelihood receiver in the absence of GNI, $z_0^2 = 2E/N_0$ is the SNR for one UWBS in the absence of GNI, and $E = \int_{-\infty}^{\infty} s^2(t)\mathrm{d}t$ is the energy of one UWBS in the sequence (3). The quantity χ_1 in (18) shows how many times SNR is decreased as a consequence of GNI influence and is represented by

$$\chi_1 = 1 + \frac{2}{N_0}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K_y(t_2 - t_1)s(t_1)s(t_2)\mathrm{d}t_1\mathrm{d}t_2 \Big/ \int_{-\infty}^{\infty} s^2(t)\mathrm{d}t. \tag{19}$$

According to (7), QLE $\hat{R}_1$ and $\hat{V}_1$ are the solutions of the following system of equations

$$\frac{\partial}{\partial R}\left[S_1(R, R_0, V, V_0) + N_1(R, V)\right]_{\hat{R}_1, \hat{V}_1} = 0,$$

$$\frac{\partial}{\partial V}\left[S_1(R, R_0, V, V_0) + N_1(R, V)\right]_{\hat{R}_1, \hat{V}_1} = 0. \tag{20}$$

If the noise function in (8) is absent, i.e. $N_1(R, V) \equiv 0$, the function (6) reaches its maximum in point (R_0, V_0). Consequently, QLE (7) is asymptotically unbiased [6].

Let's assume that SNR (18) is rather large, so that QLE (7) possess high posterior precision. Then solutions of the equations (20) may be determined using the method of small parameter [6] when the quantity $1/z_1$ is considered the small parameter. Limiting the consideration to the first approximation, we determine dispersions of QLE (7)

$$D_1(R) = \left\langle(\hat{R}_1 - R_0)^2\right\rangle = (S_{RV}^2 K_V - 2S_{RV}S_V K_{RV} + S_V^2 K_R)(S_R S_V - S_{RV}^2)^{-1}, \tag{21}$$

$$D_1(V)=\left\langle(\hat{V}_1-V_0)^2\right\rangle=(S_R^2K_V-2S_RS_{RV}K_{RV}+S_{RV}^2K_R)(S_RS_V-S_{RV}^2)^{-1}. \qquad (22)$$

Here we have the following denotations:

$$S_R=\left[\frac{\partial^2S_1(R,R_0,V,V_0)}{\partial R^2}\right],$$

$$S_V=\left[\frac{\partial^2S_1(R,R_0,V,V_0)}{\partial V^2}\right],$$

$$S_{RV}=\left[\frac{\partial^2S_1(R,R_0,V,V_0)}{\partial R\partial V}\right],$$

$$K_R=\left[\frac{\partial^2K_1(R_1,R_2,V_1,V_2)}{\partial R_1\partial R_2}\right],$$

$$K_V=\left[\frac{\partial^2K_1(R_1,R_2,V_1,V_2)}{\partial V_1\partial V_2}\right],$$

$$K_{RV}=\left[\frac{\partial^2K_1(R_1,R_2,V,V_2)}{\partial R_1\partial V_2}\right]. \qquad (23)$$

All derivatives (23) are calculated in point (R_0,V_0). Substituting (15), (16) into (23) and the result of differentiation into (21) and (22) we obtain the following expressions for dispersions of range and velocity QLE (7):

$$D_1(R)=\left\langle(\hat{R}_1-R_0)^2\right\rangle=D_0(R)\kappa_1,$$

$$D_1(V)=\left\langle(\hat{V}_1-V_0)^2\right\rangle=D_0(V)\kappa_1.$$

Here

$$D_0(R)=\frac{c^2N_0}{8F_0}\frac{N^2-1+12\left[(N-1)/2-\mu\right]^2}{N(N^2-1)}, \qquad (24)$$

$$D_0(V)=\frac{3c^2N_0}{2\theta^2F_0N(N^2-1)} \qquad (25)$$

are dispersions of range and velocity MLE in the absence of GNI [4], $F_0=\int_{-\infty}^{\infty}[\mathrm{d}s(t)/\mathrm{d}t]^2\mathrm{d}t$.

The quantity

$$\kappa_1 = 1 + \frac{2}{N_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} K_y(t_2 - t_1) \frac{ds(t_1)}{dt_1} \frac{ds(t_2)}{dt_2} dt_1 dt_2 / F_0 \tag{26}$$

shows the gain in precision of QLE (7) due to the impact of GNI.

The greatest precision of range and velocity estimates may be obtained is the GNI correlation function $K_y(\tau)$ in known in advance. In this case logarithm of the likelihood ratio functional, neglecting an insignificant summand, may be represented by [6]

$$L_2(R,V) = \sum_{k=0}^{N-1} \int_0^T x(t) v[t - 2R/c - (k-\mu)\theta(1 + 2V/c)] dt, \tag{27}$$

where function $v(t)$ is determined by solving the following integral equation

$$N_0 v(t)/2 + \int_0^T K_y(t-\tau) v(\tau) d\tau = s(t).$$

Correspondingly, MLE $(\hat{R}_2, \hat{V}_2)$ for range R_0 and velocity V_0 are expressed as

$$(\hat{R}_2, \hat{V}_2) = \operatorname{argsup} L_2(R,V). \tag{28}$$

To determine the characteristics of MLE let's represent (27) as a sum of signal and noise functions [6]

$$L_2(R,V) = S_2(R, R_0, V, V_0) + N_2(R,V).$$

Here considering satisfaction of (2), (13) and (14) the signal function is expressed as

$$S_2(R, R_0, V, V_0) = \sum_{k=0}^{N-1} \int_0^T s[t - 2R_0/c - (k-\mu)$$

$$\times \theta(1 + 2V_0/c)] v\left[t - 2R/c - (k-\mu)\theta(1 + 2V/c)\right] dt, \tag{29}$$

while the correlation function of the centered noise function is represented by

$$K_2(R_1, R_2, V_1, V_2) = \langle N_2(R_1, V_1) N_2(R_2, V_2) \rangle = S_2(R_1, R_2, V_1, V_2). \tag{30}$$

For the algorithm (27) and (28) SNR may be expressed as [6]

$$z_2^2 = S_2^2(R_0, R_0, V_0, V_0) = z^2 / \chi_2. \tag{31}$$

Here z^2 is the SNR at the output of maximum likelihood receiver (6) in the presence of WGN only. The quantity χ_2 in (31) shows how many times SNR is smaller due to the impact of GNI with known in advance characteristics and is defined by the expression

$$\chi_2 = \frac{2}{N_0} \int_{-\infty}^{\infty} s^2(t) dt \Big/ \int_{-\infty}^{\infty} s(t) v(t) dt. \tag{32}$$

Let's assume SNR (31) is rather large, so that MLE (28) possesses high posterior precision. Substituting (29) and (30) into (23) and the result of differentiation into (21) and (22) we obtain for range and velocity dispersions of MLE (28)

$$D_2(R) = \left\langle (\hat{R}_2 - R_0)^2 \right\rangle = D_0(R)\kappa_2,$$

$$D_2(V) = \left\langle (\hat{V}_2 - V_0)^2 \right\rangle = D_0(V)\kappa_2.$$

Here $D_0(R)$ and $D_0(V)$ are the range and velocity dispersions of MLE in the presence of WGN only. The quantity

$$\kappa_2 = \frac{2}{N_0} \int_{-\infty}^{\infty} \left[\frac{\mathrm{d}s(t)}{\mathrm{d}t}\right]^2 \mathrm{d}t \Big/ \int_{-\infty}^{\infty} \frac{\mathrm{d}s(t)}{\mathrm{d}t}\frac{\mathrm{d}v(t)}{\mathrm{d}t}\mathrm{d}t \tag{33}$$

shows how many times greater the dispersion of MLE (28) is due to the impact of GNI.

To calculate parameters (19), (26), (32) and (33) that determine the impact of GNI it is often convenient to use the frequency representation. Let's denote spectrum of one UWBS in the sequence (3) by

$$S(\mathrm{j}\omega) = \int_{-\infty}^{\infty} s(t)\mathrm{e}^{-\mathrm{j}\omega t}\mathrm{d}t.$$

Then expressions for parameters (19), (26), (32) and (33) may be rewritten as follows

$$\chi_1 = 1 + \frac{\int_{-\infty}^{\infty} \rho(\omega)|S(\mathrm{j}\omega)|^2 \,\mathrm{d}\omega}{\int_{-\infty}^{\infty} |S(\mathrm{j}\omega)|^2 \,\mathrm{d}\omega}, \tag{34}$$

$$\chi_2 = \left[1 - \frac{\int_{-\infty}^{\infty} \frac{\rho(\omega)|S(\mathrm{j}\omega)|^2}{1+\rho(\omega)} \mathrm{d}\omega}{\int_{-\infty}^{\infty} |S(\mathrm{j}\omega)|^2 \,\mathrm{d}\omega}\right]^{-1}, \tag{35}$$

$$\kappa_1 = 1 + \frac{\int_{-\infty}^{\infty} \omega^2 \rho(\omega)|S(\mathrm{j}\omega)|^2 \,\mathrm{d}\omega}{\int_{-\infty}^{\infty} \omega^2 |S(\mathrm{j}\omega)|^2 \,\mathrm{d}\omega}, \tag{36}$$

$$\kappa_2 = \left[1 - \frac{\int\limits_{-\infty}^{\infty} \frac{\omega^2 \rho(\omega) |S(j\omega)|^2}{1+\rho(\omega)} d\omega}{\int\limits_{-\infty}^{\infty} \omega^2 |S(j\omega)|^2 d\omega} \right]^{-1}, \tag{37}$$

where $\rho(\omega) = 2G_y(\omega) / N_0$.

Obviously, the quantities are always $\chi_i \geq 1$ and $\kappa_i \geq 1$, $i = 1, 2$, so that presence of GNI decreases the estimates effectiveness, even in the case when statistical characteristics of GNI are known in advance.

Let's determine parameters (34)–(37) for a special case of GNI of rectangular shape of spectral density (5)

$$G_y(\omega) = \frac{\gamma}{2}\left[I\left(\frac{\omega_0 - \omega}{\Omega}\right) + I\left(\frac{\omega_0 + \omega}{\Omega}\right) \right]. \tag{38}$$

Here γ is the GNI spectral density, ω_0 is the GNI central frequency, Ω is the GNI bandwidth, $I(x) = 1$ for $|x| \leq 1/2$ and $I(x) = 0$ for $|x| > 1/2$. Substituting (38) into (34)–(37) yields

$$\chi_1 = 1 + q\varepsilon, \quad \chi_2 = (1+q) / [1 + q(1-\varepsilon)], \tag{39}$$

$$\kappa_1 = 1 + q\delta, \quad \kappa_2 = (1+q) / [1 + q(1-\delta)]. \tag{40}$$

In (39) and (40) $q = \gamma / N_0$ is the ratio of GNI and WGN spectral densities,

$$\varepsilon = \int\limits_{\omega_0 - \Omega/2}^{\omega_0 + \Omega/2} |S(j\omega)|^2 d\omega \Big/ \int\limits_{0}^{\infty} |S(j\omega)|^2 d\omega$$

ε is the relative part of energy of one UWBS in the bandwidth affected by GNI, and

$$\delta = \int\limits_{\omega_0 - \Omega/2}^{\omega_0 + \Omega/2} \omega^2 |S(j\omega)|^2 d\omega \Big/ \int\limits_{0}^{\infty} \omega^2 |S(j\omega)|^2 d\omega$$

δ is the relative part of energy of the first UWBS derivative in the bandwidth affected by GNI. Comparing parameters (39) and (4) we find

$$\chi_1 / \chi_2 = 1 + q^2 \varepsilon (1-\varepsilon) / (1+q) \geq 1, \tag{41}$$

$$\kappa_1 / \kappa_2 = 1 + q^2 \delta (1-\delta) / (1+q) \geq 1. \tag{42}$$

The ratio (41) and (42) shows by how many times SNR is greater and dispersion is smaller for range and velocity estimates due to presence of prior information on GNI parameters when using the algorithm (28) instead of algorithm (7).

Thus, the obtained results allow making a grounded decision when choosing range and velocity estimation algorithms depending on the availability of prior information on GNI parameters and depending on requirements to the estimates' precision.

REFERENCES

1. L. Yu. Astanin and A. A. Kostylev, *Basics of Ultra-Wideband Radar Measurements* (Radio i Svyaz', Moscow, 1989) [in Russian].
2. H. F. Harmuth, *Nonsinusoidal Waves for Radar and Radio Communication* (Academic Press, 1981; Radio i Svyaz', 1985).
3. V. G. Radziyevskiy and P. A. Trifonov, Ultra-Wideband Signals and Noise Processing (Radiotekhnika, Moscow, 2009) [in Russian].
4. A. P. Trifonov and M. B. Bespalova, "Effectiveness of ultra-wideband detection and range and velocity measurements," Radiotekh. Elektron. **42**, No. 4, 451 (1997).
5. V. G. Radziyevskiy and A. A. Sirota, *Theoretical Basics of Radio-Electronic Intelligence* (Radiotekhnika, Moscow, 2004) [in Russian].
6. Ye. I. Kulikov and A. P. Trifonov, *Signal Parameters Estimation on the Noise Background* (Sov. Radio, Moscow, 1978) [in Russian].
7. V. I. Mudrov and V. L. Kushko, *Methods of Processing Measurements* (Radio i Svyaz', Moscow, 1983) [in Russian].