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Impact of Narrowband Interference on the Threshold Characteristics of Range and Velocity Estimates during the Probing with a Sequence of Ultrawideband Pulses[1]

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Abstract—Threshold characteristics of two algorithms for the range and velocity estimation under exposure to the Gaussian narrowband interference have been found.

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The possibilities of applying supershort (subnanosecond) pulses and their sequences in radiolocation were discussed in [1–6] and some others. Short-pulsed signals and their sequences represent a special case of ultrawideband signals (UWBS). The use of the latter has its specific character and makes it possible to enhance the capabilities of radiolocation. The characteristics of range and velocity estimates under exposure to interferences in the form of Gaussian white noise (GWN) were found in [4]. However, in addition to GWN, the deliberate interferences are also frequently present in real-life environment. Such interferences can be interpreted as a Gaussian narrowband random process [5]. The characteristics of reliable estimates of range and velocity under exposure to the Gaussian narrowband interference (GNI) were found in paper [6]. However, the expressions for the characteristics of range and velocity estimates obtained in paper [6] can be used only under the conditions of high a posteriori accuracy, when anomalous errors are not present and the impact of threshold effects is negligibly small [7, 8]. In this connection, the present paper considered the threshold properties of range and velocity estimates under exposure to both GWN and GNI.

Similar to paper [4] the probing UWBS sequence can be written in the form:

$$\tilde{s}_N(t) = \sum_{k=0}^{N-1} s_0[t-(k-\mu)\theta-\varepsilon], \tag{1}$$

where function $s_0(\cdot)$ describes the waveform of one pulse, θ is the repetition cycle, ε is the temporal position of the sequence. Parameter μ determines the point of sequence (1) that is related to its temporal position. Hence, at $\mu = 0$ the value of ε represents the temporal position of the first pulse of the sequence, at $\mu = (N-1)/2$ quantity ε gives the temporal position of the middle of the sequence, while at $\mu = N-1$ quantity ε represents the temporal position of the last pulse in the sequence.

Let us assume that probing sequence (1) is found at distance R_0 and moves with radial velocity V_0. The unknown range R_0 and velocity V_0 of target assume values from a priori domain $W = \{[R_{\min}; R_{\max}], [-V_{\max}/2; V_{\max}/2]\}$ so that

$$R_0 \in [R_{\min}; R_{\max}], \quad V_0 \in [-V_{\max}/2; V_{\max}/2],$$
$$V_{\max} << c, \tag{2}$$

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where c is the speed of light. Then, the signal received can be written in the form [4]:

$$s(t, R_0, V_0) = \sum_{k=0}^{N-1} s[t - 2R_0 / c - (k - \mu)\theta(1 + 2V_0 / c)]. \tag{3}$$

Function $s(\cdot)$ describes the waveform of one received UWBS signal and, in the general case, differs from $s_0(\cdot)$ in sequence (1) [4].

The following realization is observed on time interval $[0; T]$ under exposure to both GWN and GNI:

$$x(t) = s(t, R_0, V_0) + n(t) + y(t), \tag{4}$$

where $n(t)$ is the centered GWN with one-sided spectral density N_0, $y(t)$ is the centered GNI having correlation function $K_y(\tau) = \langle y(t) y(t + \tau)\rangle$. Processes $n(t)$ and $y(t)$ are assumed to be statistically independent.

Let us initially assume that the GNI correlation function is a priori unknown. Then for the estimation of range and velocity it is proposed to use the maximum likelihood algorithm synthesized on condition that GNI is not present. Let the pulse ratio of sequence (3) is sufficiently large so that separate pulses do not overlap and the observation interval $[0; T]$ is longer than the duration of the entire sequence, i.e., $T > N\theta$. Then, provided only GWN is present, the logarithm of the likelihood ratio functional, omitting the non-essential term, can be written in the form [4, 6, 7]:

$$L_1(R, V) = \frac{2}{N_0} \sum_{k=0}^{N-1} \int_0^T x(t) s[t - 2R / c - (k - \mu)\theta(1 + 2V / c)] dt. \tag{5}$$

The realization of the observed data $x(t)$ (4), besides GWN $n(t)$, contains GNI $y(t)$. Therefore, estimates

$$(\hat{R}_1, \hat{V}_1) = \arg\sup L_1(R, V), \quad (R, V) \in W \tag{6}$$

are not maximum likelihood estimates (MLE). These estimates can be called quasi-likelihood estimates (QLE) [9], because they coincide with MLE estimates at $y(t) \equiv 0$, i.e. in the absence of GNI.

In order to determine the characteristics of QLE (6), we shall present expression (5) in the form of a sum of signal and noise functions [7, 8]:

$$L_1(R, V) = S_1(R, R_0, V, V_0) + N_1(R, V),$$

where signal function has the form:

$$S_1(R, R_0, V, V_0) = \frac{2}{N_0} \sum_{k=0}^{N-1} \sum_{n=0}^{N-1} \int_0^T s[t - 2R_0 / c - (k - \mu)\theta(1 + 2V_0 / c)]$$

$$\times s[t - 2R / c - (n - \mu)\theta(1 + 2V / c)] dt, \tag{7}$$

while noise function $N_1(R, V) = L_1(R, V) - \langle L_1(R, V)\rangle$ is the realization of the Gaussian random field. The first two moments of the noise function have the form:

$$\langle N_1(R, V)\rangle = 0,$$

$$K_1(R_1, R_2, V_1, V_2) = \langle N_1(R_1, V_1) N_1(R_2, V_2)\rangle$$

$$=\sum_{k=0}^{N-1}\sum_{n=0}^{N-1}\left\{\frac{2}{N_0}\int_0^T s[t-2R_1/c-(k-\mu)\theta(1+2V_1/c)]\,s[t-2R_2/c-(n-\mu)\theta(1+2V_2/c)]dt\right.$$

$$\left.+\frac{4}{N_0^2}\int_0^T\int_0^T K_y(t_2-t_1)s[t_1-2R_1/c-(k-\mu)\theta(1+2V_1/c)]\,s[t_2-2R_2/c-(n-\mu)\theta(1+2V_2/c)]dt_1dt_2\right\}. \quad (8)$$

Since it is assumed that $T > N\theta$ so that the entire received sequence (3) is located inside the observation interval, the integration limits in expressions (7) and (8) can be replaced with infinite ones. As a result we obtain

$$S_1(R,R_0,V,V_0)=\sum_{k=0}^{N-1}\sum_{n=0}^{N-1}S_f\left\{2(R-R_0)/c+(n-k)\theta+2\theta\left[nV-kV_0-\mu(V-V_0)\right]/c\right\}, \quad (9)$$

$$K_1(R_1,R_2,V_1,V_2)=\sum_{k=0}^{N-1}\sum_{n=0}^{N-1}K_H\left\{2(R_2-R_1)/c+(n-k)\theta+2\theta\left[nV_2-kV_1-\mu(V_2-V_1)\right]/c\right\}, \quad (10)$$

where $S_f(\tau)=\frac{2}{N_0}\int_{-\infty}^{\infty}s(t)s(t-\tau)dt$ is the signal function (uncertainty function) [7, 8] for a single UWBS of sequence (3), while

$$K_H(\eta)=S_f(\eta)+\frac{4}{N_0^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}K_y(t_2-t_1+\eta)s(t_1)s(t_2)dt_1dt_2.$$

Let us designate the duration of one pulse of sequence (3) as τ_s and GNI correlation time as τ_y, so that $S_f(\pm\tau_s)\cong 0$ and $K_y(\pm\tau_y)\cong 0$. We shall restrict ourselves with the analysis of central peaks of the signal (9) correlation (10) functions assuming that, in addition to (2), the following condition is satisfied:

$$\max\left\{|R-R_0|,|R_1-R_2|\right\}\le c\theta/2. \quad (11)$$

Let the pulse ratio of received UWBS sequence (3) be sufficiently large so that

$$\tau_s \ll \theta,\ \tau_y \ll \theta. \quad (12)$$

Then, provided conditions (2), (11), and (12) are satisfied, functions (9) and (10) assume the form [2, 6]:

$$S_1(R,R_0,V,V_0)=\sum_{k=0}^{N-1}S_f\left[2(R-R_0)/c+2\theta(k-\mu)(V-V_0)/c\right], \quad (13)$$

$$K_1(R_1,R_2,V_1,V_2)=\sum_{k=0}^{N-1}K_H\left[2(R_2-R_1)/c+2\theta(k-\mu)(V_2-V_1)/c\right]. \quad (14)$$

From the last expression it follows, in particular, that the noise function is a realization of Gaussian uniform field.

It is obvious [7] that signal function (13) reaches its maximum at $R=R_0$ and $V=V_0$. Therefore, the signal-to-noise ratio (SNR) [7] can be written in the form:

$$z_1^2=S_1^2(R_0,R_0,V_0,V_0)/K_1(R_0,R_0,V_0,V_0). \quad (15)$$

Substituting the values of functions (13) and (14) into (15) we obtain:

$$z_1^2 = z^2 / \chi_1 = N z_0^2 / \chi_1, \tag{16}$$

where $z^2 = N z_0^2$ is the SNR at the output of the maximum likelihood receiver in the absence of GNI, $z_0^2 = 2E / N_0$ is the SNR of one UWBS in the absence of GNI, while $E = \int_{-\infty}^{\infty} s^2(t)\mathrm{d}t$ is the energy of one UWBS of sequence (3). Quantity χ_1 in formula (16) shows how many times SNR is reduced due to the GNI action; this quantity is determined by the following expression:

$$\chi_1 = 1 + \frac{\dfrac{2}{N_0} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} K_y(t_2 - t_1) s(t_1) s(t_2) \mathrm{d}t_1 \mathrm{d}t_2}{\int\limits_{-\infty}^{\infty} s^2(t)\mathrm{d}t}.$$

Let us designate the duration (length) of signal function (13) in terms of corresponding arguments as ΔR_1 and ΔV_1. Then

$$S_1(R_0 \pm \Delta R_1, R_0, V_0, V_0) \cong 0,$$

$$S_1(R_0, R_0, V_0 \pm \Delta V_1, V_0) \cong 0$$

and obviously

$$S_1(R_0 \pm \Delta R_1, R_0, V_0 \pm \Delta V_1, V_0) \cong 0.$$

Let us single out in the domain W (2) of possible values of the range and velocity the signal subdomain

$$W_S = \{[R_0 - \Delta R_1; R_0 + \Delta R_1],$$

$$[V_0 - \Delta V_1; V_0 + \Delta V_1]\}, \tag{17}$$

where signal function (13) is other than zero. If

$$(\hat{R}_1, \hat{V}_1) \in W_S, \tag{18}$$

QLE estimates (6) are reliable [7, 8].

For the case of reliable QLE (6) their dispersions were found in paper [6]:

$$D_1(R) = \langle(\hat{R}_1 - R_0)^2\rangle = D_0(R)\kappa_1,$$

$$D_1(V) = \langle(\hat{V}_1 - V_0)^2\rangle = D_0(V)\kappa_1, \tag{19}$$

where

$$D_0(R) = \frac{c^2 N_0\left(N^2 - 1 + 12\left[(N-1)/2 - \mu\right]^2\right)}{8F_0 N(N^2-1)}, \tag{20}$$

$$D_0(V) = 3c^2 N_0 / 2\theta^2 F_0 N(N^2-1), \tag{21}$$

$D_0(R)$ and $D_0(V)$ are the dispersions of the range and velocity MLE, respectively, in the absence of GNI [4], $F_0 = \int_{-\infty}^{\infty} [\mathrm{d}s(t)/\mathrm{d}t]\mathrm{d}t$. Quantity

$$\kappa_1 = 1 + \frac{\dfrac{2}{N_0}\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K_y(t_2 - t_1)\frac{\mathrm{d}s(t_1)}{\mathrm{d}t_1}\frac{\mathrm{d}s(t_2)}{\mathrm{d}t_2}\mathrm{d}t_1\mathrm{d}t_2}{F}$$

shows the loss in accuracy of reliable QLE (6) due to the effect of GNI.

If condition (18) is not satisfied, anomalous errors may appear [7, 8] that results in abrupt (threshold) deterioration of the QLE (6) accuracy. Correspondingly, the QLE dispersion becomes much larger than the value obtained from relationship (19). The threshold properties of QLE can be characterized by the probability of reliable estimate [7]:

$$P_{01} = P[(\hat{R}_1, \hat{V}_1) \in W_S]. \tag{22}$$

Let W_N be the complement of domain W_S (17) to domain W (2) so that $W = W_S \cup W_N$. Let us introduce the following designations: $H_{1S} = \sup L_1(R,V)$, $(R,V) \in W_S$ and $H_{1N} = \sup L_1(R,V)$, $(R,V) \in W_N$. Since QLE (6) is determined by the position of absolute (highest) maximum of random field (5), expression (22) can be rewritten in the form:

$$P_{01} = P\left[H_{1N} < H_{1S}\right]. \tag{23}$$

If a priori interval of possible values of the range and velocity (2) is not too small so that

$$\Delta R_1 << R_{\max} - R_{\min}, \quad \Delta V_1 << V_{\max}, \tag{24}$$

then random quantities H_{1S} and H_{1N} are roughly statistically-independent [7, 8]. This allows us to present expression (23) in the form:

$$P_{01} = \int F_{1N}(x)\mathrm{d}F_{1S}(x), \tag{25}$$

where $F_{1N}(x)$ and $F_{1S}(x)$ are the distribution functions of random quantities H_{1S} and H_{1N}, respectively.

Using the results [2, 10] of approximation of the specified distribution functions $F_{1N}(x)$ and $F_{1S}(x)$, we can present them as follows:

$$F_{1N}(x) \cong \begin{cases} \exp\left[-\dfrac{x\xi_0\kappa_1}{z_0(2\pi\chi_1)^{3/2}\sqrt{N}}\exp\left(-\dfrac{x^2}{2Nz_0^2\chi_1}\right)\right], & x \ge z_0\sqrt{N\chi_1}, \\ 0, & x < z_0\sqrt{N\chi_1}, \end{cases} \tag{26}$$

$$F_{1S}(x) \cong \Phi\left(x / z_0\sqrt{N\chi_1} - z_0\sqrt{N/\chi_1}\right), \tag{27}$$

where $\Phi(x)=\int_{-\infty}^{x}\exp(-t^2/2)\mathrm{d}t/\sqrt{2\pi}$ is the probability integral,

$$\xi_0 = 2(R_{\max}-R_{\min})V_{\max}\theta d_0^2\sqrt{N^2-1}/c^3\sqrt{3}$$

is the reduced area [10] of a priori domain W of possible values of the range and velocity in the presence of only GWN [4], $d_0^2 = F_0/E$.

Substituting approximations (26) and (27) into expression (25) we can find an approximate expression for the probability of reliable QLE (6):

$$P_{01} \cong \frac{1}{\sqrt{2\pi}}\int_1^{\infty}\exp\left[-\frac{x\xi_0\kappa_1}{\chi_1(2\pi)^{3/2}}\exp\left(-\frac{x^2}{2}\right)-\frac{\left(x-z_0\sqrt{N/\chi_1}\right)^2}{2}\right]\mathrm{d}x. \tag{28}$$

The accuracy of this expression enhances with the rise of SNR and increase of the reduced area of a priori domain of the range and velocity possible values.

Let unknown range R_0 and unknown velocity V_0 be distributed uniformly in the a priori domain W (2) of their possible values. Then, in a similar way [8], for unconditional spreading of QLE (6) with due regard for the threshold effects, we obtain the following expressions:

$$B_1(R)=\langle(\hat{R}_1-R_0)^2\rangle = P_{01}D_0(R)\kappa_1+(1-P_{01})(R_{\max}-R_{\min})^2/6, \tag{29}$$

$$B_1(V)=\langle(\hat{V}_1-V_0)^2\rangle = P_{01}D_0(V)\kappa_1+(1-P_{01})V_{\max}^2/6\,. \tag{30}$$

The comparison of expressions (29) and (30) with (19) makes it possible to determine contribution into the estimate error mean square introduced by the threshold effects under exposure to GNI. Assuming that $\kappa_1=\chi_1=1$ in expressions (28)–(30) we obtain the characteristics of the range and velocity MLE in the absence of GNI [4]. In particular, comparing expressions (29) and (30) with the results of paper [4] it is possible to determine the effect of GNI presence on the accuracy of range and velocity estimates.

Formula (28) for the probability of reliable QLE estimate is fairly cumbersome and the calculations based on this formula are possible only in case of using the numerical methods. That is why, similar to [8], we shall find a relatively simple analytical expression for the probability of anomalous errors $P_{a1}=1-P_{01}$; this expression is valid at sufficiently large SNR (16):

$$P_{a1} \cong \frac{\xi_0 z_0\kappa_1\sqrt{N}}{8(\pi\chi_1)^{3/2}}\exp\left(-\frac{z_0^2 N}{4\chi_1}\right). \tag{31}$$

This formula provides a satisfactory accuracy at $P_{a1}<0.05\ldots0.1$. Assuming that $\kappa_1=\chi_1=1$ in expression (31) we obtain the anomalous error probability

$$P_{a1} \cong \frac{\xi_0 z_0\sqrt{N}}{8\pi^{3/2}}\exp\left(-\frac{z_0^2 N}{4}\right) \tag{32}$$

for the range and velocity MLE in the absence of GNI. Comparing expression (31) and (32) we find:

$$\alpha_1=\frac{P_{a1}}{P_a}=\frac{\kappa_1}{\chi_1^{3/2}}\exp\left[\frac{z_0^2 N(\chi_1-1)}{4\chi_1}\right]. \tag{33}$$

Quantity (33) shows how many times the anomalous error probability is increased due to the GNI action.

The highest accuracy of range and velocity estimates can be ensured, if GNI correlation function $K_y(\tau)$ is known a priori. In this case the logarithm of likelihood ratio functional, omitting an inessential term, can be written as follows [7]:

$$L_2(R,V)=\sum_{k=0}^{N-1}\int_0^T x(t)v\left[t-2R/c-(k-\mu)\theta(1+2V/c)\right]\mathrm{d}t, \tag{34}$$

where function $v(t)$ is determined by solving integral equation

$$N_0 v(t)/2+\int_0^T K_y(t-\tau)v(\tau)\mathrm{d}\tau = s(t).$$

Hence, MLE $(\hat{R}_2,\hat{V}_2)$ of range R_0 and velocity V_0 has the form

$$(\hat{R}_2,\hat{V}_2)=\operatorname{argsup} L_2(R,V),\quad (R,V)\in W. \tag{35}$$

In order to determine the characteristics of MLE (27), we shall present expression (34) in the form of a sum of signal and noise functions [7]:

$$L_2(R,V)=S_2(R,R_0,V,V_0)+N_2(R,V).$$

In this case, provided conditions (2), (11), and (12) are satisfied, signal function [6] has the form:

$$S_2(R,R_0,V,V_0)=\sum_{k=0}^{N-1}\int_0^T s\left[t-2R_0/c-(k-\mu)\theta(1+2V_0/c)\right] v\left[t-2R/c-(k-\mu)\theta(1+2V/c)\right]\mathrm{d}t,$$

while the correlation function of centered noise function can be presented as follows:

$$K_2(R_1,R_2,V_1,V_2)=\langle N_2(R_1,V_1)N_2(R_2,V_2)\rangle = S_2(R_1,R_2,V_1,V_2).$$

For algorithm of expressions (34) and (35) SNR can be written in the form:

$$z_2^2=S_2(R_0,R_0,V_0,V_0)=z^2/\chi_2=Nz_0^2/\chi_2, \tag{36}$$

where z^2 is the SNR at the output of maximum likelihood receiver in the absence of GNI, while z_0^2 is the SNR of one UWBS signal in the absence of GNI. The value of χ_2 in expression (36) shows how many times the signal-to-noise ratio is reduced due to the exposure to GNI with a priori known correlation function; χ_2 is determined by the following expression:

$$\chi_2=\frac{2}{N_0}\int_{-\infty}^{\infty}s^2(t)\mathrm{d}t \Big/ \int_{-\infty}^{\infty}s(t)v(t)\mathrm{d}t.$$

Similar to expression (28), MLE estimates (35) are reliable [7, 8], if

$$(\hat{R}_2,\hat{V}_2)\in W_S. \tag{37}$$

For the case of reliable MLE (37) the dispersions of such estimates were found in paper [6]:

$$D_2(R) = \langle(\hat{R}_2 - R_0)^2\rangle = D_0(R)\kappa_2,$$

$$D_2(V) = \langle(\hat{V}_2 - V_0)^2\rangle = D_0(V)\kappa_2, \tag{38}$$

where $D_0(R)$ (20) and $D_0(V)$ (21) are the dispersions of reliable MLE estimates of range and velocity in the presence of GWN only, while the value of

$$\kappa_2 = \frac{2}{N_0}\int_{-\infty}^{\infty}\left[\frac{\mathrm{d}s(t)}{\mathrm{d}t}\right]^2 \mathrm{d}t \Bigg/ \int_{-\infty}^{\infty}\frac{\mathrm{d}s(t)}{\mathrm{d}t}\frac{\mathrm{d}v(t)}{\mathrm{d}t}\mathrm{d}t$$

shows the loss in accuracy of reliable MLE estimates (35) due to the effect of GNI with a priori known correlation function.

For MLE estimate (35), similar to expression (22), the probability of reliable estimate can be determined as follows:

$$P_{02} = P\left[\left(\hat{R}_2, \hat{V}_2\right) \in W_S\right]. \tag{39}$$

Provided the conditions similar to (24) are satisfied, for relationship (39) we obtain an expression similar to (25):

$$P_{02} = \int F_{2N}(x)\mathrm{d}F_{2S}(x), \tag{40}$$

where $F_{2N}(x)$ is the distribution function of random quantity $H_{2N} = \sup L_2(R,V)$, $(R,V) \in W_N$, while $F_{2S}(x)$ is the distribution function of random quantity $H_{2S} = \sup L_2(R,V)$, $(R,V) \in W_S$. The approximations of distribution functions $F_{2N}(x)$ and $F_{2S}(x)$ can be obtained similar to expressions (27) and (28) in the form [10]:

$$F_{2N}(x) \cong \begin{cases} \exp\left[-\dfrac{x\xi_0}{z_0\kappa_2\sqrt{N}}\left(\dfrac{\chi_2}{2\pi}\right)^{3/2}\exp\left(-\dfrac{x^2\chi_2}{2Nz_0^2}\right)\right], & x \ge z_0\sqrt{N/\chi_2}, \\ 0, & x < z_0\sqrt{N/\chi_2}, \end{cases} \tag{41}$$

$$F_{2S}(x) \cong \Phi\left(x / z_0\sqrt{N\chi_2} - z_0\sqrt{N/\chi_2}\right). \tag{42}$$

Substituting approximations (41) and (42) into (40) we can find an approximate expression for the probability of reliable MLE estimate (35) in the presence of GNI with a priori known correlation function

$$P_{02} \cong \frac{1}{\sqrt{2\pi}}\int_1^{\infty}\exp\left[-\frac{x\xi_0\chi_2}{\kappa_2(2\pi)^{3/2}}\exp\left(-\frac{x^2}{2}\right) - \frac{\left(x - z_0\sqrt{N/\chi_2}\right)^2}{2}\right]\mathrm{d}x. \tag{43}$$

The accuracy of this approximate expression improves with the rise of SNR and increase of the reduced area of a priori domain of possible values of the range and velocity.

If the unknown values of R_0 and velocity V_0 are random and distributed uniformly in the a priori domain W (2), the unconditional spreading of QLE estimates (35) with due regard for anomalous errors have the form:

$$B_2(R) = \langle(\hat{R}_2 - R_0)^2\rangle = P_{02} D_0(R) \kappa_2 + (1 - P_{02})(R_{\max} - R_{\min})^2 / 6, \tag{44}$$

$$B_2(V) = \langle(\hat{V}_2 - V_0)^2\rangle = P_{02} D_0(V) \kappa_2 + (1 - P_{02}) V_{\max}^2 / 6. \tag{45}$$

Comparing expressions (44) and (45) with (38) we can determine the contribution into the estimate error mean square introduced by the threshold effects under exposure to GNI with a priori known correlation function. Assuming that $\kappa_2 = \chi_2 = 1$ in expressions (43)–(45) we obtain the characteristics of the range and velocity MLE in the absence of GNI [4]. In particular, comparing expressions (44) and (45) with the results in paper [4] it is possible to determine the effect of GNI with a priori known correlation function on the accuracy of MLE estimates of the range and velocity.

Finally, the comparison of expressions (29) and (30) with expressions (44) and (45) makes it possible to determine the impact of the a priori information about the GNI characteristics on the accuracy of range and velocity estimates.

Formula (43) for the probability of reliable QLE estimate (35) is fairly cumbersome and the calculations based on this formula are possible only by using the numerical methods. That is why, similar to expression (31), we shall find a relatively simple analytical expression for the probability of anomalous errors $P_{a2} = 1 - P_{02}$. This expression is valid at sufficiently large values of SNR (36):

$$P_{a2} \cong \frac{\xi_0 z_0 \sqrt{\chi_2 N}}{8\pi^{3/2} \kappa_2} \exp\left(-\frac{z_0^2 N}{4\chi_2}\right). \tag{46}$$

Let us compare the probabilities of anomalous errors of MLE estimates in the absence of GNI (32) and in the presence of GNI with a priori known correlation function (46). Comparing expression (32) and (46) we find:

$$\alpha_2 = \frac{P_{a2}}{P_a} = \frac{\sqrt{\chi_2}}{\kappa_2} \exp\left[\frac{z_0^2 N(\chi_2 - 1)}{4\chi_2}\right]. \tag{47}$$

The value of α_2 (47) shows how many times the probability of the anomalous error of the range and velocity MLE is increased due to the effect of GNI with a priori known correlation function.

Next, let us compare the probability of anomalous error (31) of MLE estimate (6) under exposure to GNI with a priori unknown correlation function and the probability of anomalous error (46) of MLE estimate (35) under exposure to GNI with a priori known correlation function. Comparing expressions (31) and (46) we find:

$$\alpha = \frac{P_{a1}}{P_{a2}} = \frac{\kappa_1 \kappa_2}{\chi_1^{3/2} \sqrt{\chi_2}} \exp\left[\frac{z_0^2 N(\chi_1 - \chi_2)}{4\chi_1 \chi_2}\right]. \tag{48}$$

Quantity α (48) shows how many times the probability of the anomalous error of the range and velocity estimates is reduced due to the presence of a priori information about the GNI characteristics and the use of estimate algorithm (35) instead of algorithm (6).

The analysis of results obtained indicates that the presence of narrowband interference leads to an exponential relative rise of the anomalous error probability with an increase of the number of pulses in the probing sequence. The found characteristics of estimates allow us, with due regard for the threshold effects, to make a sound selection of the estimate algorithm for location systems depending on the available a priori information about a narrowband interference and also on the requirements specifying the realization simplicity of the algorithm and on the requirements to the accuracy of estimates.

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