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Quazi-Likelihood Estimation of Motion Parameters during the Target Probing with a Sequence of Optical Pulses[1]

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Abstract—Characteristics of quasi-likelihood estimates of the target range, velocity, and acceleration have been obtained during the probing with a sequence of optical pulses. The losses in accuracy of quasi-likelihood estimates as compared with the accuracy of maximum likelihood estimates were also found.

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The optical detection-and-ranging systems [1–4] make a wide use of sequences of optical pulses. Potential accuracy of the range, velocity and acceleration estimates was investigated in paper [3]. In this case it was assumed that the intensity waveform of the pulse sequence scattered by target is a priori known. However, target fluctuations in real conditions and also physical effects accompanying the light scattering and propagation result in distortion of the signal intensity waveform. Since in practice the signal intensity waveform is often known inexactly, it is important to know the effect of incomplete knowledge of the signal waveform on the estimate characteristics.

The present paper investigates the estimation of range, velocity and acceleration, when the intensity waveform of the signal scattered by target is known inexactly. To this end, we used the method of quasi-likelihood estimation [4, 5].

The concept of the method of quasi-likelihood estimation relies on the fact that the estimation algorithm synthesis is based on certain anticipated (expected) signal $s_1(t,\vec{l})$ rather than received signal $s(t,\vec{l})$. Here $\vec{l}=(R,V,A)$ is the vector of estimated parameters: range R, velocity V and acceleration A. Quasi-likelihood estimate is used when the signal waveform is known inexactly. In addition, this estimate can be used as an alternative to the maximum likelihood estimate of a signal with unknown inconclusive parameters. The quasi-likelihood algorithm makes it possible to significantly simplify the technical implementation of the receiver, namely, to eliminate the elements responsible for the receiver "tuning" to the unknown inconclusive parameters. It entails possible losses in accuracy of estimates of inconclusive parameters.

Let us assume that a sequence of optical pulses is radiated with the following intensity:

$$s_N(t)=\sum_{k=0}^{N-1}\hat{s}(t-(k-\mu)\theta-\lambda), \tag{1}$$

where $\hat{s}(t)$ is the function describing the intensity of individual optical pulse, θ is the pulse period, λ is the time position of sequence. Parameter μ determines the point of sequence related to its time position λ. Hence, at $\mu=0$ quantity λ represents the time position of the first pulse, at $\mu=(N-1)/2$ it represents the time position of the middle of sequence (1), while at $\mu=(N-1)$—the time position of the last pulse.

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We shall assume that the signal received (subjected to processing) is the result of scattering of a sequence of optical pulses (1) by an object located at range R_0 and moving with velocity V_0 and acceleration A_0. Let us use the description of physical processes in photodetector presented in papers [1, 2]. Then the intensity of signal received has the form [1, 3]:

$$s(t,\vec{l}_0) = s(t,R_0,V_0,A_0) = \sum_{k=0}^{N-1} s(t - 2R_0 / c - (k-\mu)(1+2V_0 / c)\theta - A_0(k-\mu)^2\theta^2 / c), \tag{2}$$

where function $s(t)$ describes the intensity waveform of one scattered optical pulse of the sequence, and in the general case it is different from $\hat{s}(t)$ in expression (1); c is the velocity of light, $|V_0| << c$ and $N\theta|A_0| << c$. The true values of unknown parameters R, V, and A of the received sequence of optical pulses are marked with zero subscript.

Let us assume that the signal with intensity (2) is observed on the time interval $[0; T]$ against the background of optical noise representing a stationary Poisson process with intensity $\nu > 0$. Physical mechanisms of generation of optical noise and the techniques of its description are presented, e.g., in paper [6]. Thus signal $\pi(t)$ accessible for processing represents a Poisson process with intensity $\beta(t,\vec{l}) = s(t,\vec{l}) + \nu$, where the value of vector parameter $\vec{l} = (R,V,A)$ is subject to estimation, while quantity ν is possibly unknown. In case of using the receiver with direct photodetection, the process $\pi(t)$ is equal to the number of photoelectrons at the photodetector output during time $[0; t]$. Correspondingly, the intensity of this process $\beta(t,\vec{l})$ represents the average number of photoelectrons per unit of time, so that $\beta(t,\vec{l})\mathrm{d}t$ is the average number of photoelectrons on time interval $[t; t+\mathrm{d}t]$ [1, 2].

The synthesis of the receiver is performed for the anticipated signal having intensity $\beta_1(t,\vec{l}) = s_1(t,\vec{l}) + \nu_1$, where ν_1 is the anticipated intensity of optimal noise.

If the waveform of received signal $s(t,\vec{l})$ and noise intensity ν had been known a priori, the estimation of vector $\vec{l}$ would have been possible by using the maximum likelihood method [7]. To this end, it is necessary to use the position of the largest maximum of the logarithm of likelihood ratio functional [3]:

$$L_F(\vec{l}) = \int_0^T \ln\left(1 + s(t,\vec{l}) / \nu\right)\mathrm{d}\pi(t) - \int_0^T s(t,\vec{l})\mathrm{d}t. \tag{3}$$

If the signal waveform is known inexactly, the expected signal $s_1(t,\vec{l})$ and expected noise intensity ν_1 are used as reference signal $s(t,\vec{l})$ and noise intensity ν, respectively. Thus, we obtain the following expression for decision making statistic:

$$L(\vec{l}) = \int_0^T \ln\left(1 + s_1(t,\vec{l}) / \nu_1\right)\mathrm{d}\pi(t) - \int_0^T s_1(t,\vec{l})\mathrm{d}t. \tag{4}$$

The value of vector $\vec{l}$ corresponding to the largest maximum of decision making statistic (4) is taken as estimate $\vec{l}_m$. The obtained estimate shall be called quasi-likelihood estimate. Indeed, in case the received and expected signals ($s(t,\vec{l})$ and $s_1(t,\vec{l})$) coincide and the true and expected intensities of optical noise (ν and ν_1) also coincide, the decision making statistic (4) coincides with the logarithm of likelihood ratio functional (3). Hence, the quasi-likelihood estimate transforms into the maximum likelihood estimate.

In order to find characteristics of the quasi-likelihood estimate, we shall present the decision making statistic (4) in the form of signal and noise functions [7]:

$$L(l) = S(l_0,l) + N(l) + C. \tag{5}$$

The signal function in expression (5) can be determined from relationship

$$S(\vec{l}_0,\vec{l}) = \langle L(\vec{l}_0)\rangle - C,$$

where symbol $\langle\cdot\rangle$ designates the conditional mathematical expectation on the assumption that received signal $\pi(t)$ corresponds to value $\vec{l}_0$ of parameter $\vec{l}$ of signal intensity $\beta(t,\vec{l}) = s(t,\vec{l}) + \nu$, while quantity C can be determined from expression

$$C = -\int_0^T s_1(t,\vec{l})\mathrm{d}t + \nu\int_0^T \ln\Bigl(1 + s_1(t,\vec{l})/\nu_1\Bigr)\mathrm{d}t.$$

Since parameters $\vec{l} = (R,V,A)$ are nonpower, quantity C does not depend on estimated parameters $\vec{l}$.

Thus, for the signal function we have the following expression

$$S(\vec{l}_0,\vec{l}) = \int_0^T s(t,\vec{l}_0)\ln\Bigl(1 + s_1(t,\vec{l})/\nu_1\Bigr)\mathrm{d}t. \tag{6}$$

Noise function $N(\vec{l})$ is determined by expression

$$N(\vec{l}) = L(l) - \langle L(\vec{l})\rangle = L(\vec{l}) - S(\vec{l}_0,\vec{l}) - C.$$

Hence, from expressions (5) and (6) we obtain the following relationship for noise function:

$$N(\vec{l}) = \int_0^T \ln\Bigl(1 + s_1(t,\vec{l})/\nu_1\Bigr)(\pi'(t) - s(t,\vec{l}_0) - \nu)\mathrm{d}t. \tag{7}$$

It should be noted that the mathematical expectation of the noise function is equal to zero, while its correlation function has the form:

$$B_N(\vec{l}_1,\vec{l}_2) = \langle N(\vec{l}_1)N(\vec{l}_2)\rangle = \int_0^T [s(t,\vec{l}_0) + \nu]\ln[1 + s_1(t,\vec{l}_1)/\nu_1]\ln[1 + s_1(t,\vec{l}_2)/\nu_1]\mathrm{d}t. \tag{8}$$

Let signal function $\hat{S}(\vec{l}_0,\vec{l})$ (6) at fixed $\vec{l}_0$ reach its highest value at point $\vec{l}_*$ and have only one strongly pronounced maximum. Then the signal-to-noise ratio at the output of quasi-likelihood receiver can be written as follows [7]:

$$z^2 = S^2(\vec{l}_0,\vec{l}_*)/B_N(\vec{l}_*,\vec{l}_*). \tag{9}$$

Henceforth we assume that the signal-to-noise ratio is sufficiently large, so that quasi-likelihood estimate possesses a high a posteriori accuracy [7]. In addition, let the received ($s(t,\vec{l})$ and expected $s_1(t,\vec{l})$) signals be differentiable with respect to all estimated parameters. Then quasi-likelihood estimate $\vec{l}_m$ can be found from the solution of the system of equations [7]:

$$\left[\frac{\partial L(\vec{l})}{\partial l_i}\right]_{\vec{l}=\vec{l}_m} = 0, \quad i = 1,2,3. \tag{10}$$

For solving the system of equations (10) we shall employ the small parameter method [7], where the quantity inverse to the signal-to-noise ratio will be used as a small parameter, i.e., small parameter $\varepsilon = 1/z$ (9). Restricting our consideration to the first approximation, we shall obtain the following expression for quasi-likelihood estimate:

$$\vec{l}_m = \vec{l}_* + \mathbf{I}^{-1}\vec{n}, \tag{11}$$

where due to formula (5) vector random quantity $\vec{n}$ consists of the following coordinates:

$$n_i = \left.\frac{\partial N(\vec{l})}{\partial l_i}\right|_{\vec{l}=\vec{l}_*}, \quad i = 1,2,3.$$

In expression (11) $\mathbf{I}$ is the quasi-information matrix defined by expression

$$\mathbf{I} = \left\| -\left.\frac{\partial^2 S(\vec{l}_0, \vec{l})}{\partial l_i \partial l_j}\right|_{\vec{l}=\vec{l}_*} \right\|, \quad i, j = 1,2,3. \tag{12}$$

Substituting expression (6) into (12) we obtain an explicit expression for quasi-information matrix:

$$\mathbf{I} = \left\| \int_0^T \frac{s(t, \vec{l}_0)}{(s_1(t, \vec{l}_*) + \nu_1)^2} \left.\frac{\partial s_1(t, \vec{l})}{\partial l_i}\right|_{\vec{l}=\vec{l}_*} \left.\frac{\partial s_1(t, \vec{l})}{\partial l_j}\right|_{\vec{l}=\vec{l}_*} \mathrm{d}t \right.$$

$$\left. - \int_0^T \frac{s(t, \vec{l}_0)}{s_1(t, \vec{l}_*) + \nu_1} \left.\frac{\partial^2 s_1(t, \vec{l})}{\partial l_i \partial l_j}\right|_{\vec{l}=\vec{l}_*} \right\| \mathrm{d}t, \quad i, j = 1, 2, 3. \tag{13}$$

If $\nu = \nu_1$ and the waveforms of received signal $s(t, \vec{l})$ and expected signal $s_1(t, \vec{l})$ coincide, the quasi-information matrix coincides with the Fisher information matrix [3].

Let us introduce into consideration matrix

$$\mathbf{I}^0 = \left\| \int_0^T \frac{s(t, \vec{l}_0) + \nu}{(s_1(t, \vec{l}_*) + \nu_1)^2} \left.\frac{\partial s_1(t, \vec{l})}{\partial l_i}\right|_{\vec{l}=\vec{l}_*} \left.\frac{\partial s_1(t, \vec{l})}{\partial l_j}\right|_{\vec{l}=\vec{l}_*} \mathrm{d}t \right\|, \quad i, j = 1,2,3, \tag{14}$$

that shall be called shortened information matrix. It should be noted that the shortened matrix consists of the second partial derivatives of correlation function $B_N(\vec{l}_1, \vec{l}_2)$ (8) of noise function $N(\vec{l})$ (7):

$$\mathbf{I}^0 = \left\| \frac{\partial^2 B_N(\vec{l}_1, \vec{l}_2)}{\partial l_{1i} \partial l_{2j}} \right\|_{\vec{l}_1=\vec{l}_*, \vec{l}_2=\vec{l}_*}, \quad i, j = 1,2,3.$$

Matrices $\mathbf{I}$ and $\mathbf{I}^0$ allow us to obtain the following expression for the correlation matrix of quasi-likelihood estimates [7]:

$$\mathbf{K} = \mathbf{I}^{-1}\mathbf{I}^0\mathbf{I}^{-1}. \tag{15}$$

Let us render concrete the expected signal $s_1(t,\vec{l})$ by writing it in the form similar to the signal received (2):

$$s_1(t,\vec{l}) = s_1(t,R,V,A) = \sum_{k=0}^{N-1} s_1(t - 2R/c - (k-\mu)(1+2V/c)\theta - A(k-\mu)^2\theta^2/c). \tag{16}$$

Here function $s_1(t)$ describes the waveform of intensity of one expected optical pulse, and in the general case $s_1(t) \neq s(t)$ from expression (2).

We shall assume that the duration of functions $s(t)$ and $s_1(t)$ is less than the pulse-repetition cycle θ, because the pulse ratio of sequences (2) and (16) is no less than 3…4. In addition, functions $s(t)$ and $s_1(t)$ should be differentiable. This ensures the differentiability of signals $s(t,\vec{l})$ (2) and $s_1(t,\vec{l})$ (16) with respect to all estimated parameters. Substituting expressions (2) and (16) into (13) and (14) we obtain that quasi-information matrices can be presented in the form:

$$\mathbf{I} = \frac{1}{c^2}\begin{pmatrix} 4\hat{M}_0 & 4\theta\hat{M}_1 & 2\theta^2\hat{M}_2 \\ 4\theta\hat{M}_1 & 4\theta^2\hat{M}_2 & 2\theta^3\hat{M}_3 \\ 2\theta^2\hat{M}_2 & 2\theta^3\hat{M}_3 & \theta^4\hat{M}_4 \end{pmatrix}, \tag{17}$$

$$\mathbf{I}^0 = \frac{1}{c^2}\begin{pmatrix} 4\hat{M}_0^0 & 4\theta\hat{M}_1^0 & 2\theta^2\hat{M}_2^0 \\ 4\theta\hat{M}_1^0 & 4\theta^2\hat{M}_2^0 & 2\theta^3\hat{M}_3^0 \\ 2\theta^2\hat{M}_2^0 & 2\theta^3\hat{M}_3^0 & \theta^4\hat{M}_4^0 \end{pmatrix}, \tag{18}$$

where

$$\hat{M}_n = \sum_{k=0}^{N-1}\alpha_k (k-\mu)^n, \quad \hat{M}_n^0 = \sum_{k=0}^{N-1}\alpha_k^0 (k-\mu)^n,$$

$$\alpha_k^0 = \int_0^T \frac{s(t+\Delta_k)+\nu}{(s_1(t)+\nu_1)^2}\left(\frac{\mathrm{d}s_1(t)}{\mathrm{d}t}\right)^2 \mathrm{d}t,$$

$$\alpha_k = \int_0^T \frac{1}{s_1(t)+\nu_1}\frac{\mathrm{d}s(t+\Delta_k)}{\mathrm{d}t}\frac{\mathrm{d}s_1(t)}{\mathrm{d}t}\mathrm{d}t, \tag{19}$$

$$\Delta_k = 2(R_* - R_0)/c + 2(k-\mu)\theta(V_* - V_0)/c + (k-\mu)^2\theta^2(A_* - A_0)/c,$$

where R_*, V_*, and A_* are the values of target motion parameters that ensure the maximum value of function (6).

Thus, the dispersion and correlation of quasi-likelihood estimates can be generally obtained from expression (15) by substituting therein expressions (17) and (18). This involves the need of inversion and multiplication of 3×3 matrices that may require cumbersome manipulations. In addition, the determination of parameters R_*, V_*, and A_* involves the need of numerical solution of the system of transcendental equations $[\partial S(\vec{l}_0,\vec{l})/\partial l_i]_{\vec{l}=\vec{l}_*} = 0, i = 1,2,3$ [7]. Biases of quasi-likelihood estimates of signal parameters (11) have the form:

$$b(R) = \langle R_m - R_0\rangle = R_* - R_0,$$

$$b(V) = \langle V_m - V_0 \rangle = V_* - V_0,$$

$$b(A) = \langle A_m - A_0 \rangle = A_* - A_0,$$

and in the general case they are not equal to zero. Moreover, since biases of estimates do not depend on the signal-to-noise ratio, but are determined by the waveform of intensity of received $s(t)$ and expected $s_1(t)$ signals, in the general case quasi-likelihood estimates are not consistent [7]. The comparison of results of calculations by formula (15) with the correlation matrix of maximum likelihood estimates (obtained in paper [3]) make it possible to determine the losses in accuracy of quasi-likelihood estimates of range, velocity and acceleration as compared with the accuracy of maximum likelihood estimates.

Let us consider a particular case of consistent quasi-likelihood estimates. From the analysis of signal function (6) it follows that the quasi-likelihood estimates of range, velocity and acceleration are consistent and unbiased, if the intensities of received $s(t)$ and expected $s_1(t)$ signals satisfy the following conditions:

— both these functions have maximum in one and the same point t_0;

— they decrease on the time interval (t_0, ∞);

— they are even with respect to t_0.

If these conditions are fulfilled, then $R_* = R_0$, $V_* = V_0$, and $A_* = A_0$, quasi-likelihood estimates are unbiased and consistent, and in relationships (19) $\Delta_k = 0$, $k = 0, \ldots, N-1$.

The specified conditions are satisfied, in particular, by quasi-rectangular pulses having the form [18]:

$$s_2(t,\tau,\delta) = a \begin{cases} \exp\left[-\dfrac{\pi}{2\delta^2} \left(\dfrac{t}{\tau} - \dfrac{1-\delta}{2} \right)^2 \right], & \dfrac{t}{\tau} \geq \dfrac{1-\delta}{2}, \\ 1, & \dfrac{|t|}{\tau} \leq \dfrac{1-\delta}{2}, \\ \exp\left[-\dfrac{\pi}{2\delta^2} \left(\dfrac{t}{\tau} + \dfrac{1-\delta}{2} \right)^2 \right], & \dfrac{t}{\tau} \leq -\dfrac{1-\delta}{2}, \end{cases} \qquad (20)$$

$$s_3(t,\tau,\delta) = a \begin{cases} \left\{ 1 + \left[\dfrac{\pi}{2\delta} \left(\dfrac{t}{\tau} - \dfrac{1-\delta}{2} \right) \right]^2 \right\}^{-1}, & \dfrac{t}{\tau} \geq \dfrac{1-\delta}{2}, \\ 1, & \dfrac{|t|}{\tau} \leq \dfrac{1-\delta}{2}, \\ \left\{ 1 + \left[\dfrac{\pi}{2\delta} \left(\dfrac{t}{\tau} + \dfrac{1-\delta}{2} \right) \right]^2 \right\}^{-1}, & \dfrac{t}{\tau} \leq -\dfrac{1-\delta}{2}, \end{cases} \qquad (21)$$

where $\tau = \int_{-\infty}^{\infty} s^2(t) \mathrm{d}t / [\max s(t)]^2$ is the equivalent pulse duration, $\delta \, (0 < \delta \leq 1)$ is the parameter equal to the relative fraction of pulse energy concentrated at its edges.

Examples of the functions satisfying the specified conditions are also presented in paper [4].

In the case of a consistent estimate, quantities α_k and α_k^0 become independent on k, while numbers $\hat{M}_n$ and $\hat{M}_n^0$ turn into the following numbers:

$$\hat{M}_n = \alpha M_n, \quad \hat{M}_n^0 = \alpha^0 M_n,$$

where $M_n = \sum_{k=0}^{N-1} (k - \mu)^n$, $\alpha = \int_0^T \dfrac{1}{s_1(t) + \nu_1} \dfrac{\mathrm{d}s(t)}{\mathrm{d}t} \dfrac{\mathrm{d}s_1(t)}{\mathrm{d}t} \mathrm{d}t$, $\alpha^0 = \int_0^T \dfrac{s(t) + \nu}{(s_1(t) + \nu_1)^2} \left(\dfrac{\mathrm{d}s_1(t)}{\mathrm{d}t} \right)^2 \mathrm{d}t.$

Thus, for expressions (17) and (18) we obtain the following relationships:

$$\mathbf{I} = \frac{\alpha}{c^2}\begin{pmatrix} 4M_0 & 4\theta M_1 & 2\theta^2 M_2 \\ 4\theta M_1 & 4\theta^2 M_2 & 2\theta^3 M_3 \\ 2\theta^2 M_2 & 2\theta^3 M_3 & \theta^4 M_4 \end{pmatrix},$$

$$\mathbf{I}^0 = \frac{\alpha^0}{c^2}\begin{pmatrix} 4M_0 & 4\theta M_1 & 2\theta^2 M_2 \\ 4\theta M_1 & 4\theta^2 M_2 & 2\theta^3 M_3 \\ 2\theta^2 M_2 & 2\theta^3 M_3 & \theta^4 M_4 \end{pmatrix}.$$

Next, from formula (15) we have the following expression for the correlation matrix of consistent quasi-likelihood estimates of range, velocity and acceleration:

$$\mathbf{K} = c^2\frac{\alpha^0}{\alpha^2}\frac{1}{(2M_1M_3 + M_0M_4)M_2 - M_2^3 - M_0M_3^2 - M_1^2M_4}$$

$$\times\begin{pmatrix} \frac{1}{4}(M_2M_4 - M_3^2) & \frac{1}{4\theta}(M_2M_3 - M_1M_4) & \frac{1}{2\theta^2}(M_1M_3 - M_2^2) \\ \frac{1}{4\theta}(M_2M_3 - M_1M_4) & \frac{1}{4\theta^2}(M_0M_4 - M_2^2) & \frac{1}{2\theta^3}(M_1M_2 - M_0M_3) \\ \frac{1}{2\theta^2}(M_1M_3 - M_2^2) & \frac{1}{2\theta^3}(M_1M_2 - M_0M_3) & \frac{1}{\theta^4}(M_0M_2 - M_1^2) \end{pmatrix}. \quad (22)$$

Substituting $s_1(t) = s(t)$ and $\nu_1 = \nu$ into expression (22), we obtain the correlation matrix of maximum likelihood estimates of range, velocity and acceleration [3]:

$$\mathbf{K}_F = \frac{c^2}{\hat{\alpha}}\frac{1}{((2M_1M_3 + M_0M_4)M_2 - M_2^3 - M_0M_3^2 - M_1^2M_4)}$$

$$\times\begin{pmatrix} \frac{1}{4}(M_2M_4 - M_3^2) & \frac{1}{4\theta}(M_2M_3 - M_1M_4) & \frac{1}{2\theta^2}(M_1M_3 - M_2^2) \\ \frac{1}{4\theta}(M_2M_3 - M_1M_4) & \frac{1}{4\theta^2}(M_0M_4 - M_2^2) & \frac{1}{2\theta^3}(M_1M_2 - M_0M_3) \\ \frac{1}{2\theta^2}(M_1M_3 - M_2^2) & \frac{1}{2\theta^3}(M_1M_2 - M_0M_3) & \frac{1}{\theta^4}(M_0M_2 - M_1^2) \end{pmatrix}, \quad (23)$$

where $\hat{\alpha} = \int_0^T \frac{1}{s(t)+\nu}\left(\frac{\mathrm{d}s(t)}{\mathrm{d}t}\right)^2 \mathrm{d}t.$

Comparing expressions (22) and (23) we can see that they differ only in their coefficients preceding matrices. Thus, ratio χ of corresponding dispersions and correlations of the quasi-likelihood consistent estimate and the maximum likelihood estimate is the same for all estimated parameters of motion (range, velocity and acceleration) and has the form:

$$\chi = \frac{D^*}{D} = \frac{\hat{\alpha}\alpha^0}{\alpha^2},$$

where D^* is the dispersion of quasi-likelihood estimate of one of the parameters R, V or A, while D is the dispersion of the maximum likelihood estimate of the same parameter.

For the analysis of quantity χ it is expedient to pass to dimensionless variables. To this end, we shall present the intensity waveform of individual pulses in the form $s(t) = af(t)$, $s_1(t) = a_1 f_1(t)$, where symbol a designates the maximum of signal $s(t)$, while symbol a_1 designates the maximum of signal $s_1(t)$. Thus the maxima of functions $f(t)$ and $f_1(t)$ are equal to unity. Let us introduce the dimensionless quantities having the meaning of the ratio of intensities of signal–background

$$q = \frac{a}{\nu}, \quad q_1 = \frac{a_1}{\nu_1}.$$

Using the specified designations the ratio χ of the corresponding dispersions and correlations of the consistent quasi-likelihood estimate and the maximum likelihood estimate assumes the form:

$$\chi = \frac{D^*}{D} = \frac{\hat{\alpha}\alpha^0}{\alpha^2} = \frac{\int_0^T \frac{1}{1+qf(t)}\left(\frac{\mathrm{d}f(t)}{\mathrm{d}t}\right)^2 \mathrm{d}t \int_0^T \frac{1+qf(t)}{\left(1+q_1 f_1(t)\right)^2}\left(\frac{\mathrm{d}f_1(t)}{\mathrm{d}t}\right)^2 \mathrm{d}t}{\left(\int_0^T \frac{1}{1+q_1 f_1(t)} \frac{\mathrm{d}f(t)}{\mathrm{d}t}\frac{\mathrm{d}f_1(t)}{\mathrm{d}t}\mathrm{d}t\right)^2}. \tag{24}$$

Let us introduce auxiliary functions:

$$g(t) = \frac{1}{\sqrt{1+qf(t)}}\frac{\mathrm{d}f(t)}{\mathrm{d}t}, \qquad g_1(t) = \frac{\sqrt{1+qf(t)}}{1+q_1 f_1(t)}\frac{\mathrm{d}f_1(t)}{\mathrm{d}t}.$$

Expressing relationship (24) in terms of these functions we obtain:

$$\chi = \int_0^T g^2(t)\mathrm{d}t \int_0^T g_1^2(t)\mathrm{d}t \Big/ \left(\int_0^T g(t)g_1(t)\mathrm{d}t\right)^2.$$

Hence, due to the Bunyakovskii–Schwarz inequality it follows that $\chi \geq 1$ and $\chi = 1$ only at $g_1(t) = \mathrm{const}\, g(t)$.

Let us render concrete expression (24) for the case of weak optical pulses, when $a_0 << \nu$ and $a_1 << \nu_1$. To this end, we shall pass to the limit in expression (24) at $q \to 0$ and $q_1 \to 0$. As a result, we obtain

$$\chi_0 = \chi\big|_{q_0=q_1=0} = \frac{\int_0^T \left(\frac{\mathrm{d}f(t)}{\mathrm{d}t}\right)^2 \mathrm{d}t \int_0^T \left(\frac{\mathrm{d}f_1(t)}{\mathrm{d}t}\right)^2 \mathrm{d}t}{\left(\int_0^T \frac{\mathrm{d}f(t)}{\mathrm{d}t}\frac{\mathrm{d}f_1(t)}{\mathrm{d}t}\mathrm{d}t\right)^2}. \tag{25}$$

As follows from relationship (25), the loss in estimate accuracy for weak optical pulses does not depend on the difference of their maximum intensities (a and a_1). Figures 1–4 present the relationship $\chi_0(\kappa)$ of the loss (25) in accuracy of the quasi-likelihood estimate for signals (20) and (21) in comparison with the

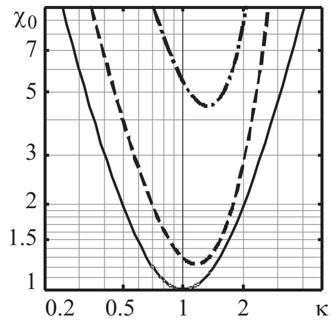

Fig. 1.

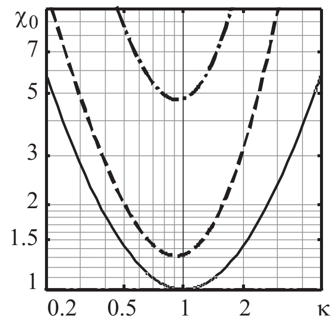

Fig. 2.

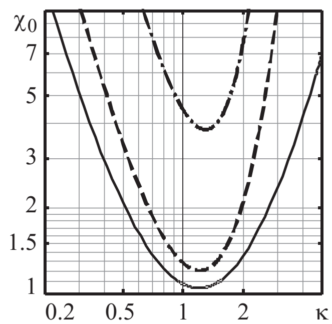

Fig. 3.

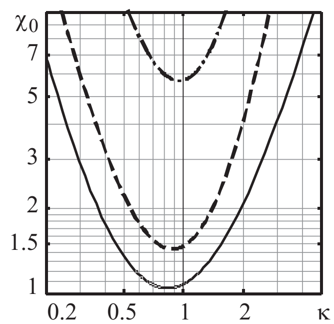

Fig. 4.

accuracy of maximum likelihood estimate as a function of the duration ratio $\kappa = \tau / \tau_0$ of the expected and received signals. Solid lines were calculated for the parameter value $\delta = 1$, dashed lines—for $\delta = 0.5$ and chain-dotted lines—for $\delta = 0.1$. For Fig. 1 we selected the received signal $s(t,\tau_0) = s_2(t,\tau_0,\delta = 1)$ and expected signal $s_1(t,\tau) = s_2(t,\tau,\delta)$. For Fig. 2 we selected $s(t,\tau_0) = s_3(t,\tau_0,\delta = 1)$ and $s_1(t,\tau) = s_3(t,\tau,\delta)$. For Fig. 3 we selected $s(t,\tau_0) = s_2(t,\tau_0,\delta = 1)$ and $s_1(t,\tau) = s_3(t,\tau,\delta)$ and for Fig. 4 we selected $s(t,\tau_0) = s_3(t,\tau_0,\delta = 1)$ and $s_1(t,\tau) = s_2(t,\tau,\delta)$. As follows from plots in Figs. 1–4, the loss in accuracy of quasi-likelihood estimate of motion parameters can be significant.

Examples of the calculation of the accuracy loss for the quasi-likelihood estimate are also presented in paper [4], where the acceleration was assumed known a priori.

The determined characteristics of quasi-likelihood estimates allow us to make a reasonable choice of the algorithm of estimation and the intensity waveform of expected signal depending on the available a priori information and the admissible loss in the accuracy of estimate.

REFERENCES

1. V. I. Vorob'ev, *Optical Detection and Raging for Radio Engineers* (Radio i Svyaz', Moscow, 1983) [in Russian].
2. N. A. Dolinin, A. F. Terpugov, *Statistical Methods in Optical Detection and Raging* (TGU, Tomsk, 1982) [in Russian].
3. A. P. Trifonov, M. B. Bespalova, M. V. Maksimov, "Estimation of the range, speed and acceleration during the probing with an optical pulse sequence," Radiotekhnika, No. 4, 99 (2001).
4. A. P. Trifonov, M. B. Bespalova, "Quazi-likelihood estimation of the range and speed during the probing with optical pulse sequence," Izv. Vyssh. Uchebn. Zaved., Radioelektron. **39**(8), 23 (1996) [Radioelectron. Commun. Syst. **39**(8), 17 (1996)].
5. V. I. Mudrov, V. L. Kushko, *Measurement Processing Procedures: Quasi-Likelihood Estimates* (Radio i Svyaz', Moscow, 1983) [in Russian].
6. Kh. V. Khindrikus, *Noises in Laser Information Systems* (Radio i Svyaz', Moscow, 1987) [in Russian].
7. E. I. Kulikov, A. P. Trifonov, *Estimation of Signal Parameters against the Background of Interferences* (Sov. Radio, Moscow, 1978) [in Russian].
8. M. S. Yarlykov, *Application of the Markov Nonlinear Filtration Theory in Radio Engineering* (Sov. Radio, Moscow, 1980) [in Russian].