Potential Accuracy of Joint Signal Parameters Estimates for a Small-Sized Target in Bistatic Radar System

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Abstract—Analytical expressions that characterize accuracy of joint signal parameters estimation for a small-sized target in bistatic radar system are obtained. Influence of true parameters values on probabilistic characteristics of their estimates is analyzed.

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Nowadays practical interest to bistatic radars, which configuration expects availability of distant in space one transmitting and one receiving stations [1, 2], has significantly increased.

Configuration of bistatic radar is depicted in Fig. 1. Transmitter T, receiver R and target G form a so-called bistatic triangle. We'll call a line between T and R the base line, and a distance TR the base L of radar. Further we consider a case of planar problem, when bistatic triangle's plane T–G–R does not change its orientation in space.

The present article aims to determine potential accuracy of jointly estimating velocity of a small-sized target and time when target crosses the base line of a bistatic radar system.

Let's assume that we have a small-sized target that creates shadow. We'll also assume that receiver has monochromatic wave with frequency f_0 at its input. Using Kirchhoff method and Babine principle, one can obtain the following expression for received signal [1]:

$$s_{\rm R}(t) = A\cos\left[2\pi\left(f_0 t - \frac{L}{\lambda}\right)\right] + \frac{2A\mu}{\nu^2}\sin\left\{2\pi\left[f_0 t - \frac{L}{\lambda} - \frac{V^2(t-\tau)^2}{2\nu^2}\right]\right\},\tag{1}$$

where A is amplitude multiplier, λ is wavelength of probing signal; L is radar's base, $v = \sqrt{\lambda d_T d_R} / L$ is radius of first Fresnel zon, d_T and d_R are distances from target to transmitter and receiver respectively at time τ when target crosses the base line, μ is area of a small-sized target, V is absolute value of target's velocity.

Expression (1) is obtained using approximation of Fresnel diffraction of probing wave on the small-sized target in the shape of a sphere. Under such configuration of bistatic radar system target is considered to be small-sized when its area μ and first Fresnel zone's radius v satisfy the following inequality: $\sqrt{\mu} \ll v$.

Formula (1) may be also re-written as follows:

$$s_{\rm R}(t) = e(t) + s_{\rm sh}(t), \tag{2}$$

where

$$e(t) = A\cos\left[2\pi\left(f_0t - \frac{L}{\lambda}\right)\right],$$

$$s_{\rm sh}(t) = \frac{2A\mu}{\nu^2}\sin\left\{2\pi\left[f_0t - \frac{L}{\lambda} - \frac{V^2(t-\tau)^2}{2\nu^2}\right]\right\}.$$
(3)



As follows from (2), the received signal $s_{\rm R}(t)$ is a sum of non-informative oscillation e(t) and informative signal $s_{\rm sh}(t)$. Informative signal $s_{\rm sh}(t)$ is a consequence of shadow created by target. It contains information on target's velocity V, time τ when target crosses the base line and first Fresnel zone's radius v.

Due to target's movement with respect to transmitter and receiver one may observe the Doppler effect. It shows up in the fact that expression for informative signal $s_{sh}(t)$ contains target's velocity. From formula (3) it is easy to notice that informative signal appears to be frequency modulated according to linear law.

Phase of signal $s_{sh}(t)$ depends on first Fresnel zone's radius, target's velocity and time when target crosses the base line. Informative signal's amplitude in inversely proportional to second power of first Fresnel zone's radius and does not depend on parameters V and τ . Thus, parameter v is in expressions for amplitude and phase of informative signal $s_{sh}(t)$.

Let's consider a problem of estimating parameters of received bistatic radar's signal on the noise background. We'll assume that some realization of additive mixture that contains useful signal and noise is observed on a fixed time interval [0, T]. By processing the observed realization it is necessary to estimate the values of searched parameters. We'll assume that the estimated signal's parameters do not depend on time.

There exist various estimation approaches [3, 4] depending on prior knowledge of estimated parameters. In this work maximum likelihood estimate for a quasi-determined signal's parameters, which provides maximum maximorum of logarithm of likelihood ratio's functional.

Let's determine potential accuracy of joint conditional estimations of three parameters: v, V, and τ . Let's introduce into consideration a column-vector of estimated parameters $\vec{l} = (v, V, \tau)^{T}$, where the superscript "T" denotes transposing.

Additive mixture of useful signal $s_{\rm R}(t) \equiv s(t, \vec{l}_0)$ and noise n(t) may be expressed as follows:

$$x(t) = s(t, l_0) + n(t), \quad 0 \le t \le T,$$

where $\vec{l}_0 = (v_0, V_0, \tau_0)^T$ is a column-vector of estimated parameters' true values; and *T* is observation interval. We'll assume that noise n(t) is represented by a realization of Gaussian random process with zero mean and correlation function

$$K(t_1, t_2) = (N_0 / 2)\delta(t_1 - t_2),$$

where N_0 is physical spectral power density of noise, and $\delta(*)$ is Dirac's delta function.

Expression for functional of likelihood ratio may be represented as follows:

$$\hat{\Lambda}(\vec{l}) = \exp\left\{\frac{2}{N_0} \int_0^T x(t) s(t, \vec{l}) dt - \frac{1}{N_0} \int_0^T s^2(t, \vec{l}) dt\right\}.$$

Using expression for output signal of optimal receiver [3], logarithm of likelihood ratio functional may be written as follows:

$$Z(\vec{l}) = S(\vec{l}, \vec{l}_0) - \frac{1}{2}S(\vec{l}, \vec{l}) + N(\vec{l}),$$

where we have introduced into consideration

$$S(\vec{l}_{1},\vec{l}_{2}) = \frac{2\rho_{0}^{2}}{T[1-\theta]} \frac{v_{0}^{4}}{v_{1}^{2}v_{2}^{2}} \frac{\aleph}{1-\frac{v_{0}}{V_{0}T}\sqrt{\frac{2\aleph(1-\aleph)}{1-\theta}}}$$

$$\times \int_{0}^{T} \sin\left\{2\pi \left[f_{0}t - \frac{L}{\lambda} - \frac{V_{1}^{2}(t-\tau_{1})^{2}}{2v_{1}^{2}}\right]\right\} \sin\left\{2\pi \left[f_{0}t - \frac{L}{\lambda} - \frac{V_{2}^{2}(t-\tau_{2})^{2}}{2v_{2}^{2}}\right]\right\} dt$$

$$+ \frac{\rho_{0}^{2}(1-\aleph)}{1-\frac{v_{0}}{V_{0}T}\sqrt{\frac{2\aleph(1-\aleph)}{1-\theta}}} \left[1-\sqrt{2}\frac{\mu}{T}\left(\frac{1}{v_{1}V_{1}} + \frac{1}{v_{2}V_{2}}\right)\right], \quad (4)$$

$$N(\vec{l}) = \frac{2}{N_{0}}\int_{0}^{T} s(t,\vec{l})n(t)dt \quad (5)$$

signal and noise components of logarithm of likelihood ratio functional respectively;

$$\vec{l}_1 = (v_1, V_1, \tau_1)^{\mathrm{T}}, \quad \vec{l}_2 = (v_2, V_2, \tau_2)^{\mathrm{T}},$$
$$\rho_0 \equiv \rho(\vec{l}_0) = \left\{ \frac{A^2 T}{N_0} \left[1 - \frac{v_0}{V_0 T} \sqrt{\frac{2\aleph(1 - \aleph)}{1 - \theta}} \right] / (1 - \aleph) \right\}^{1/2}$$

is signal-to-noise ratio with respect to voltage at optimal receiver's output when $\vec{l} = \vec{l}_0$ [3, 4], which corresponds to signal function's maximum. Here

$$\Re = 1 - \left\{ 1 + \left(\frac{2\mu}{v_0^2}\right)^2 (1-\theta) \right\}^{-1},$$

$$\theta = \frac{v_0}{2V_0 T} \left[\cos(4\pi\eta) \left\{ I_C \left(2\frac{V_0}{v_0} g \right) - I_C \left[2\frac{V_0}{v_0} (g-T) \right] \right\} \right] + \sin(4\pi\eta) \left\{ I_S \left(2\frac{V_0}{v_0} g \right) - I_S \left[2\frac{V_0}{v_0} (g-T) \right] \right\} \right],$$

$$\eta = \frac{v_0^2}{2V_0^2} \left(f_0^2 + 2\frac{V_0^2}{v_0^2} \tau_0 f_0 \right) - \frac{L}{\lambda}, \quad g = \left(\tau_0 + \frac{v_0^2}{V_0^2} f_0 \right),$$

$$I_C(x) = \int_0^x \cos(\pi t^2 / 2) dt \text{ and } I_S(x) = \int_0^x \sin(\pi t^2 / 2) dt \text{ are Fresnel integrals.}$$
(6)

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Let's estimate infinitesimal order of quantity θ for some values of target's velocity, probing wave frequency and bistatic radar system's parameters. For example, for V = 20 m/s, $f_0 = 900 \text{ MHz}$, L = 200 m and $T = 3 \text{ s we obtain } |\theta| \sim 10^{-6}$. Consequently, we'll neglect the dependence on θ in formula (4).

Maximum likelihood estimate $\vec{l}_m = (v_m, V_m, \tau_m)^T$ may be found as a solution of a system of likelihood equations

$$\frac{\partial Z(\vec{l}\,)}{\partial l_i}\Big|_{\vec{l}=\vec{l}_m} = 0, \quad i=1,2,3.$$

$$\tag{7}$$

We'll search for solution of this system assuming absence of abnormal errors and considering that estimate \vec{l}_m is a point inside some priory defined area of possible values for vector \vec{l} . Solving system of equations (7) using small parameter method (we use the quantity inversely proportional to signal-to-noise ratio as the small parameter) we obtain that in first approximation the maximum likelihood estimates are unbiased and their correlation matrix is reciprocal to Fisher matrix [3]:

$$F = \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 S}{\partial v_1 \partial v_2} & \frac{\partial^2 S}{\partial v_1 \partial v_2} & \frac{\partial^2 S}{\partial v_1 \partial v_2} \\ \frac{\partial^2 S}{\partial V_1 \partial v_2} & \frac{\partial^2 S}{\partial V_1 \partial V_2} & \frac{\partial^2 S}{\partial V_1 \partial \tau_2} \\ \frac{\partial^2 S}{\partial \tau_1 \partial v_2} & \frac{\partial^2 S}{\partial \tau_1 \partial V_2} & \frac{\partial^2 S}{\partial \tau_1 \partial \tau_2} \end{pmatrix}_{\vec{l}_1 = \vec{l}_2 = \vec{l}_0.}$$
(8)

Considering (4) one may state that Fisher matrix F components have the following appearance:

$$\begin{split} F_{11} &= F_{11}' + H, \qquad F_{11}' = 2\rho_0^2 \aleph \Psi_4 \; \frac{\left(V_0 T\right)^4}{v_0^6} \bigg(1 - \frac{v_0}{V_0 T} \sqrt{2\aleph(1 - \aleph)} \bigg)^{-1}, \\ &\qquad H = \frac{4\rho_0^2 \aleph}{v_0^2} \bigg(1 - \frac{v_0}{V_0 T} \sqrt{2\aleph(1 - \aleph)} \bigg)^{-1}, \\ &\qquad F_{12} = F_{21} = -F_{11}' \frac{v_0}{V_0}, \qquad F_{13} = F_{31} = F_{11}' \frac{v_0}{T} \frac{\Psi_3}{\Psi_4}, \\ &\qquad F_{22} = F_{11}' \bigg(\frac{v_0}{V_0} \bigg)^2, \qquad F_{23} = F_{32} = -F_{11}' \frac{v_0^2}{V_0 T} \frac{\Psi_3}{\Psi_4}, \qquad F_{33} = F_{11}' \frac{v_0^2}{T^2} \frac{\Psi_2}{\Psi_4}, \end{split}$$

where

$$\Psi_n = \frac{\left[\left(1 - \tau_0 / T \right)^{n+1} + \left(-1\right)^n \left(\tau_0 / T \right)^{n+1} \right]}{2(n+1)},$$

$$n = 2,3,4.$$
(9)

Let's calculate algebraic complements for the first row of Fisher matrix:

$$A_{11} = 4\rho_0^4 \aleph^2 \frac{(V_0 T)^6}{v_0^8} (\Psi_4 \Psi_2 - \Psi_3^2) \left(1 - \frac{v_0}{V_0 T} \sqrt{2\aleph(1 - \aleph)} \right)^{-2},$$
$$A_{12} = -\frac{V_0}{v_0} A_{11}, \quad A_{13} = 0.$$

Determinant of matrix F may be represented as follows:

$$\det F = \sum_{i=1}^{3} (-1)^{1+i} F_{1i} A_{1i} = A_{11} H$$

or

$$\det F = \frac{16\rho_0^6 \aleph^3}{v_0^4} \left(\frac{V_0 T}{v_0}\right)^6 \left(\Psi_4 \Psi_2 - \Psi_3^2\right) \left(1 - \frac{v_0}{V_0 T} \sqrt{2\aleph(1 - \aleph)}\right)^{-1}.$$
 (10)

Considering the last expression, in a general case determinant of Fisher matrix has non-zero value. As was mentioned above, the value of first Fresnel zone's radius is present in expressions for amplitude and phase of signal (3). However, during detection of received signal (1) its amplitude multiplier will be subject to changes connected with parameters of detector's components. As a result revealing the dependence of amplitude of informative signal $s_{\rm sh}(t)$ on the value of first Fresnel zone's radius in practice appears to be a complex task. If we decide not to account for dependence of informative signal's amplitude on quantity v during calculation of Fisher matrix (8) components, then only expression for matrix F element F_{11} will change. Then

$$F_{11} = F_{11}'$$

and the new Fisher matrix F' becomes singular: det $F' \equiv 0$. Thus, determinant of Fisher matrix (8) has non-zero value only due to consideration of dependence of informative signal $s_{sh}(t)$ amplitude on first Fresnel zone's radius.

Dispersions of joint estimates of parameters v, V, and τ appear to be diagonal elements of correlation estimates matrix, which is reciprocal to Fisher matrix F. Determinant of Fisher matrix equals $(\det F)^{-1}$ and for values $\mu / v_0^2 < 1$ becomes rather large. Analysis of determinant (10) considering (6) shows that accuracy of joint estimates of parameters v, V, and τ of signal (1) is low and get even worse with decreasing the ratio of target's area to second power of first Fresnel zone's radius μ / v_0^2 .

A situation is possible when first Fresnel zone's radius is known and is not a measured parameter. Let's determine accuracy of joint estimate of two parameters, namely target's velocity and time when it crosses the base line, using the maximum likelihood criterion.

When estimating parameters V and τ signal (4) and noise (5) functions will get the following appearance

$$S(\vec{\sigma}_{1},\vec{\sigma}_{2}) = \frac{2\rho_{0}^{2}}{T} \frac{\aleph}{1 - \frac{v_{0}}{V_{0}T}\sqrt{2\aleph(1 - \aleph)}}$$
$$\times \int_{0}^{T} \sin\left\{2\pi \left[f_{0}t - \frac{L}{\lambda} - \frac{V_{1}^{2}(t - \tau_{1})^{2}}{2v_{0}^{2}}\right]\right\} \sin\left\{2\pi \left[f_{0}t - \frac{L}{\lambda} - \frac{V_{2}^{2}(t - \tau_{2})^{2}}{2v_{0}^{2}}\right]\right\} dt$$

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$$+\frac{\rho_0^2(1-\aleph)}{1-\frac{\nu_0}{V_0T}\sqrt{2\aleph(1-\aleph)}} \left[1-\sqrt{2}\frac{\mu}{T\nu_0}\left(\frac{1}{V_1}+\frac{1}{V_2}\right)\right],$$
$$N(\vec{\sigma}) = \frac{2}{N_0}\int_0^T s(t,\vec{\sigma})n(t)dt,$$

where $\vec{\sigma} = (V, \tau)^{\mathrm{T}}$, $\vec{\sigma}_{\alpha} = (V_{\alpha}, \tau_{\alpha})^{\mathrm{T}}$, $\alpha = 1, 2$.

Fisher matrix F for parameters V and τ is represented by

$$\Phi = \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 S}{\partial V_1 \partial V_2} & \frac{\partial^2 S}{\partial V_1 \partial \tau_2} \\ \frac{\partial^2 S}{\partial \tau_1 \partial V_2} & \frac{\partial^2 S}{\partial \tau_1 \partial \tau_2} \end{pmatrix}_{\vec{\sigma}_1 = \vec{\sigma}_2 = \vec{\sigma}_0}$$
(11)

We'll find first and second moments of errors for joint estimates of parameters V and τ as well as correlation of estimates using the following expressions:

$$\begin{split} b_V &= \langle V_m - V_0 \rangle, \quad b_\tau = \langle \tau_m - \tau_0 \rangle, \\ D_V &= \langle V_m^2 \rangle - \langle V_m \rangle^2, \quad D_\tau = \langle \tau_m^2 \rangle - \langle \tau_m \rangle^2, \\ K_{V\tau} &= \langle (V_m - V_0)(\tau_m - \tau_0) \rangle, \end{split}$$

where statistical averaging operation $\langle * \rangle$ is performed for all possible noise realizations under fixed values of estimated parameters, b_V and b_{τ} are conditional biases, D_V and D_{τ} are conditional dispersions of estimates of parameters V and τ , respectively, $K_{V\tau}$ is correlation of estimates.

Matrix (11) is non-singular

$$\det \Phi = 4\rho_0^4 \aleph^2 \frac{(V_0 T)^6}{v_0^8} (\Psi_4 \Psi_2 - \Psi_3^2) \left(1 - \frac{v_0}{V_0 T} \sqrt{2\aleph(1 - \aleph)}\right)^{-2},$$

while correlation matrix K for estimates $V_m \,\mu \,\tau_m$, which is reciprocal to it, is given by

$$K = \begin{pmatrix} D_V & K_{V\tau} \\ K_{V\tau} & D_{\tau} \end{pmatrix}, \quad \Phi \times K = E,$$

where *E* is a unit 2×2 matrix.

Calculations indicate that estimates V_m and τ_m are unbiased. We'll use dispersions of estimates

$$D_V = \frac{1}{2\aleph T^4 \rho_0^2} \frac{\nu_0^4}{V_0^2} \frac{\Psi_2}{\Psi_4 \Psi_2 - \Psi_3^2} \left(1 - \frac{\nu_0}{V_0 T} \sqrt{2\aleph(1 - \aleph)} \right), \tag{12}$$

¹ With respect to true values of estimated parameters.



$$D_{\tau} = \frac{1}{2\aleph T^2 \rho_0^2} \frac{\nu_0^4}{V_0^4} \frac{\Psi_4}{\Psi_4 \Psi_2 - \Psi_3^2} \left(1 - \frac{\nu_0}{V_0 T} \sqrt{2\aleph(1 - \aleph)} \right)$$
(13)

and correlation coefficient

$$r_{V\tau} = \frac{K_{V\tau}}{\sqrt{D_V D_\tau}} = \frac{-\Psi_3}{\sqrt{\Psi_2 \Psi_4}}$$

as parameters that characterize accuracy of joint estimation. Here

$$K_{V\tau} = -\frac{1}{2\aleph\rho_0^2} \frac{\nu_0^4}{(V_0 T)^3} \frac{\Psi_3}{\Psi_4 \Psi_2 - \Psi_3^2} \left(1 - \frac{\nu_0}{V_0 T} \sqrt{2\aleph(1 - \aleph)} \right)$$

is correlation of estimates.

According to (12), (13), dispersions of joint estimates of target's velocity and time when it crosses the base line asymptotically converge to zero with increasing signal-to-noise ratio ρ_0 , duration of observation interval *T* and target's velocity V_0 . With decreasing the value of \aleph dispersions D_V and D_{τ} increase. Consequently, considering (6), accuracy of joint estimates of parameters *V* and τ decreases as the value of ratio μ / v_0^2 becomes smaller.

Let's introduce parameter $\gamma = \tau / T$, which is time when target crosses the base line normalized by duration of observation interval. $\gamma_0 = \tau_0 / T$ will denote true value of parameter γ . When $0 \le \gamma_0 \le 1$ target reaches radar's base line within the duration of observation interval [0,T]. Moreover $\gamma_0 = 0$, 0.5, 1 correspond to the base line crossing time in the beginning, in the middle and in the end of observation interval, respectively. Negative values of parameter γ_0 correspond to cases, when during observation interval [0,T] target moves away from radar's base line, and when $\gamma_0 > 1$ it moves towards it.

We'll re-write formula (9) as follows:

$$\Psi_n = \frac{\left[(1 - \gamma_0)^{n+1} + (-1)^n \gamma_0^{n+1} \right]}{2(n+1)}, \quad n = 2, 3, 4.$$

Dependence of correlation coefficient $r_{V\tau}$ of target's velocity and base line crossing time estimates on γ_0 is depicted in Fig. 2. Analysis of the presented dependence allows making a conclusion that when $\gamma_0 = 0.5$ estimates are uncorrelated. With increasing the value of $|\gamma_0 - 0.5|$ correlation coefficient between estimates of parameters V and τ grows. This leads to increased dispersions D_V and D_{τ} , i.e. to decreased accuracy of joint estimation of parameters V and τ .

Correlation coefficient of estimates increases with increase of parameter γ_0 , so that $r_{V\tau} < 0$ when $\gamma_0 < 0.5$ and $r_{V\tau} > 0$ when $\gamma_0 > 0.5$. Let's find a physical meaning of the sign of correlation coefficient $r_{V\tau}$.



In cases $\gamma_0 < 0$ or $\gamma_0 > 1$ target does not reach radar's base line on the observation interval [0,*T*]. When $\gamma_0 < 0$ target moves away from receiver R, and when $\gamma_0 > 1$ approaches it. In this case the sign of correlation coefficient $r_{V_{\tau}}$ means direction of target on the observation interval [0,*T*].

When $0 \le \gamma_0 \le 1$ target crosses radar's base line at time $\tau_0 = \gamma_0 T$. On the time interval $[0, \gamma_0 T]$ target approaches receiver and on the interval $[\gamma_0 T, T]$ it moves away from it. Distance $\Pi = V_0 T$, which target passed during time of observation, may be represented as $\Pi = \Pi_1 + \Pi_2$, where $\Pi_1 = \gamma_0 V_0 T$ and $\Pi_2 = (1 - \gamma_0) V_0 T$ are distances covered by target before and after crossing the base line.

When $0 < \gamma_0 < 0.5$ distance Π_2 is greater. When $0.5 < \gamma_0 < 1$ distance Π_1 is greater. In the mentioned values intervals of γ_0 , correlation coefficient $r_{V\tau}$ has the opposite sign:

$$r_{V\tau} < 0$$
 when $0 < \gamma_0 < 0.5$ and $r_{V\tau} > 0$ when $0.5 < \gamma_0 < 1$.

Thus, when $0 \le \gamma_0 \le 1$ the sign of correlation coefficient characterizes target's movement to $(r_{V\tau} > 0)$ or away $(r_{V\tau} < 0)$ from receiver and the greater distance Π_1 or Π_2 .

Until now we considered accuracy of joint estimation of target's velocity and base line crossing time. One may show that dispersions d_V and d_τ for separate estimates of parameters V and τ are given by

$$d_V = \frac{1}{F_{22}} = \frac{1}{2\rho_0^2 \otimes T^4 \Psi_4} \frac{\nu_0^4}{V_0^2} \left(1 - \frac{\nu_0}{V_0 T} \sqrt{2\aleph(1 - \aleph)} \right), \tag{14}$$

$$d_{\tau} = \frac{1}{F_{33}} = \frac{1}{2\rho_0^2 \otimes T^2 \Psi_2} \frac{\nu_0^4}{V_0^4} \left(1 - \frac{\nu_0}{V_0 T} \sqrt{2\aleph(1 - \aleph)} \right).$$
(15)

Calculations show that similarly to the case of joint estimations separate estimates of parameters V and τ are unbiased, and their dispersions asymptotically converge to zero with increasing signal-to-noise ratio ρ_0 , duration of observation interval T and target's velocity V_0 . Accuracy of separate estimation of parameters V and τ gets worse with decreasing the value of ratio μ / v_0^2 .

According to (12)–(15), ratios D_V / d_V and D_τ / d_τ of dispersions of joint and separate estimates remain the same and are functions of only γ_0 :

$$\chi = D_{\tau} / d_{\tau} = D_V / d_V = \frac{\Psi_2 \Psi_4}{\Psi_2 \Psi_4 - \Psi_3^2} = \frac{1}{1 - r_{Vt}^2(\gamma_0)}.$$
(16)

According to (16), parameter χ increases with increasing the absolute value of correlation coefficient $r_{V\tau}$.

Dependence of parameter χ on γ_0 is depicted in Fig. 3. When $\gamma_0 = 0.5$ parameter $\chi = 1$, i.e. dispersions of joint estimates D_V and D_{τ} coincide with those of separate estimates d_V and d_{τ} . Parameter χ grows with increasing $|\gamma_0 - 0.5|$.

In the case of a small-sized target accuracy of joint estimation of first Fresnel zone's radius, target's velocity and base line crossing time is low and decreases as target's area gets smaller. When estimating two parameters, namely V and τ , accuracy of joint estimation improves with increasing value of V_0T / v_0 . If target crosses radar's base line in the middle of observation interval, joint estimates of parameters V and τ are uncorrelated. Dispersions D_V and D_{τ} of joint estimates asymptotically converge to zero with increasing signal-to-noise ratio ρ_0 , duration of observation interval T and target's velocity V_0 . Accuracy of joint estimates of parameters V and τ gets worse with decreasing the ratio of target's area to second power of first Fresnel zone's radius.

The obtained results may be used when solving various radio engineering problems that deal with classification of moving targets using diffraction signals, when performing automatic control of objects' velocity and their crossing of some conditional boundary, in radio monitoring systems, etc.

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