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ANALYSIS AND SYNTHESIS OF SIGNALS AND IMAGES

Efficiency of Optimal Joint Detection and Estimation of Image Areas with Spatial Noise

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Abstract— Bayesian and maximum likelihood algorithms are considered. A comparative analysis of their effectiveness is performed. The results are specified for an image in the shape of an ellipse with linearly varying intensity.

Keywords: maximum likelihood algorithm, Bayesian algorithm, characteristics of algorithms, statistical modeling.

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INTRODUCTION

Recently, the problem of optimal image processing becomes more and more urgent. This is due not only to the extension of the range of image processing tasks, but also to the increasing performance of computing means, which allow implementing increasingly complex algorithms. In most tasks, it is required to obtain the maximum achievable performance of algorithms. Tasks of nonuniform image processing with unknown area with incomplete prior information about the image parameters are considered in [1–4]. The optimal use of complete a priori information can improve the performance of algorithms for joint detection and estimation of image areas. Therefore, it is of interest to synthesize and analyze optimal algorithms of image processing in the presence of full a priori information on the image parameters.

Assume that in the region G, there is a random-field realization:

$$\xi(x,y)=\gamma_0 s(x,y;\chi_0)+n(x,y). \tag{1}$$

Here

$$s(x,y;\chi_0)=F(x,y)I(x,y;\chi_0) \tag{2}$$

is a useful image with intensity $F(x,y)$ which occupies a region $\Omega(\chi_0)$ of area χ_0. The shape of the region occupied by the image is described by the indicator $I(x,y;\chi)=1$ for $x,y\in\Omega$ and $I(x,y;\chi)=0$ for $x,y\notin\Omega$. In (1), $n(x,y)$ is Gaussian spatial white noise with one-sided spectral density N_0, the unknown area of the image χ_0 takes values from the a priori interval $[\chi_{\min},\chi_{\max}]$, and $\gamma_0=1$ or $\gamma_0=0$ is a discrete parameter that reflects the presence or absence of a useful image.

IMAGE PROCESSING ALGORITHMS

In analyzing the random-field realization (1), it is necessary to synthesize various image processing algorithms (2). Trifonov and Zimovets [1] considered the problem of detecting a nonuniform image of unknown area and synthesized the image detection algorithm (2) ignoring the accuracy of estimation of the unknown area χ_0. Trifonov and Zimovets [2] synthesized and analyzed an algorithm for estimating the area χ_0 of a nonuniform image provided that it is always present in the accepted realization ($\gamma_0 \equiv 1$). The most common case of image processing is presented in [3], where the synthesis and analysis of an algorithm for joint detection and estimation of the areas of nonuniform images are described.

Image processing algorithms (2) synthesized by the method of maximum likelihood were considered in [1–4]. In accordance with this method, image processing (2) requires forming the logarithm of the likelihood ratio functional (LRF)

$$L(\chi) = \frac{2}{N_0} \iint\limits_G \xi(x, y) s(x, y; \chi) dx dy - \frac{1}{N_0} \iint\limits_G s^2(x, y; \chi) dx dy, \quad \chi \in [\chi_{\min}, \chi_{\max}]. \tag{3}$$

The problem of detecting the image (2) discussed in [1] involves making a decision $\hat{\gamma}$ on the presence of the image in the accepted realization of (1) ($\hat{\gamma} = 1$) or its absence ($\hat{\gamma} = 0$). In this case, the algorithm for estimating the parameter γ is written as

$$\sup_{\chi \in [\chi_{\min}, \chi_{\max}]} L(\chi) \underset{\hat{\gamma}=0}{\overset{\hat{\gamma}=1}{\gtrless}} h, \tag{4}$$

where h is the given threshold. In the case where the image is permanently present in the accepted realization (1) ($\gamma_0 \equiv 1$), the maximum likelihood estimate $\hat{\chi}$ of the area χ_0 of the useful image is defined as the position of the absolute maximum $L(\chi)$ [2]:

$$\hat{\chi} = \arg \sup_{\chi \in [\chi_{\min}, \chi_{\max}]} L(\chi). \tag{5}$$

If it is necessary not only to make a decision on the presence or absence of the image but also to estimate its size, it is better to use the maximum likelihood algorithm of image processing [3], which involves the joint use of the two decision rules

$$\hat{\gamma} = \begin{cases} 1, & L(\tilde{\chi}) \geq h, \\ 0, & L(\tilde{\chi}) < h; \end{cases} \qquad \hat{\chi} = \begin{cases} \tilde{\chi}, & L(\tilde{\chi}) \geq h, \\ 0, & L(\tilde{\chi}) < h, \end{cases} \tag{6}$$

where $\tilde{\chi} = \arg\sup L(\chi)$. In contrast to (5), the value of $\tilde{\chi}$ is defined for an arbitrary value of γ_0 in (1). Using the classical method of maximum likelihood, the threshold in the detection algorithm (4) and the joint detection and estimation algorithm (6) should be chosen to be zero ($h = 0$).

It is well known that, in the class of linear estimates, the maximum likelihood algorithm provides an effective estimate of the measured parameter. It has been shown [5] that, in the estimation of regular parameters, the maximum likelihood method is asymptotically efficient. In the image model (2), the unknown area χ_0 is a discontinuous parameter. In this case, the question of optimality of the maximum likelihood method remains open. Therefore, of interest is to study image processing algorithms synthesized on the basis of the Bayesian approach.

If the a priori probability p_1 of the presence of the image in the accepted realization (1) and the a priori probability density $W_{pr}(\chi)$ of the image area (2) are known, the algorithm for processing the image (2) can be synthesized using the Bayesian approach, which allows minimizing the average risk for a given loss function. Thus, it provides strict optimality under the given criterion [5].

For image detection, an important characteristic of the processing algorithm is the average detection-error probability. Using a simple loss function in which the cost of correct decisions is zero and the costs of first and second kind errors are identical for synthesis of a Bayesian detection algorithm allows us to minimize the average detection-error probability, which coincides with the average risk value. Then, the algorithm for detecting the image present with probability p_1 in the accepted realization (1) of unknown area χ_0 distributed with a priori probability density $W_{pr}(\chi_0)$ in the region $[\chi_{\min}, \chi_{\max}]$ is given by

$$L \underset{\hat{\gamma}_{\mathrm{B}}=0}{\overset{\hat{\gamma}_{\mathrm{B}}=1}{\gtrless}} C, \tag{7}$$

where $L = \int\limits_{\chi_{\min}}^{\chi_{\max}} W_{pr}(\chi) \exp[L(\chi)] d\chi$; $C = (1 - p_1)/p_1$. Thus, the decision on the presence of the image in the observed data realization (1) is made if $\hat{\gamma}_{\mathrm{B}} = 1$, and the decision on its absenceis made if $\hat{\gamma}_{\mathrm{B}} = 0$. One of the main characteristics of the algorithm for estimating the image (2), provided that the image is always present in the accepted realization (1) ($\gamma_0 \equiv 1$), is the variance of the estimate. The use of the quadratic loss function

$$C(\chi, \chi_0) = (\chi - \chi_0)^2$$

for the synthesis of the Bayesian algorithm of area estimation makes it possible to obtain an estimate with minimal variance. Also it should be noted that the estimate

$$\hat{\chi}_{\mathrm{B}} = \int\limits_{\chi_{\min}}^{\chi_{\max}} \chi \exp[L(\chi)] W_{pr}(\chi) d\chi \Big/ \int\limits_{\chi_{\min}}^{\chi_{\max}} \exp[L(\chi)] W_{pr}(\chi) d\chi, \tag{8}$$

obtained using the Bayesian approach and the quadratic loss function is certainly unbiased. More generally, when it is required not only to detect the image but to estimate its size, the choice of loss function is quite difficult. This is because it is necessary to simultaneously estimate two parameters and make a choice between the accuracies of estimation of each of them. In this case, it is useful to apply an additive loss function that is quadratic in the estimated parameter:

$$\Pi = \begin{Vmatrix} 1 - C_0(1 - g_0\hat{\chi}^2) & 1 \\ 1 & 1 - C_1(1 - g_1(\hat{\chi} - \chi)^2) \end{Vmatrix}, \tag{9}$$

where C_0, C_1, g_0, and g_1 are some constants.

The Bayesian algorithm for joint detection and estimation of the image area synthesized using the loss function (9) takes the form

$$\hat{\gamma}_{\mathrm{B}} = \begin{cases} 1, & d_0 \ge C, \\ 0, & d_0 < C; \end{cases} \qquad \hat{\chi}_{\mathrm{B}} = \begin{cases} \tilde{\chi}, & d_0 \ge C, \\ 0, & d_0 < C, \end{cases} \tag{10}$$

where

$$\hat{\chi} = \frac{d_1}{d_0}; \quad C = \frac{p_0 C_0}{p_1 C_1} \Big(1 - \frac{g_1}{d_0} \int\limits_{\chi_{\min}}^{\chi_{\max}} \Big(\frac{d_1}{d_0} - \chi\Big)^2 W_{pr}(\chi) \exp[L(\chi)] d\chi\Big)^{-1};$$

$$d_k = \int\limits_{\chi_{\min}}^{\chi_{\max}} \chi^k W_{pr}(\chi) \exp[L(\chi)] d\chi.$$

Qualitative analysis of the structure of these algorithms leads to the conclusion that the algorithms synthesized on the basis of the Bayesian approach are notably more complicated in hardware or software implementation in comparison with the algorithms synthesized using the maximum likelihood method. The decision to use algorithms (7), (8), and (10) for image processing can be made only after analyzing the quantitative indicators of their effectiveness. The effectiveness of the detection algorithm (4) is characterized by the average error probability

$$P_e = (1 - p_1)\alpha + p_1\beta, \tag{11}$$

where α is the probability of an error of the first kind (false alarm) and $\beta = \int_{\chi_{\min}}^{\chi_{\max}} \beta(\chi_0) W_{pr}(\chi_0) d\chi_0$ is the absolute probability of an error of the second kind (non-detection). The exact expressions for α and the conditional probability of the second-kind error $\beta(\chi_0)$ are obtained in [1]. The main characteristic describing the area estimation accuracy obtained using algorithm (5) is the absolute scattering

$$V(\hat{\chi}) = \int\limits_{\chi_{\min}}^{\chi_{\max}} V(\hat{\chi} \mid \chi) W_{pr}(\chi) d\chi. \tag{12}$$

The exact expression for the conditional variance $V(\hat{\chi} \mid \chi_0)$ is taken from [2]. In the joint detection and estimation algorithm, it is necessary to consider the accuracy of estimates for two parameters at once. The accuracy of estimates of the parameters γ and χ is characterized by the average error probability (11) and the absolute variance

$$V(\hat{\chi}) = p_0 V(\tilde{\chi} \mid 0) + p_1 \int_{\chi_{\min}}^{\chi_{\max}} [V(\tilde{\chi} \mid \chi) + \chi^2 \beta(\chi)] W_{pr}(\chi) d\chi, \tag{13}$$

where $V(\tilde{\chi} \mid 0)$ is the variance of the area estimation in the absence of the image, and for the absent image, $\chi_0 \equiv 0$, and $V(\tilde{\chi} \mid \chi_0)$ is the variance of the area estimate in the presence of the image. The exact expression for $V(\tilde{\chi} \mid 0)$, $V(\tilde{\chi} \mid \chi_0)$, α, and $\beta(\chi_0)$ were obtained in [3]. It should be noted that the efficiency of algorithms (4) and (6) may be improved in the presence of complete a priori information. In this case, the structure of the algorithm remains unchanged. Only the threshold h changes, which in algorithm (4) can be chosen so as to minimize the average error probability:

$$h_P = \arg\inf_h P_e,$$

and in algorithm (6), it is chosen so as to minimize the estimate variance:

$$h_m = \arg\inf_h V(\hat{\chi}).$$

Unlike in algorithms (4)–(6), obtaining the characteristics of algorithms (7), (8), and (10) synthesized using the Bayesian approach is analytically impossible. Therefore, a quantitative comparison of the effectiveness of the synthesized algorithms requires statistical modeling of algorithms (7), (8), and (10).

RESULTS OF STATISTICAL MODELING

Statistical simulation of algorithms requires forming the logarithm of the LRF $L(\chi)$ (3). We substitute the random-field realization (1) in the expression for $L(\chi)$; then, $L(\chi)$ is of the form

$$L(\chi) = \gamma_0 S(\chi, \chi_0) - Q(\chi)/2 + N(\chi). \tag{14}$$

Here

$$N(\chi) = \frac{2}{N_0} \iint_G n(x, y) s(x, y; \chi) dx dy$$

is the noise function;

$$Q(\chi) = \frac{2}{N_0} \iint_G s^2(x, y; \chi) dx dy$$

is the signal/noise ratio for the images of area χ;

$$S(\chi_0, \chi) = \frac{2}{N_0} \iint_G s(x, y; \chi_0) s(x, y; \chi) dx dy = \min[Q(\chi), Q(\chi_0)]$$

is the signal function.

Passing to the normalized variable $\eta = \chi/\chi_{\max}$ in (14) and considering that $N(\eta)$ is a Gaussian Markov process with zero mathematic expectation and the correlation function $\langle N(\eta_1) N(\eta_2) \rangle = \min[\tilde{Q}(\eta_1), \tilde{Q}(\eta_2)]$, where $\tilde{Q}(\eta) \equiv Q(\eta \chi_{\max})$, we represent this process as

$$N(\eta) = \int_0^{\tilde{Q}(\eta)} n(x) dx.$$

Here, $n(x)$ is Gaussian white noise with zero mathematical expectation and the correlation function $\langle n(x_1)n(x_2)\rangle = \delta(x_1 - x_2)$. In modeling with a step $\Delta\eta$, we generated samples of the function $N(\eta)$ on the basis of which the realization of the logarithm of the LRF $L(\eta)$ was approximated by a step function with maximum relative RMS error $\Delta = 0.02$. According to [6], the discrete samples of the logarithm of the LRF can be written as

$$L(\eta_{\min} + i\Delta\eta) = \gamma_0 S(\eta_{\min} + i\Delta\eta, \eta_0) +$$

$$+ \sum_{j=1}^{i} \sqrt{\tilde{Q}(\eta_{\min} + j\Delta\eta) - \tilde{Q}(\eta_{\min} + (j-1)\Delta\eta)}\, n_j + \sqrt{\tilde{Q}(\eta_{\min})}\, n_0 - \tilde{Q}(\eta_{\min} + i\Delta\eta)/2,$$

where n_j are independent Gaussian random variables with zero mathematical expectation and single variances; $i = \overline{0, N_{\max}}$ ($N_{\max} = \mathrm{ent}((1 - \eta_{\min})/\Delta\eta)$, $\mathrm{ent}(\cdot)$ is the integer part of the number); $\Delta\eta = \eta_{\min}\Delta^2$.

The general results obtained were specified for an image in the form of an ellipse whose intensity varies linearly along the x axis. The equation of the contour limiting the region $\Omega(\chi)$ occupied by an image of area χ is given by

$$x^2/a^2 + y^2/b^2 = \chi. \tag{15}$$

Here $ab = 1/\pi$. If we denote the eccentricity of the ellipse (15) by $\varepsilon = \sqrt{a^2 - b^2}/a$, the image intensity can be written as

$$F(x, y) = \frac{4F_H}{\sqrt{(1-q)^2 + 4(1+q)^2}} \Big[\frac{(1-q)\sqrt{\pi}(1-\varepsilon^2)^{1/4}}{2\sqrt{\chi_{\max}}} x + \frac{1+q}{2}\Big], \tag{16}$$

where the parameter F_H characterizes the amplitude of the intensity, the parameter $q = \dfrac{F(-a_{\max}, 0)}{F(a_{\max}, 0)}$ defines the slope of the image intensity, and the parameter $a_{\max}$ defines the large semiaxis of the ellipse of the maximum area $\chi_{\max}$. The function (16) describing the intensity of the image is normalized so that the energy of the image with maximum area $E_{\max} = \iint\limits_{\Omega(\chi_{\max})} F^2(x, y)\,dx\,dy = F_H^2 \chi_{\max}$ is independent fn the slope of the image intensity. This independence makes it possible to compare the efficiency of image processing algorithms with different intensity slopes. Substituting (16) into the expression for $\tilde{Q}(\eta)$, we find the signal/noise ratio:

$$\tilde{Q}(\eta) = z_H^2 \Big[\frac{(1-q)^2}{16}\eta^2 + \frac{(1+q)^2}{4}\eta\Big] \Big/ \Big[\frac{(1-q)^2}{16} + \frac{(1+q)^2}{4}\Big], \tag{17}$$

where $\eta = \chi/\chi_{\max}$ is the normalized area, $\eta \in [1/g; 1]$; $g = \chi_{\max}/\chi_{\min}$ is the dynamic range of the unknown area; $z_H^2 = 2F_H^2\chi_{\max}/N_0 = 2E_{\max}/N_0$ is the signal/noise ratio for a uniform image with intensity F_H and area $\chi_{\max}$.

In the process of modeling, the a priori probability density $W_{pr}(\chi)$ of the unknown area was assumed to be uniform:

$$W_{pr}(\chi) = \begin{cases} 1/(\chi_{\max} - \chi_{\min}), & \chi_{\min} \le \chi \le \chi_{\max}, \\ 0, & \chi < \chi_{\min},\ \chi > \chi_{\max}. \end{cases}$$

Therefore, in the presence of the image ($\gamma_0 = 1$), the true value of the normalized space η_0 was chosen to be random and uniformly distributed on the interval $[1/g; 1]$. The parameter γ_0 was also chosen to be random and taking the values 0 and 1 with probabilities $(1 - p_1)$ and p_1 respectively. In the modeling process, we implemented $5 \cdot 10^4$ test cycles for each z_H.

Figures 1–4 show the performance characteristics of the algorithms for processing the nonuniform ellipse-shaped image (15) with linearly varying intensity ($q = 10$) depending on the signal/noise ratio z_H. The dynamic range of the selected area is $g = 5$. Dependences of the average error probability P_e (11) on the signal/noise ratio z_H for the detection algorithms are shown in Fig. 1 and for the joint detection and

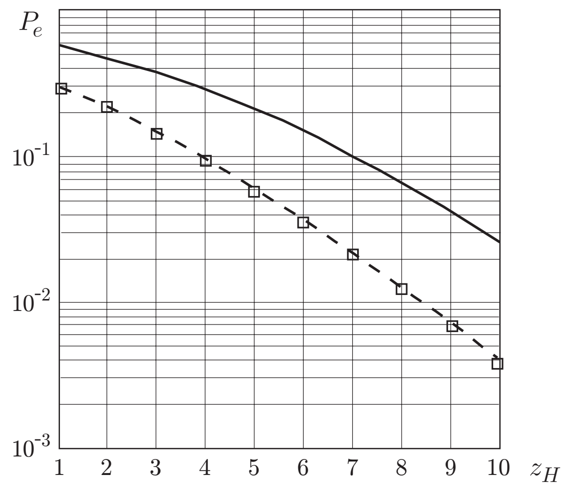

Fig. 1

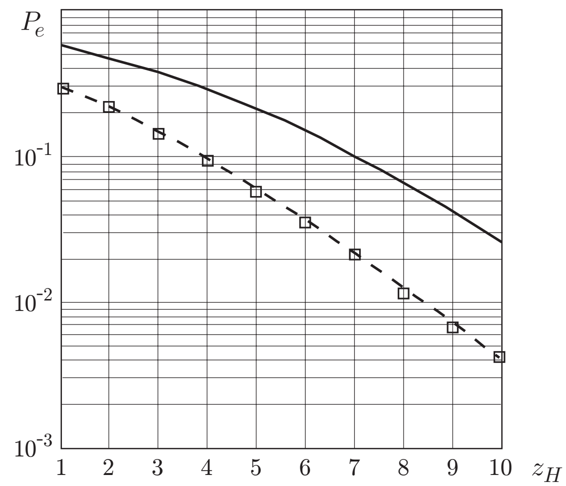

Fig. 2

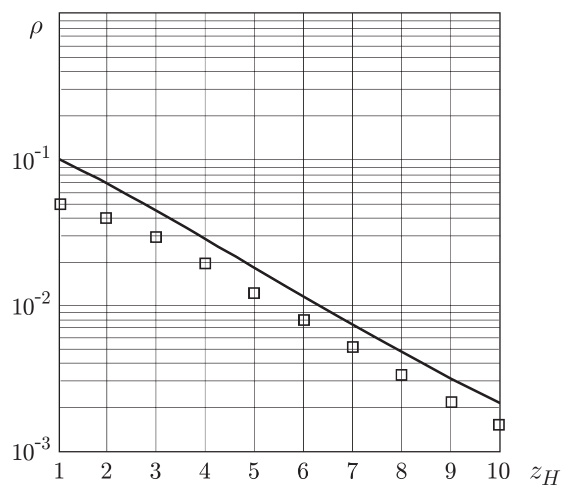

Fig. 3

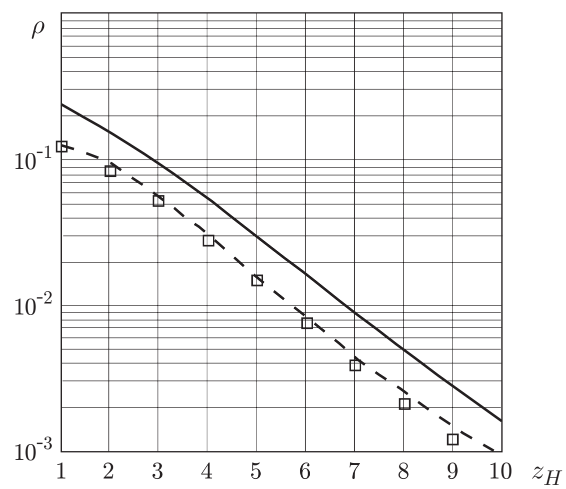

Fig. 4

estimation algorithms in Fig. 2. Dependences of the normalized variance $\rho = V(\hat{\chi})/\chi^2_{\max}$ (12) and (13) on the signal/noise ratio z_H for the estimation algorithms are shown in Fig. 3 and for the joint detection and estimation algorithms in Fig. 4. Solid curves in the figures correspond to the classical maximum likelihood algorithms ($h = 0$, dashed curves to the maximum likelihood algorithms with optimized threshold, and squares show the results of modeling the Bayesian algorithms.

CONCLUSIONS

With the same amount of a priori information and uniform a priori distribution of the unknown area, the characteristics of the Bayesian and maximum likelihood algorithms for detection and joint detection and estimation almost coincide. Hence, instead of the relatively complex Bayesian algorithms, one can use the simpler maximum likelihood algorithms with optimized threshold. In estimating nonuniform image areas in the case where an image is always present in the accepted realization, the use of the Bayesian approach provides a gain in estimation accuracy.

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