ISSN 0735-2727, Radioelectronics and Communications Systems, 2013, Vol. 56, No. 2, pp. 73–81. © Allerton Press, Inc., 2013.
Original Russian Text © A.P. Trifonov, A.V. Kurbatov, 2013, published in Izv. Vyssh. Uchebn. Zaved., Radioelektron., 2013, Vol. 56, No. 2, pp. 24–32.

Quazi-Likelihood Estimation of Motion Parameters of Rapidly Fluctuating Target during the Probing with a Sequence of Optical Pulses[1]

A. P. Trifonov and A. V. Kurbatov
Voronezh State University, Voronezh, Russia

Received in final form December 10, 2012

Abstract—Characteristics of efficient estimates of the range, velocity, and acceleration of a rapidly fluctuating target have been obtained during the probing with a sequence of optical pulses. The losses in estimation accuracy of the range, velocity, and acceleration caused by the presence of non-informative parameters have been also found.

DOI: 10.3103/S0735272713020027

The optical detection-and-ranging systems make a wide use of sequences of optical pulses [1–4]. The potential accuracy of estimates of such parameters as range, velocity and acceleration was investigated in paper [3]. It was assumed in that study that all parameters of the sequence scattered by target, except the estimated ones, are known a priori. However, target fluctuations in real conditions and the physical effects accompanying the light scattering and propagation in different media result in the situation where the intensity of individual optical pulses may depend on the finite number of non-informative parameters, the estimation of which is not needed.

Despite the estimation of non-informative parameters is not needed, their presence, however, affects the estimation accuracy of informative parameters, such as range, velocity and acceleration. The dependence of the intensity of the scattered sequence of optical pulses on non-informative parameters is determined by the pattern of target fluctuations. In the event of slow fluctuations of target the values of parameters are equal for all pulses of the sequence. In the event of a rapidly fluctuating target the parameters for individual pulses are different.

The present paper considers the case of a rapidly fluctuating target. The efficiency of estimates of the target range, velocity and acceleration is discussed upon condition that the scattered sequence of optical pulses has a finite number of arbitrary non-informative parameters.

Let us assume that an optical pulse is radiated with the following pulse intensity:

$$s_N(t) = \sum_{k=0}^{N-1} \hat{s}(t-(k-\mu)\theta-\lambda), \tag{1}$$

where $\hat{s}(\cdot)$ is the function describing the intensity of individual optical pulse, θ is the pulse repetition period, λ is the time position of the sequence. Parameter μ determines the point of sequence related to its time position λ. Hence, at $\mu = 0$ quantity λ represents the time position of the first pulse, at $\mu = (N-1)/2$ it represents the time position of the middle of sequence (1), while at $\mu = N-1$—the time position of the last pulse.

Owing to the scattering of probing sequence (1) by a target, which range R_0, velocity V_0 and acceleration A_0 are to be estimated, the intensity of the signal received can be written as follows [1, 3]:

[1] The study was carried out with support of RFFI (Project No. 13-08-00735) and FGP "Research and Academic-Teaching Staff of the Innovation Russia" (No. 14.V37.21.2032).

$$s(t, R_0, V_0, A_0, \vec{L}_0) = \sum_{k=0}^{N-1} s(t - 2R_0 / c - (k-\mu)(1+2V_0/c)\theta - A_0(k-\mu)^2\theta^2 / c, \vec{l}_{0k}), \tag{2}$$

where function $s(t, \vec{l}_{0k})$ describes the intensity waveform of one scattered optical pulse of sequence (2); in the general case it is different from $\hat{s}(t)$ in expression (1), where c is the velocity of light, $|V_0| \ll c$ and $N\theta|A_0| \ll c$.

Unlike the case considered in paper [3], the pulse sequence depends on non-informative parameters, and in this case the values of parameters are different for various pulses of the sequence. Vector

$$\vec{l}_{0k} = \| l_{0k1}, l_{0k2}, \ldots, l_{0kp} \|$$

consists of p non-informative parameters unknown due to the effect of target fluctuations and propagation medium. In case vector $\vec{l}_{0k}$ does not depend on k, i.e., the waveform of all pulses in the sequence is the same, pulse sequence (2) is commonly called “slowly fluctuating”. If vector $\vec{l}_{0k}$ changes with the variation of k, pulse sequence (2) is commonly called “rapidly fluctuating”.

From the viewpoint of physics the need of considering rapidly fluctuating sequences is related to the fact that during the time passing between the sending of individual pulses of the sequence parameters of a single pulse may significantly change due to target fluctuations. The complete set of non-informative parameters $\vec{l}_{0k}$, $k = 0, 1, \ldots, N-1$ shall be written in the form $\vec{L}_0 = (\vec{l}_{00}, \vec{l}_{01}, \ldots, \vec{l}_{0N-1})$ or in more detail:

$$\vec{L}_0 = \| l_{001}, l_{002}, \ldots, l_{00p}; l_{011}, l_{012}, \ldots, l_{01p}; \ldots; l_{0,N-1,1}, l_{0N-12}, \ldots, l_{0N-1p} \|.$$

From now on subscript “0” will be used to mark the true values of unknown parameters of the received sequence of optical pulses with intensity (2).

Let us assume that the signal with intensity (2) is observed on the time interval [0; T] against the background of optical noise with intensity $\nu > 0$. Thus, an accessible for processing realization of the Poisson process will have intensity

$$\beta(t, R_0, V_0, A_0, \vec{L}_0) = s(t, R_0, V_0, A_0, \vec{L}_0) + \nu. \tag{3}$$

According to paper [5] for calculating the potential accuracy of simultaneous estimates of unknown parameters of the optical pulse sequence with intensity (2), first, we need to determine the uncertainty function:

$$H(R_1, R_2, V_1, V_2, A_1, A_2, \vec{L}_1, \vec{L}_2)$$

$$= \int_0^T \beta(t, R_0, V_0, A_0, \vec{L}_0) \ln\left(\frac{\beta(t, R_1, V_1, A_1, \vec{L}_1)}{\nu}\right) \ln\left(\frac{\beta(t, R_2, V_2, A_2, \vec{L}_2)}{\nu}\right) dt. \tag{4}$$

We shall assume that the duration of observation interval [0; T] is more than that of the entire sequence of optical pulses, i.e., $T > N\theta$, while the pulse ratio of the sequence is no less than two, so that individual pulses do not overlap. Then, substituting relationship (2) into (3) and expression (3) into (4) we obtain:

$$H(R_1, R_2, V_1, V_2, A_1, A_2, \vec{L}_1, \vec{L}_2) = \sum_{k=0}^{N-1} H_k(R_1, R_2, V_1, V_2, A_1, A_2, \vec{l}_{1k}, \vec{l}_{2k}), \tag{5}$$

where

$$H_k(R_1,R_2,V_1,V_2,A_1,A_2,\vec{l}_{1k},\vec{l}_{2k}) = \int_0^T (\nu + s(t-2R_0/c-(k-\mu)(1+2V_0/c)\theta$$

$$-A_0(k-\mu)^2\theta^2/c,\vec{l}_{0k}))\ln(1+(s(t-2R_1/c-(k-\mu)(1+2V_1/c)\theta - A_1(k-\mu)^2\theta^2/c,\vec{l}_{1k}))/\nu)$$

$$\times\ln(1+(s(t-2R_2/c-(k-\mu)(1+2V_2/c)\theta - A_2(k-\mu)^2\theta^2/c,\vec{l}_{2k}))/\nu)dt. \quad (6)$$

Let us consider a regular case [6] where the intensities of individual pulses are differentiable with respect to t and all parameters l_{ki}, $k=0,\dots,N-1$, $i=1,\dots,p$. In this case the potential estimation accuracy of both informative and non-informative parameters of the sequence of optical pulses with intensity (2) is characterized by the following correlation matrix of joint-efficient estimates:

$$\mathbf{K}_p = \mathbf{I}^{-1}, \quad (7)$$

where $\mathbf{I}$ is the Fisher information matrix [6], which can be presented in the form of a block matrix:

$$\mathbf{I} = \begin{Vmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^{\mathrm{T}} & \mathbf{D} \end{Vmatrix}, \quad (8)$$

where “T” is the transposition operation, while the blocks have the form:

$$\mathbf{A} = \begin{Vmatrix} \dfrac{\partial^2 H}{\partial R_1 \partial R_2} & \dfrac{\partial^2 H}{\partial R_1 \partial V_2} & \dfrac{\partial^2 H}{\partial R_1 \partial A_2} \\ \dfrac{\partial^2 H}{\partial V_1 \partial R_2} & \dfrac{\partial^2 H}{\partial V_1 \partial V_2} & \dfrac{\partial^2 H}{\partial V_1 \partial A_2} \\ \dfrac{\partial^2 H}{\partial A_1 \partial R_2} & \dfrac{\partial^2 H}{\partial A_1 \partial V_2} & \dfrac{\partial^2 H}{\partial A_1 \partial A_2} \end{Vmatrix}, \quad \mathbf{B} = \begin{Vmatrix} \dfrac{\partial^2 H}{\partial R_1 \partial l_{2mj}} \\ \dfrac{\partial^2 H}{\partial V_1 \partial l_{2mj}} \\ \dfrac{\partial^2 H}{\partial A_1 \partial l_{2mj}} \end{Vmatrix}, \quad \mathbf{D} = \begin{Vmatrix} \dfrac{\partial^2 H}{\partial l_{1ki} \partial l_{2mj}} \end{Vmatrix}.$$

Here all derivatives are calculated at $R_1 = R_2 = R_0$, $V_1 = V_2 = V_0$, $A_1 = A_2 = A_0$, $l_{1ki} = l_{0ki}$ and $l_{2mj} = l_{0mj}$.

It should be noted that matrix $\mathbf{B}$ has the size of 3×P, where $P = Np$, while matrix $\mathbf{D}$ has the size of P×P. We shall consider that the internal numeration in rows of these matrixes is organized as follows: matrix $\mathbf{B}$ consists of N blocks B_m having size 3×p, namely,

$$\mathbf{B} = \| \mathbf{B}_0, \mathbf{B}_1, \dots, \mathbf{B}_{N-1} \|,$$

where

$$\mathbf{B}_m = \begin{Vmatrix} \dfrac{\partial^2 H}{\partial R_1 \partial l_{2m1}} & \dfrac{\partial^2 H}{\partial R_1 \partial l_{2m2}} & \cdots & \dfrac{\partial^2 H}{\partial R_1 \partial l_{2mp}} \\ \dfrac{\partial^2 H}{\partial V_1 \partial l_{2m1}} & \dfrac{\partial^2 H}{\partial V_1 \partial l_{2m2}} & \cdots & \dfrac{\partial^2 H}{\partial V_1 \partial l_{2mp}} \\ \dfrac{\partial^2 H}{\partial A_1 \partial l_{2m1}} & \dfrac{\partial^2 H}{\partial A_1 \partial l_{2m2}} & \cdots & \dfrac{\partial^2 H}{\partial A_1 \partial l_{2mp}} \end{Vmatrix},$$

while matrix **D** has the form:

$$\mathbf{D} = \begin{Vmatrix} \mathbf{D}_{00} & \cdots & \mathbf{D}_{0N-1} \\ \vdots & \ddots & \vdots \\ \mathbf{D}_{N-10} & \cdots & \mathbf{D}_{N-1N-1} \end{Vmatrix},$$

where blocks

$$\mathbf{D}_{km} = \begin{Vmatrix} \dfrac{\partial^2 H}{\partial l_{1k1}\partial l_{2m1}} & \cdots & \dfrac{\partial^2 H}{\partial l_{1k1}\partial l_{2mp}} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial^2 H}{\partial l_{1kp}\partial l_{2m1}} & \cdots & \dfrac{\partial^2 H}{\partial l_{1kp}\partial l_{2mp}} \end{Vmatrix}$$

have the size of $p \times p$. Since function (5) consists of N terms, each containing non-informative parameters corresponding to only one pulse (with its number k), partial derivatives $\dfrac{\partial^2 H}{\partial l_{1ki}\partial l_{2mj}}$ for pairs of parameters l_{1k} and l_{2mj} with $k \neq m$ are equal to zero. This means that matrix **D** is a block-diagonal matrix with blocks $\mathbf{D}_{kk}$ on the diagonal.

Substituting expression (6) into (5) and (5) into (8) and differentiating, we obtain

$$\mathbf{A} = \frac{1}{c^2}\sum_{k=0}^{N-1}\alpha_k \begin{Vmatrix} 4 & 4\theta(k-\mu) & 2\theta^2(k-\mu)^2 \\ 4\theta(k-\mu) & 4\theta^2(k-\mu)^2 & 2\theta^3(k-\mu)^3 \\ 2\theta^2(k-\mu)^2 & 2\theta^3(k-\mu)^3 & \theta^4(k-\mu)^4 \end{Vmatrix},$$

$$\mathbf{B}_k = -\frac{1}{c}\begin{Vmatrix} 2 \\ 2\theta(k-\mu) \\ \theta^2(k-\mu)^2 \end{Vmatrix}\vec{\beta}_k,$$

$$\vec{\beta}_k = \|\beta_{k1} \quad \beta_{k2} \quad \ldots \quad \beta_{kp}\|,$$

$$\mathbf{D}_{kk} = (D_{kikj})_{i,j=1,2,\ldots,p},$$

where

$$\alpha_k = \int_{-\infty}^{+\infty} \frac{1}{\nu + s(t,\vec{l}_{0k})}\left[\frac{\partial s(t,\vec{l}_{0k})}{\partial t}\right]^2 \mathrm{d}t,$$

$$\beta_{ki} = \int_{-\infty}^{+\infty} \frac{1}{(\nu + s(t,\vec{l}_{0k})}\left[\frac{\partial s(t,\vec{l}_k)}{\partial t}\frac{\partial s(t,\vec{l}_k)}{\partial l_{ki}}\right]_{\vec{l}_{0k}} \mathrm{d}t,$$

$$D_{ki\,kj} = \int_{-\infty}^{+\infty} \frac{1}{\nu + s(t,\vec{l}_{0k})} \left[\frac{\partial s(t,\vec{l}_k)}{\partial l_{ki}} \frac{\partial s(t,\vec{l}_k)}{\partial l_{kj}} \right]_{\vec{l}_{0k}} \mathrm{d}t,$$

$$\mathbf{D}_{kk} = (D_{ki\,kj})_{i,\,j=1,2,\ldots,p}.$$

Unknown parameters $\vec{l}_{0k}$ are non-informative. That is why there is no need to determine all elements of correlation matrix (7). It is sufficient to find the elements of this matrix that are located at the crossing of the first three rows and columns of matrix (7), which characterize the potential accuracy of the estimates of range, velocity and acceleration.

Let us designate the matrix formed from the elements of matrix (7) located at the crossing of its first three rows and columns as **K**, and its inverse matrix as **F**. Thus,

$$\mathbf{K} = \mathbf{F}^{-1}.$$

Matrix **F** can be found by using the Frobenius formula [7]. Indeed, assuming that matrix **D** in expression (8) is nondegenerate, we obtain

$$\mathbf{F} = \mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{B}^{\mathrm{T}}.$$

Let us calculate this matrix. We shall introduce the following designation for elements of matrix Δ_{kk} inverse with respect to matrix $\mathbf{D}_{kk} = \| D_{ki\,kj} \|_{i,\,j=1,2,\ldots,p}$:

$$\mathbf{D}_{kk}^{-1} = \| D_{ki\,kj} \|^{-1} = \| \Delta_{ki\,kj} \|_{i,\,j=1,\ldots,p} = \Delta_{kk}.$$

Let us also introduce designation

$$\rho_{pk} = \frac{1}{\alpha_k} \vec{\beta}_k \Delta_{kk} \vec{\beta}_k^T = \frac{1}{\alpha_k} \sum_{i=1}^{p} \sum_{j=1}^{p} \beta_{ki} \Delta_{ki\,kj} \beta_{kj}.$$

Owing to the block diagonality of matrix **D** we have

$$\mathbf{B}\mathbf{D}^{-1}\mathbf{B}^{\mathrm{T}} = \sum_{k=0}^{N-1} \mathbf{B}_k \mathbf{D}_{kk}^{-1} \mathbf{B}_k^{\mathrm{T}}$$

$$= \sum_{k=0}^{N-1} \left(-\frac{1}{c} \left\| \begin{matrix} 2 \\ 2\theta(k-\mu) \\ \theta^2 (k-\mu)^2 \end{matrix} \right\| \vec{\beta}_k \right) \Delta_{kk} \left(-\frac{1}{c} \vec{\beta}_k^T \| 2,\, 2\theta(k-\mu),\, \theta^2 (k-\mu)^2 \| \right)$$

$$= \frac{1}{c^2} \sum_{k=0}^{N-1} \alpha_k \rho_{pk} \left\| \begin{matrix} 4 & 4\theta(k-\mu) & 2\theta^2 (k-\mu)^2 \\ 4\theta(k-\mu) & 4\theta^2 (k-\mu)^2 & 2\theta^3 (k-\mu)^3 \\ 2\theta^2 (k-\mu)^2 & 2\theta^3 (k-\mu)^3 & \theta^4 (k-\mu)^4 \end{matrix} \right\|.$$

It follows that

$$\mathbf{F} = \begin{Vmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{Vmatrix} = \frac{1}{c^2}\sum_{k=0}^{N-1}\alpha_k(1-\rho_{pk})$$

$$\times\begin{Vmatrix} 4 & 4\theta(k-\mu) & 2\theta^2(k-\mu)^2 \\ 4\theta(k-\mu) & 4\theta^2(k-\mu)^2 & 2\theta^3(k-\mu)^3 \\ 2\theta^2(k-\mu)^2 & 2\theta^3(k-\mu)^3 & \theta^4(k-\mu)^4 \end{Vmatrix}. \quad (9)$$

Let us introduce designation

$$M_n = \sum_{k=0}^{N-1}\alpha_k(1-\rho_{pk})(k-\mu)^n.$$

Using this designation we can write

$$\mathbf{F} = \frac{1}{c^2}\begin{Vmatrix} 4M_0 & 4\theta M_1 & 2\theta^2 M_2 \\ 4\theta M_1 & 4\theta^2 M_2 & 2\theta^3 M_3 \\ 2\theta^2 M_2 & 2\theta^3 M_3 & \theta^4 M_4 \end{Vmatrix}.$$

It follows that

$$\mathbf{K} = \frac{c^2}{(2M_1M_3+M_0M_4)M_2 - M_2^3 - M_0M_3^2 - M_1^2M_4}$$

$$\times\begin{Vmatrix} \frac{1}{4}(M_2M_4 - M_3^2) & \frac{1}{4\theta}(M_2M_3 - M_1M_4) & \frac{1}{2\theta^2}(M_1M_3 - M_2^2) \\ \frac{1}{4\theta}(M_2M_3 - M_1M_4) & \frac{1}{4\theta^2}(M_0M_4 - M_2^2) & \frac{1}{2\theta^3}(M_1M_2 - M_0M_3) \\ \frac{1}{2\theta^2}(M_1M_3 - M_2^2) & \frac{1}{2\theta^3}(M_1M_2 - M_0M_3) & \frac{1}{\theta^4}(M_0M_2 - M_1^2) \end{Vmatrix}.$$

Let us first assume that the target velocity and acceleration are known a priori and it is necessary to estimate only the target range. Then, due to relationship (9) the dispersion of the efficient estimate of range for a rapidly fluctuating sequence has the form:

$$D(R|R_0,\vec{L}_0) = \frac{1}{F_{11}} = \frac{c^2}{4}\frac{1}{\sum_{k=0}^{N-1}\alpha_k(1-\rho_{pk})}.$$

In the case $\rho_{pk} = 0$ corresponding to the absence of non-informative parameters:

$$D(R|R_0) = \frac{c^2}{4}\frac{1}{\sum_{k=0}^{N-1}\alpha_k}.$$

Thus, the loss in accuracy of the estimate of range R caused by the presence of non-informative parameters is characterized by the following quantity:

$$\chi(R|R_0)=\frac{D(R|R_0,\vec{L}_0)}{D(R|R_0)}=\frac{\sum_{k=0}^{N-1}\alpha_k}{\sum_{k=0}^{N-1}\alpha_k(1-\rho_{pk})}.$$

Let us assume further that the target range and acceleration are known a priori and it is necessary to estimate only the target velocity. Then, due to relationship (9) the dispersion of the efficient estimate of velocity has the form:

$$D(V|V_0,\vec{L}_0)=\frac{1}{F_{22}}=\frac{c^2}{4\theta^2}\frac{1}{\sum_{k=0}^{N-1}\alpha_k(1-\rho_{pk})(k-\mu)^2}.$$

In the case $\rho_{pk}=0$ corresponding to the absence of non-informative parameters:

$$D(V|V_0)=\frac{c^2}{4\theta^2}\frac{1}{\sum_{k=0}^{N-1}\alpha_k(k-\mu)^2}.$$

It follows that the loss in accuracy of the estimate of velocity V caused by the presence of non-informative parameters is characterized by the following quantity:

$$\chi(V|V_0)=\frac{D(V|V_0,\vec{L}_0)}{D(V|V_0)}=\frac{\sum_{k=0}^{N-1}\alpha_k(k-\mu)^2}{\sum_{k=0}^{N-1}\alpha_k(1-\rho_{pk})(k-\mu)^2}.$$

Let us assume now that the target range and velocity are known a priori and it is necessary to estimate only the target acceleration. Then, due to relationship (9) the dispersion of the efficient estimate of acceleration has the form:

$$D(A|A_0,\vec{L}_0)=\frac{1}{F_{33}}=\frac{c^2}{\theta^4}\frac{1}{\sum_{k=0}^{N-1}\alpha_k(1-\rho_{pk})(k-\mu)^4}.$$

In the case $\rho_{pk}=0$ corresponding to the absence of non-informative parameters:

$$D(A|A_0)=\frac{c^2}{\theta^4}\frac{1}{\sum_{k=0}^{N-1}\alpha_k(k-\mu)^4}.$$

Hence, on the assumption that the range and velocity are known the loss in accuracy of the estimate of acceleration A caused by the presence of non-informative parameters is characterized by the following quantity:

$$\chi(A|A_0)=\frac{D(A|A_0,\vec{L}_0)}{D(A|A_0)}=\frac{\sum_{k=0}^{N-1}\alpha_k(k-\mu)^4}{\sum_{k=0}^{N-1}\alpha_k(1-\rho_{pk})(k-\mu)^4}.$$

The obtained expressions are essentially simplified if all non-informative parameters are nonpower [8] or the true values of non-informative parameters are the same for all pulses of sequence (2). The last assumption corresponds to the case, where a slowly fluctuating sequence is processed as a rapidly fluctuating one. Therefore,

$$\alpha_0=\alpha_1=\ldots=\alpha_k=\ldots=\alpha,$$

$$\rho_{p0}=\rho_{p1}=\ldots=\rho_{pk}=\ldots=\rho_p,$$

while expressions for the loss of estimate accuracy assume the form:

$$\chi(R|R_0)=\chi(V|V_0)=\chi(A|A_0)=\chi=(1-\rho_p)^{-1}.$$

Thus, the loss in accuracy of estimates due to the presence of non-informative parameters proves to be the same for all parameters of target motion.

Let us determine the loss in estimation accuracy of the target motion parameters while using optical pulses with the intensity of the form

$$s(t,\vec{l})=a\left\{\eta(t)\left[1-\exp\left(-\frac{t}{\Delta}\right)\right]-\eta(t-\tau)\left[1-\exp\left(-\frac{t-\tau}{\Delta}\right)\right]\right\},$$

$$\eta(t)=\begin{cases}1, & t\ge 0,\\ 0, & t<0,\end{cases}\qquad \vec{l}=\{a,\tau,\Delta\}, \tag{10}$$

where τ is the pulse duration, Δ characterizes the duration of pulse front, a is the amplitude factor.

In estimating the parameters of target motion by using the pulses sequence of form (10) each pulse can have up to 3 non-informative parameters ($p\le 3$): a,τ, and Δ.

Figure 1 presents the relationship of the loss in estimation accuracy χ as a function of parameter $y=\Delta/\tau$ characterizing the ratio of the duration of pulse front to the pulse duration for different sets of non-informative parameters at $q=a/\nu=10$. Curve *1* illustrates the loss in estimation accuracy when parameter a is non-informative. As can be seen, the presence of non-informative parameter a does not deteriorate the characteristics of efficient estimates of motion parameters. Curves *2* and *3* illustrate the loss in estimation accuracy when parameters τ and Δ, respectively, are non-informative; curve *4* corresponds to non-informative parameters a and τ, curve *5* corresponds to non-informative parameters a and Δ, *6* corresponds to τ and Δ, and curve *7*—to all three parameters (a, τ, and Δ).

Comparison of the curves makes it possible to determine the impact of the presence of various non-informative parameters of pulse (10) on the accuracy of efficient estimation of motion parameters under conditions of rapid fluctuations of the target. As can be seen from Fig. 1, with an increase of parameter y the accuracy loss caused by the presence of certain non-informative parameters (curves *2*, *3*, *5*, and *6*) decreases, while the accuracy loss caused by certain others (curves *4* and *7*) rises up to the value of 4.

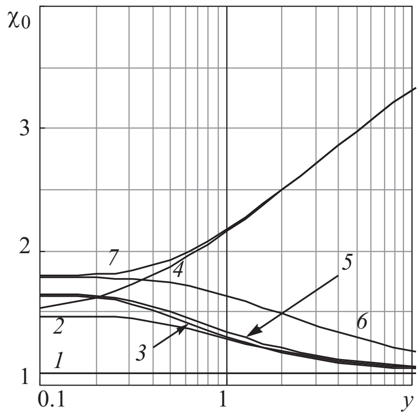

Fig. 1.

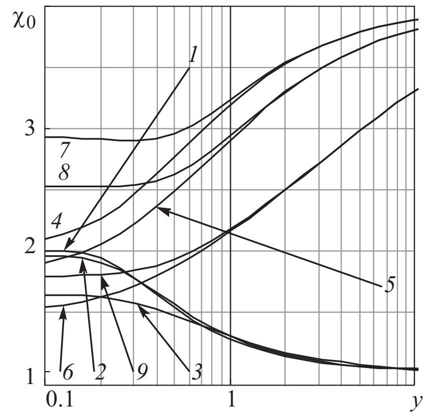

Fig. 2.

Figure 2 displays the relationship of the loss in estimation accuracy χ as a function of parameter $y = \Delta / \tau$ for certain sets of non-informative parameters at different values of parameter $q = a / \nu$. Curves *1*, *2*, and *3* illustrate the loss in estimation accuracy, where parameter Δ is non-informative, at the values of parameter q = 0.1, 1, 10, respectively. Curves *4*, *5*, and *6* correspond to the case, where parameters a and τ are non-informative, at the values of parameter q = 0.1, 1, 10. Curves *7*, *8*, and *9* correspond to the case, where parameters a, τ and Δ are non-informative, at the values of parameter q = 0.1, 1, 10.

It can be seen that for the sets of non-informative parameters $\{a, \tau\}$ and $\{a, \tau, \Delta\}$ an increase of parameter q leads to the reduction of the loss value. In accordance with Figs. 1 and 2 the presence of non-informative parameters of pulse (10) may lead to a four-fold rise of the dispersions of efficient estimates of target motion parameters.

Thus, the obtained results make it possible to find the losses in accuracy of estimates of the range, velocity and acceleration caused by rapid fluctuations of the target.

REFERENCES

1. V. I. Vorob'ev, *Optical Detection and Raging for Radio Engineers* (Radio i Svyaz', Moscow, 1983) [in Russian].
2. N. A. Dolinin and A. F. Terpugov, *Statistical Methods in Optical Detection and Raging* (TGU, Tomsk, 1982) [in Russian].
3. A. P. Trifonov, M. B. Bespalova, and M. V. Maksimov, "Estimation of Range, Speed and Acceleration during the Probing with an Optical Pulse Sequence," Radiotekhnika, No. 4, 99 (2001).
4. A. P. Trifonov, M. B. Bespalova, and A. V. Kurbatov, "Quazi-Likelihood Estimation of Motion Parameters during the Target Probing with a Sequence of Optical Pulses," Izv. Vyssh. Uchebn. Zaved., Radioelektron. **56** (1), 23 (2013) [Radioelectron. Commun. Syst. **56** (1), 20 (2013)], DOI: 10.3103/S0735272713010020.
5. C. Helstrom, "Quantum Detection and Estimation Theory," Dep. of Applied Electrophysics, University of California (1975).
6. A. P. Trifonov and Yu. S. Shinakov, *Simultaneous Discrimination of Signals and Estimation of their Parameters against the Background of Interferences* (Radio i Svyaz', Moscow, 1986) [in Russian].
7. F. R. Gantmakher, *Theory of Matrices* (Nauka, Moscow, 1988) [in Russian].
8. E. I. Kulikov and A. P. Trifonov, *Estimation of Signal Parameters against the Background of Interferences* (Sov. Radio, Moscow, 1978) [in Russian].