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Characteristics of Quazi-Likelihood Estimation of the Image Area in the Presence of Spatial Noise[1]

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Abstract—Asymptotic expressions for the characteristics of quasi-likelihood estimation of image area have been obtained by using the local Markov approximation method. It was shown that the accuracy of area estimation is determined by the magnitude of intensity jump over the boundary limiting the area occupied by image.

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The problem of area estimation of images, the intensity distribution of which is exactly known a priori was considered in papers [1–3] and others. In real tasks the conditions of distant formation of radio and optical images generally do not ensure the a priori knowledge of exact distribution of image intensity. In this connection it may be interesting to consider the area estimation problem for a non-uniform image, the intensity distribution of which is not exactly known.

Let us assume that the following realization of random field is available for processing in domain G:

$$\xi(x,y) = S_0(x,y,\chi_0) + n(x,y), \quad x,y \in G, \tag{1}$$

where

$$S_0(x,y,\chi_0) = F_0(x,y) I(x,y,\chi_0), \tag{2}$$

$S_0(x,y,\chi_0)$ is the useful image with intensity $F_0(x,y)$ that occupies domain $\Omega(\chi_0)$ having area χ_0. The shape of domain $\Omega(\chi)$ with image area χ is determined by the following indicator:

$$I(x,y,\chi) = \begin{cases} 1, & x,y \in \Omega(\chi), \\ 0, & x,y \overline{\in} \Omega(\chi). \end{cases}$$

Term $n(x,y)$ in expression (1) represents a realization of the Gaussian spatial white noise with one-sided spectral density N_0, while the unknown area of image χ_0 assumes values from a priori interval $[\chi_{\min}, \chi_{\max}]$.

In many applied problems of image processing the distribution of image intensity $F_0(x,y)$ is known inaccurately. Therefore, for synthesizing an estimation algorithm of the useful image area by using the maximum likelihood method we shall employ the image

$$S(x,y,\chi) = F(x,y) I(x,y,\chi), \tag{3}$$

where $F(x,y)$ specifies the expected (predicted) distribution of image intensity. It is noted that in the general case $F(x,y) \neq F_0(x,y)$.

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For obtaining estimate χ_m of area χ_0 of image (2) in accordance with the maximum likelihood method [4] it is necessary to form the logarithm of likelihood ratio functional (LRF) [5]:

$$L(\chi)=\frac{2}{N_0}\iint_G \xi(x,y)S(x,y;\chi)\mathrm{d}x\mathrm{d}y-\frac{1}{N_0}\iint_G S^2(x,y;\chi)\mathrm{d}x\mathrm{d}y, \tag{4}$$

for all values $\chi\in[\chi_{\min},\chi_{\max}]$. The realization of observed data (1) contains image (2) that in the general case differs from image (3), for which the LRF logarithm is formed. Hence, estimate χ_m of area χ_0 of image (2) defined as position of the absolute (largest) maximum of function (4)

$$\chi_m=\mathrm{argsup}L(\chi),\quad \chi\in[\chi_{\min},\chi_{\max}] \tag{5}$$

is not a maximum likelihood estimate (MLE). This estimate may be called quasi-likelihood estimate (QLE) [6], since it coincides with MLE provided $F(x,y)\equiv F_0(x,y)$.

In order to determine characteristics of the area quasi-likelihood estimate (5), we shall present expression (4) in the form of signal component and noise function [7]:

$$L(\chi)=S(\chi)+N(\chi), \tag{6}$$

$$S(\chi)=S(\chi_0,\chi)-Q(\chi)/2, \tag{7}$$

$$N(\chi)=\frac{2}{N_0}\iint_G n(x,y)S(x,y,\chi)\mathrm{d}x\mathrm{d}y, \tag{8}$$

$$Q(\chi)=\frac{2}{N_0}\iint_G S^2(x,y,\chi)\mathrm{d}x\mathrm{d}y, \tag{9}$$

$$S(\chi_0,\chi)=\frac{2}{N_0}\iint_G S_0(x,y,\chi_0)S(x,y,\chi)\mathrm{d}x\mathrm{d}y, \tag{10}$$

where $S(\chi_0,\chi)$ is the signal function. Noise function $N(\chi)$ is the realization of centered Gaussian random process having correlation function:

$$B(\chi_1,\chi_2)=\langle N(\chi_1),N(\chi_2)\rangle=\frac{2}{N_0}\iint_G S(x,y,\chi_1)S(x,y,\chi_2)\mathrm{d}x\mathrm{d}y. \tag{11}$$

Let signal component $S(\chi)$ (7) reach its maximum at certain point

$$\tilde{\chi}=\mathrm{argsup}S(\chi),\quad \chi\in[\chi_{\min},\chi_{\max}]. \tag{12}$$

Then the signal-to-noise ratio (SNR) [4] assumes the form:

$$z^2=S^2(\tilde{\chi})/B(\tilde{\chi},\tilde{\chi})=S^2(\tilde{\chi})/Q(\tilde{\chi}). \tag{13}$$

Let us assume that signal-to-noise ratio (13) is sufficiently large, so that QLE of the area possesses high a posteriori accuracy. In this case for determining the QLE characteristics it is sufficient to analyze the behavior of functions (7), (9), (10), and (11) in the small neighborhood of point $\tilde{\chi}$ (12).

Let us introduce designation $\Omega_{\min}(\chi_1,\chi_2)$ for that domain of two ($\Omega(\chi_1)$ and $\Omega(\chi_2)$), which has a smaller area. Next, substituting expressions (2) and (3) into (7), (9)–(11) we have

$$S(\chi_0,\chi)=\frac{2}{N_0}\iint\limits_{\Omega_{\min}(\chi_0,\chi)} F_0(x,y)F(x,y)\mathrm{d}x\mathrm{d}y, \tag{14}$$

$$Q(\chi)=\frac{2}{N_0}\iint\limits_{\Omega(\chi)} F^2(x,y)\mathrm{d}x\mathrm{d}y, \tag{15}$$

$$B(\chi_1,\chi_2)=\frac{2}{N_0}\iint\limits_{\Omega_{\min}(\chi_1,\chi_2)} F^2(x,y)\mathrm{d}x\mathrm{d}y. \tag{16}$$

For investigating the local behavior of functions (14)–(16) in small neighborhood of point $\widetilde{\chi}$ (10) we shall introduce the following auxiliary function:

$$\widetilde{S}(\chi_1,\chi_2)=\frac{2}{N_0}\iint\limits_{\Omega_{\min}[\chi_1,\chi_2]} \widetilde{F}(x,y)\mathrm{d}x\mathrm{d}y, \tag{17}$$

where $\widetilde{F}(x,y)\geq 0$ and function $\widetilde{F}(x,y)$ is limited in the entire observation domain *G*. Substituting appropriate functions in place of $\widetilde{F}(x,y)$ into expression (17), we can obtain expressions (14), (15), and (16). Let us designate

$$\Delta=\min[\chi_1-\widetilde{\chi},\chi_2-\widetilde{\chi}].$$

Then, in accordance with the definition

$$\Omega_{\min}(\chi_1,\chi_2)=\Omega(\widetilde{\chi}+\Delta)$$

expression (17) can be rewritten in the form:

$$\widetilde{S}(\chi_1,\chi_2)=\frac{2}{N_0}\iint\limits_{\Omega(\widetilde{\chi}+\Delta)} \widetilde{F}(x,y)\mathrm{d}x\mathrm{d}y. \tag{18}$$

Let us consider the asymptotic behavior of relationship (18) assuming that

$$\delta=\max\left[|\chi_1-\widetilde{\chi}|,|\chi_2-\widetilde{\chi}|\right]\to 0. \tag{19}$$

Then it is obvious that $\Delta\to 0$. Let us designate the domain of unit area having the shape of image area as Ω_E. Let us also assume that equations $x=f(\varphi)$ and $y=\psi(\varphi)$ of the boundary C_E limiting domain Ω_E $(0\leq\varphi\leq 2\pi)$ are specified. Then the equations of boundary $C(\chi)$ limiting domain $\Omega(\chi)$ have the form:

$$x=f(\varphi)\sqrt{\chi},\quad y=\psi(\varphi)\sqrt{\chi}. \tag{20}$$

Using expression (20) we can rewrite expression (18) in the polar coordinate system:

$$\widetilde{S}(\chi_1,\chi_2)=\frac{2}{N_0}\int\limits_0^{2\pi}\mathrm{d}\varphi\int\limits_0^{\sqrt{(\widetilde{\chi}+\Delta)(f^2(\varphi)+\psi^2(\varphi))}}\widetilde{F}(\rho\cos\varphi,\rho\sin\varphi)\rho d\rho. \tag{21}$$

Expanding expression (21) into power series in terms of Δ and limiting our consideration to the first two terms of the series, we obtain an asymptotic expansion for function (18) provided condition (19) is satisfied:

$$\widetilde{S}(\chi_1,\chi_2)=\widetilde{S}(\widetilde{\chi},\widetilde{\chi})+\widetilde{A}(\widetilde{\chi})\min[\chi_1-\widetilde{\chi},\chi_2-\widetilde{\chi}]+o(\delta), \tag{22}$$

where

$$\widetilde{A}(\widetilde{\chi})=\frac{1}{N_0}\int_0^{2\pi}\widetilde{F}\left(\sqrt{\widetilde{\chi}(f^2(\varphi)+\psi^2(\varphi))}\cos\varphi,\right.$$

$$\left.\sqrt{\widetilde{\chi}(f^2(\varphi)+\psi^2(\varphi))}\sin\varphi\right)(f^2(\varphi)+\psi^2(\varphi))\mathrm{d}\varphi$$

$$=\frac{1}{N_0\widetilde{\chi}}\int_{C(\widetilde{\chi})}\widetilde{F}(x,y)(x\mathrm{d}y-y\mathrm{d}x)=\frac{1}{N_0\widetilde{\chi}}\oint_{C(\widetilde{\chi})}\widetilde{F}(x,y)\sqrt{x^2+y^2}\mathrm{d}s, \tag{23}$$

$\widetilde{A}(\widetilde{\chi})$ is the coefficient, the value of which is determined by only values of function $\widetilde{F}(x,y)$ in expression (18) over boundary $C(\widetilde{\chi})$ limiting domain $\Omega(\widetilde{\chi})$ with area $\widetilde{\chi}$. Indeed, in accordance with expression (23) the value of coefficient $\widetilde{A}(\widetilde{\chi})$ does not depend on the values of function $\widetilde{F}(x,y)$ in expression (18), which it assumes in the internal points of domain $\Omega(\widetilde{\chi})$. Henceforth we shall limit ourselves with the use of main terms of asymptotic expansion (22). Then the following approximate expressions will be valid for signal component (7) and correlation function (11) of noise function (8) in the small neighborhood of point $\widetilde{\chi}$:

$$S(\chi)=S(\chi_0,\widetilde{\chi})-Q(\widetilde{\chi})/2+A_0(\widetilde{\chi})\min[\chi_0-\widetilde{\chi},\chi-\widetilde{\chi}]-A(\widetilde{\chi})(\chi-\widetilde{\chi})/2, \tag{24}$$

$$B(\chi_1,\chi_2)=B(\widetilde{\chi},\widetilde{\chi})+A(\widetilde{\chi})\min[\chi_1-\widetilde{\chi},\chi_2-\widetilde{\chi}], \tag{25}$$

where

$$A_0(\widetilde{\chi})=\frac{1}{N_0\widetilde{\chi}}\int_{C(\widetilde{\chi})}F_0(x,y)F(x,y)(x\mathrm{d}y-y\mathrm{d}x),$$

$$A(\widetilde{\chi})=\frac{1}{N_0\widetilde{\chi}}\int_{C(\widetilde{\chi})}F^2(x,y)(x\mathrm{d}y-y\mathrm{d}x). \tag{26}$$

In accordance with expression (25) the realization of noise function $N(\chi)$ (8) is continuous with unitary probability [7]. Hence, given the unlimited rise of SNR z (13), QLE (5) is a consistent estimate, if

$$\widetilde{\chi}=\chi_0. \tag{27}$$

Using asymptotic expansion (24) we can find the conditions under which QLE (5) is consistent and condition (27) is obeyed. From relationship (24) for the derivative of the signal component at point χ_0 on the right ($\chi\geq\chi_0$) we obtain:

$$\mathrm{d}S(\chi)/\mathrm{d}\chi\big|_{\chi_0+0}=-A(\chi_0)/2.$$

In accordance with expression (26) condition $A(\chi_0)>0$ is always fulfilled, hence signal component (7) in small neighborhood of χ_0 at $\chi\geq\chi_0$ is a decreasing function. For the derivative of signal component (7) at point χ_0 on the left, i.e. at $\chi\leq\chi_0$, we obtain from expression (24):

$$\mathrm{d}S(\chi)/\mathrm{d}\chi\big|_{\chi_0-0}=\rho,$$

where

$$\rho = \frac{1}{N_0 \chi_0} \int_{C(\chi_0)} F(x,y)\left\{F_0(x,y) - \frac{1}{2}F(x,y)\right\}(x\mathrm{d}y - y\mathrm{d}x). \tag{28}$$

If

$$\rho > 0 \tag{29}$$

signal component $S(\chi)$ (7) in small neighborhood of χ_0 at $\chi \le \chi_0$ is an increasing function that reaches its maximum at point χ_0. Thus expression (29) is a necessary and sufficient condition of QLE (5) consistency and then from condition (29) it follows that condition (27) is fulfilled. The test of necessary and sufficient condition (29) of QLE consistency involves the need of calculating the contour integral (28) that is not always an easy task. That is why we shall show another simple and easily tested sufficient condition of QLE consistency. It is obvious that condition (29) is always satisfied if

$$F_0(x,y) > F(x,y)/2, \quad (x,y) \in G.$$

If this condition is not satisfied, it is necessary to test the necessary and sufficient condition (29).

Let us realize the obtained general relationships for a particular case of uniform observed image

$$F_0(x,y) \equiv D_0 = \text{const} \tag{30}$$

and uniform expected image

$$F(x,y) \equiv D = \text{const}. \tag{31}$$

If conditions (30) and (31) are satisfied, approximate expressions (24) and (25) that are asymptotically exact in small neighborhood of point $\tilde{\chi}$ transform into exact formulas [1], while the necessary and sufficient condition of QLE consistency (29) assumes the form

$$D_0 > D/2. \tag{32}$$

Next we shall assume that condition (32) of QLE (5) consistency is obeyed and equality (27) holds. Then asymptotic expansions of signal component (24) and correlation function (25) of the noise function can be rewritten in the form:

$$S(\chi) = S(\chi_0, \chi_0) - \frac{Q(\chi_0)}{2} + (\chi - \chi_0)\begin{cases} a_1, \chi < \chi_0, \\ -a_2, \chi > \chi_0, \end{cases}$$

$$B(\chi_1, \chi_2) = B(\chi_0, \chi_0) + 2a_2 \min[\chi_1 - \chi_0, \chi_2 - \chi_0], \tag{33}$$

where

$$a_1 = \rho, \quad a_2 = A(\chi_0)/2. \tag{34}$$

In accordance with expression (33) the process described by expressions (4) and (6) in the small neighborhood of the true value of area χ_0 is Gaussian Markov random process [7]. In this neighborhood the drift and diffusion coefficients of process $L(\chi)$ have the form [7]:

$$K_1 = \begin{cases} a_1, \chi < \chi_0, \\ -a_2, \chi > \chi_0, \end{cases} \quad K_2 = 2a_2. \tag{35}$$

The established properties of signal component (7) and noise function (8) allow the local Markov approximation method [9] to be used for finding the characteristics of QLE χ_m (5). Using this method for solving the Fokker–Planck–Kolmogorov equation [8] with coefficients (35) we find asymptotically (with the rise of signal-to-noise ratio z) the exact expressions for the bias (systematic error) and dispersion (average squared error) of QLE (5):

$$b(\chi_m|\chi_0) = \langle \chi_m - \chi_0 \rangle = \frac{z_1^2(2R+1) - z_2^2 R(R+2)}{2z_1^2 z_2^2 (1+R)^2}, \tag{36}$$

$$V(\chi_m|\chi_0) = \langle (\chi_m - \chi_0)^2 \rangle = \frac{z_2^4 R(2R^2+6R+5) + z_1^4(5R^2+6R+2)}{2z_1^4 z_2^4 (1+R)^3}, \tag{37}$$

where $z_1^2 = a_1^2 / 2a_2$, $z_2^2 = a_2 / 2$, $R = a_2 / a_1$.

In expressions (36) and (37) averaging is performed at the fixed true value χ_0 of the estimated area. Therefore, these expressions determine the conditional bias and dispersion of the area QLE. The obtained results make it possible, as a particular case, to find the MLE $\hat{\chi}$ characteristics of area χ_0 of image (2). The realization of the maximum likelihood algorithm involves the need of a priori knowledge of image intensity distribution $F_0(x, y)$. Then, substituting $S(x, y, \chi)$ (3) with $S_0(x, y, \chi)$ (2) in expression (4) we can obtain MLE $\hat{\chi}$ from expression (5). In a similar way the MLE characteristics can be determined by assuming $F(x, y) \equiv F_0(x, y)$ in expressions (36) and (37). As a result for the bias and dispersion of area MLE of non-uniform image we get:

$$b(\hat{\chi}|\chi_0) = 0,$$

$$V(\hat{\chi}|\chi_0) = 26 / A_0^2, \tag{38}$$

where similar to expression (23)

$$A_0 = \frac{1}{N_0} \int_0^{2\pi} F_0^2\left(\sqrt{\chi_0 (f^2(\varphi) + \psi^2(\varphi))} \cos\varphi, \right.$$

$$\left. \sqrt{\chi_0 (f^2(\varphi) + \psi^2(\varphi))} \sin\varphi \right) (f^2(\varphi) + \psi^2(\varphi)) \mathrm{d}\varphi$$

$$= \frac{1}{N_0 \chi_0} \int_{C(\chi_0)} F_0^2(x, y)(x\mathrm{d}y - y\mathrm{d}x) = \frac{1}{N_0 \chi_0} \oint_{C(\chi_0)} F_0^2(x, y) \sqrt{x^2 + y^2} \mathrm{d}s. \tag{39}$$

As follows from expression (2), the non-uniform image intensity over boundary $C(\chi_0)$ limiting domain $\Omega(\chi_0)$ occupied by image undergoes a jump from value $F_0(x_c, y_c)$ on the internal side of boundary $C(\chi_0)$ to zero on the external side of the boundary. Here x_c and y_c are the coordinates of the point belonging to boundary $C(\chi_0)$. Formulas (38) and (39) indicate that the dispersion of area MLE depends only on the contour integral over $C(\chi_0)$ that "summates" the values of squared intensity jump during the passage through boundary $C(\chi_0)$ over all points of this boundary.

Correspondingly, the asymptotic value of dispersion (38) of area MLE does not depend on the values of image intensity $F_0(x, y)$ assumed inside boundary $C(\chi_0)$, i.e. at internal points of domain $\Omega(\chi_0)$. Hence, it follows in particular that the selection of the expected (predicted) intensity distribution $F(x, y)$ used for obtaining the area QLE should be performed in a way ensuring the maximum closeness of this function to the true intensity $F_0(x,y)$ over boundary $C(\chi_0)$ limiting the image.

As an example we shall consider the estimation of area of the image with linearly changing intensity. Let the image domain represent a circle with the center at the origin of coordinates and radius $\sqrt{\chi_0/\pi}$. Let us also assume that the intensity of observed image changes linearly along the x-axis, so that

$$F_0(x,y) = \sqrt{\frac{E_0}{\chi_0}\frac{1}{\left\{\frac{(1-q_0)^2}{16}+\frac{(1+q_0)^2}{4}\right\}}}\left(\frac{(1-q_0)}{2\sqrt{\frac{\chi_0}{\pi}}}x+\frac{(1+q_0)}{2}\right), \tag{40}$$

where $E_0 = \iint_{\Omega(\chi_0)} F_0^2(x,y)\mathrm{d}x\mathrm{d}y$ is the energy of observed image, parameter

$$q_0 = \frac{b_0}{a_0} \tag{41}$$

characterizes the slope of the function specifying the intensity,

$$a_0 = F_0(\sqrt{\chi_0/\pi}, 0),$$

$$b_0 = F_0(-\sqrt{\chi_0/\pi}, 0).$$

We assume that the expected image has the following intensity:

$$F(x,y) = \sqrt{\frac{E_0}{\chi_0}\frac{1}{\left\{\frac{(1-q)^2}{16}+\frac{(1+q)^2}{4}\right\}}}\left(\frac{(1-q)}{2\sqrt{\frac{\chi_0}{\pi}}}x+\frac{(1+q)}{2}\right),$$

that is different form the intensity of observed image only by its parameter q. Let us consider the effect of the difference between the values of parameters q_0 and q on the estimation accuracy of circle area with linearly changing intensity. Substituting expressions (40) and (41) into (34) and (39) we find:

$$a_1 = \frac{2}{N_0}\frac{E_0}{\chi_0}\frac{\left\{\frac{1}{8}(1-q_0)(1-q)+\frac{1}{4}(1+q_0)(1+q)\right\}}{\sqrt{\left\{\frac{(1-q_0)^2}{16}+\frac{(1+q_0)^2}{4}\right\}\left\{\frac{(1-q)^2}{16}+\frac{(1+q)^2}{4}\right\}}} - \frac{1}{N_0}\frac{E_0}{\chi_0}\frac{\left\{\frac{(1-q)^2}{8}+\frac{1}{4}(1+q)^2\right\}}{\left\{\frac{(1-q)^2}{16}+\frac{(1+q)^2}{4}\right\}},$$

$$a_2 = \frac{1}{N_0}\frac{E((1-q)^2/8+(1+q)^2/4)}{\chi_0((1-q)^2/16+(1+q)^2/4)}, \tag{42}$$

$$A_0 = \frac{2}{N_0}\frac{E((1-q_0)^2/8+(1+q_0)^2/4)}{\chi_0((1-q_0)^2/16+(1+q_0)^2/4)}. \tag{43}$$

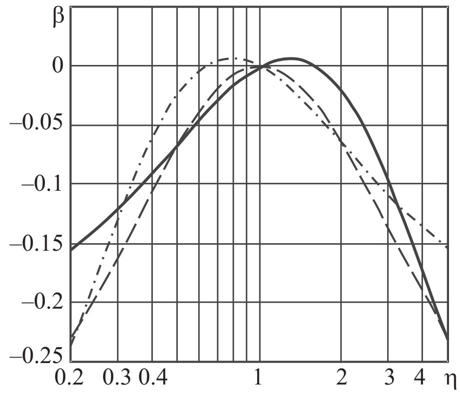

Fig. 1.

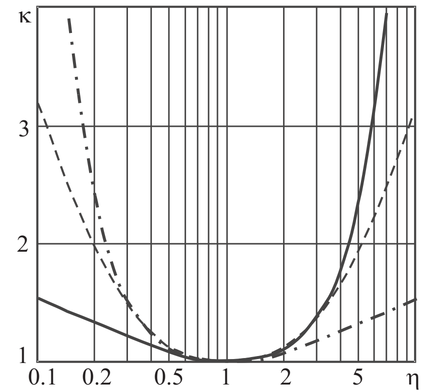

Fig. 2.

Substituting expression (42) into (36) and (37) we find characteristics of the area QLE. Next, substituting expression (43) into (38) we obtain the MLE characteristics of the circle area:

$$b(\hat{\chi}|\chi_0)=0, \quad V(\hat{\chi}|\chi_0)=\frac{26}{A_0^2}.$$

Figure 1 presents the relationship of the normalized bias

$$\beta(\eta)=\frac{b(\chi_m|\chi_0)}{\sqrt{V(\chi_m|\chi_0)}}$$

of the area QLE of the image with linearly changing intensity as a function of parameter $\eta = q/q_0$ at different values of q_0. Solid curve corresponds to value $q_0 = 0.5$; dashed line corresponds to $q_0 = 1$, and dash-dotted line corresponds to $q_0 = 2$. Figure 2 presents the relationship of loss $\kappa(\eta)=V(\chi_m|\chi_0)/V(\hat{\chi}|\chi_0)$ in the accuracy of QLE as compared with the accuracy of area MLE of image with linearly changing intensity. Designations of curves in Fig. 2 are the same as in Fig. 1.

From the analysis of curves in Figs. 1, 2 it follows that at $0.5 < \eta < 2$, i.e. $0.5q_0 < q < 2q_0$ QLE is practically unbiased, while its dispersion is only slightly higher than the MLE dispersion. Therefore, the area QLE of the image with linearly changing intensity is not actually inferior in terms of accuracy to MLE if parameter q of the expected image is different from the true value of parameter q_0 of the observed image by a factor of no more than two.

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