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Effect of Noninformative Parameters on the Estimation Efficiency of Target Motion upon Sensing with an Optical Pulse Train

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Abstract—Distance, velocity, and acceleration are efficiently estimated using a finite number of arbitrary unknown noninformative parameters. A decrease in the estimation accuracy due to the presence of noninformative parameters is determined.

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INTRODUCTION

Optical location systems widely employ optical pulse trains [1–4]. A potential estimation accuracy of distance, velocity and acceleration was analyzed in [4] on the assumption that all of the parameters of the pulse train scattered by the target are a priori unknown except for the estimated parameters. Under real conditions, the fluctuations of a target and the physical effects related to the scattering and propagation of light in various media lead to the scenario in which the intensity of single optical pulses may depend on a finite number of unknown noninformative parameters that need not to be estimated [5]. However, such parameters may affect the estimation accuracy of informative parameters (distance, velocity, and acceleration). Thus, we consider the estimation efficiency of distance, velocity, and acceleration in the presence of a finite number of noninformative parameters of the scattered pulse train.

We assume that the intensity of an optical pulse train is given by

$$s_N(t) = \sum_{k=0}^{N-1} s_0\left(t - (k-\mu)\theta - \lambda\right), \quad (1)$$

where $s_0(t)$ is the intensity function of a single optical pulse, λ is the time position of the train, and θ is the pulse repetition period. Parameter μ determines the point of the train that is related to time position λ. In particular, λ is the position of the first pulse if $\mu = 0$ and the position of the center of the train if

$$\mu = (N-1)/2 \quad (2)$$

When $\mu = N-1$, λ is the position of the last pulse.

If pulse train (2) is scattered by a target whose parameters (distance R_0, velocity V_0, and acceleration A_0) must be estimated, the signal intensity is represented as

$$s(t, R_0, V_0, A_0, \vec{l}_0) = \sum_{k=0}^{N-1} s\left[t - 2R_0/c - (k-\mu) \times (1 + 2V_0/c)\theta - A_0(k-\mu)^2\theta^2/c, \vec{l}_0\right]. \quad (3)$$

Here, $s(t, \vec{l}_0)$ is the intensity function of a single pulse in scattered train (3), which differs, in general, from intensity $s_0(t)$ in expression (1); $\vec{l}_0 = \| l_{01}, \ldots, l_{0p} \|$ is a vector of p noninformative parameters that are unknown due to the fluctuations of the target and propagation medium; and c is the velocity of light. Note that the following inequalities are satisfied: $|V_0| \ll c$ and $N\theta|A_0| \ll c$. Hereafter, zero subscripts denote true values of the unknown parameters of the received optical pulse train with intensity (3).

The signal with intensity (3) is observed over time interval $[0, T]$ in the presence of optical noise with intensity ν. Hence, we process a sample of the Poisson process with intensity

$$\beta(t, R_0, V_0, A_0, \vec{l}_0) = s(t, R_0, V_0, A_0, \vec{l}_0) + \nu. \quad (4)$$

The analysis of [3] shows that the calculation of the potential accuracy of the joint estimate for the unknown parameters of an optical pulse train with intensity (3) must employ the ambiguity function

$$H\left(R_1, R_2, V_1, V_2, A_1, A_2, \vec{l}_1, \vec{l}_2\right) = \int_0^T \beta\left(t, R_0, V_0, A_0, \vec{l}_0\right) \ln\left[\frac{\beta\left(t, R_1, V_1, A_1, \vec{l}_1\right)}{\nu}\right] \times \ln\left[\frac{\beta\left(t, R_2, V_2, A_2, \vec{l}_2\right)}{\nu}\right] dt. \quad (5)$$

We assume that observation interval $[0, T]$ is longer than the optical pulse train ($T > N\theta$) and the on-off ratio of pulse train (3) is no less than 2, so that the pulses are not overlapped. We substitute expression (3) in expression (4) and expression (4) in expression (5) to obtain

$$H(R_1, R_2, V_1, V_2, A_1, A_2, \vec{l}_1, \vec{l}_2) = \sum_{k=0}^{N-1} H_k(R_1, R_2, V_1, V_2, A_1, A_2, \vec{l}_1, \vec{l}_2), \tag{6}$$

where

$$H_k\left(R_1, R_2, V_1, V_2, A_1, A_2, \vec{l}_1, \vec{l}_2\right) = \int_0^T \{\nu + s[t - 2R_0/c - (k-\mu)\theta(1 + 2V_0/c) - A_0(k-\mu)^2\theta^2/c, \vec{l}_0]\} \times \ln\{1 + s[t - 2R_1/c - (k-\mu)\theta(1 + 2V_1/c) - A_1(k-\mu)^2\theta^2/c, \vec{l}_1]/\nu\} \times \ln\{1 + s[t - 2R_2/c - (k-\mu)\theta(1 + 2V_2/c) - A_2(k-\mu)^2\theta^2/c, \vec{l}_2]/\nu\} dt. \tag{7}$$

We consider a regular scenario [6] in which the intensities of single optical pulses are differentiable with respect to t and all of parameters l_i ($i = 1, \ldots, p$). In this case, the potential estimation accuracy of all unknown parameters for the optical pulse train with intensity (3) is characterized by the correlation matrix of the joint efficient estimates [3, 6]

$$\mathbf{K}_p = \mathbf{I}^{-1}. \tag{8}$$

Here, $\mathbf{I}$ is the Fisher information matrix [3] that is represented as a block matrix:

$$\mathbf{I} = \begin{Vmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^{\mathrm{T}} & \mathbf{D} \end{Vmatrix}, \tag{9}$$

where

$$\mathbf{A} = \begin{Vmatrix} \frac{\partial^2 H}{\partial R_1 \partial R_2} & \frac{\partial^2 H}{\partial R_1 \partial V_2} & \frac{\partial^2 H}{\partial R_1 \partial A_2} \\ \frac{\partial^2 H}{\partial V_1 \partial R_2} & \frac{\partial^2 H}{\partial V_1 \partial V_2} & \frac{\partial^2 H}{\partial V_1 \partial A_2} \\ \frac{\partial^2 H}{\partial A_1 \partial R_2} & \frac{\partial^2 H}{\partial A_1 \partial V_2} & \frac{\partial^2 H}{\partial A_1 \partial A_2} \end{Vmatrix}, \quad \mathbf{B} = \begin{Vmatrix} \vec{B}_1 \\ \vec{B}_2 \\ \vec{B}_3 \end{Vmatrix},$$

$$\vec{B}_1 = \left\Vert \frac{\partial^2 H}{\partial R_1 \partial l_{2i}} \right\Vert, \quad \vec{B}_2 = \left\Vert \frac{\partial^2 H}{\partial V_1 \partial l_{2i}} \right\Vert, \quad \vec{B}_3 = \left\Vert \frac{\partial^2 H}{\partial A_1 \partial l_{2i}} \right\Vert,$$

$$\mathbf{D} = \left\Vert \frac{\partial^2 H}{\partial l_{1i} \partial l_{2j}} \right\Vert.$$

Here, $\vec{B}_1$, $\vec{B}_2$, and $\vec{B}_3$ are row vectors and T denotes transposition. The derivatives of ambiguity function (6) in formulas (9) are calculated at $R_1 = R_2 = R_0$, $V_1 = V_2 = V_0$, $A_1 = A_2 = A_0$, and $\vec{l}_1 = \vec{l}_2 = \vec{l}_0$. Substituting expression (7) in expression (6) and expression (6) in expression (9), we derive

$$\mathbf{A} = \frac{\alpha}{c^2} \begin{Vmatrix} 4M_0 & 4\theta M_1 & 2\theta^2 M_2 \\ 4\theta M_1 & 4\theta^2 M_2 & 2\theta^3 M_3 \\ 2\theta^2 M_2 & 2\theta^3 M_3 & \theta^4 M_4 \end{Vmatrix}, \quad \mathbf{D} = M_0 \Vert D_{ij} \Vert,$$

$$\vec{B}_1 = -\frac{2}{c} M_0 \Vert \beta_i \Vert, \quad \vec{B}_2 = -\frac{2\theta}{c} M_1 \Vert \beta_i \Vert,$$

$$\vec{B}_3 = -\frac{\theta^2}{c} M_2 \Vert \beta_i \Vert,$$

$$\alpha = \int_0^T \frac{1}{\nu + s(t, \vec{l}_0)} \left[\frac{\partial s(t, \vec{l}_0)}{\partial t} \right]^2 dt, \quad M_n = \sum_{k=0}^{N-1} (k-\mu)^n, \tag{10}$$

$$\beta_i = \int_0^T \frac{1}{\nu + s(t, \vec{l}_0)} \left[\frac{\partial s(t, \vec{l})}{\partial t} \frac{\partial s(t, \vec{l})}{\partial l_i} \right]_{\vec{l} = \vec{l}_0} dt,$$

$$D_{ij} = \int_0^T \frac{1}{\nu + s(t, \vec{l}_0)} \left[\frac{\partial s(t, \vec{l})}{\partial l_i} \frac{\partial s(t, \vec{l})}{\partial l_j} \right]_{\vec{l} = \vec{l}_0} dt, \quad i, j = 1, \ldots, p.$$

We assume that unknown parameters $\vec{l}_0$ are noninformative parameters and it is not necessary to calculate all of the elements of correlation matrix (8). It is suffice to find the elements at the intersections of the first three rows and columns of matrix (8), which characterize the potential estimation accuracy of distance, velocity, and acceleration.

Thus, matrix (8) contains estimation accuracies of all unknown parameters but we will determine only the estimation accuracies of the distance, velocity, and acceleration. We do not calculate elements of matrix $\mathbf{K}_p$ that characterize the estimation accuracy of noninformative parameters $\vec{l}$.

We use the following notation for the matrix that consists of the elements from the first three rows and columns of matrix (8):

$$\mathbf{K} = \mathbf{F}^{-1}. \tag{11}$$

Matrix $\mathbf{F}$ in formula (11) can be obtained using the Frobenius formula [7]. Assuming that matrix $\mathbf{D}$ in expressions (9) and (10) is nonsingular, we obtain

$$\mathbf{F} = \mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{B}^{\mathrm{T}} = \begin{Vmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{Vmatrix}, \tag{12}$$

where

$$\begin{aligned} F_{11} &= 4\alpha(1-\rho_p)\,M_0/c^2, \\ F_{12} &= F_{21} = 4\alpha\theta(1-\rho_p)\,M_1/c^2, \\ F_{22} &= 4\alpha\theta^2\,(M_2-\rho_p M_1^2/M_0)/c^2, \\ F_{13} &= F_{31} = 2\alpha\theta^2(1-\rho_p)\,M_2/c^2, \\ F_{33} &= \alpha\theta^4\,(M_4-\rho_p M_2^2/M_0)/c^2, \\ F_{23} &= F_{32} = 2\alpha\theta^3(M_3-\rho_p M_1 M_2/M_0)/c^2. \end{aligned} \tag{13}$$

Here,

$$\rho_p = \sum_{i=1}^{p}\sum_{j=1}^{p}\beta_i\beta_j\,\Delta_{ij}/\alpha, \tag{14}$$

Δ_{ij} are elements of inverse matrix $\Delta = \|\Delta_{ij}\|$, such that $\|\Delta_{ij}\| = \|D_{ij}\|^{-1}$, $i, j = 1,\ldots, p$.

Formulas (11)–(14) make it possible to characterize individual and joint estimates of distance, velocity, and acceleration in the presence of a finite number of arbitrary noninformative parameters.

For this purpose, the rows and columns that correspond to the a priori known informative parameters must be deleted in matrix (12). Thus, the dimension of the matrix decreases to the number of the unknown informative parameters.

First, we consider the variance of the distance estimates. We assume that velocity V_0 and acceleration A_0 of the target are a priori known and only the distance must be estimated. In accordance with formula (11), the variance of the efficient estimate of the distance is given by

$$D(R \mid R_0,\vec{l}_0) = 1/F_{11} = c^2/4\alpha N(1-\rho_p). \tag{15}$$

In the absence of the noninformative parameters, the variance of the efficient estimate of the distance can be represented as [4]

$$D(R \mid R_0) = c^2/4\alpha N. \tag{16}$$

Expressions (15) and (16) show that a decrease in the accuracy of the efficient estimate of the distance due to the presence of the noninformative parameters is characterized by quantity

$$\chi_R = D(R \mid R_0,\vec{l}_0)/D(R \mid R_0) = (1-\rho_p)^{-1}. \tag{17}$$

Formula (17) shows that a decrease in the accuracy of the efficient estimate of the distance does not depend on parameter μ (i.e., the point of pulse train (3) that is related to the arrival time of the optical pulse train).

If velocity V_0 is a priori unknown and acceleration A_0 is known, the variance of the joint efficient estimate of the distance is written as

$$D(R|R_0,V_0,\vec{l}_0) = \frac{F_{22}}{F_{11}F_{22}-F_{12}^2} = \frac{c^2(1-\rho_p r_1^2)}{4\alpha N(1-\rho_p)(1-r_1^2)}. \tag{18}$$

Here,

$$r_1 = r_1(\mu) = -M_1/\sqrt{M_0 M_2} \tag{19}$$

is the correlation coefficient of the joint efficient estimates of the distance and velocity in the absence of the noninformative parameters. Assuming that $\rho_p = 0$ in expression (18), we obtain the variance of the joint efficient estimate of the distance for the a priori unknown velocity in the absence of the noninformative parameters [4].

For a priori unknown acceleration A_0 and a priori known velocity V_0, the variance of the joint efficient estimate of the distance is given by

$$D(R|R_0,A_0,\vec{l}_0) = \frac{F_{33}}{F_{11}F_{33}-F_{13}^2} = \frac{c^2(1-\rho_p r_2^2)}{4\alpha N(1-\rho_p)(1-r_2^2)}. \tag{20}$$

Here,

$$r_2 = r_2(\mu) = -M_2/\sqrt{M_0 M_4} \tag{21}$$

is the correlation coefficient of the joint efficient estimates of the distance and acceleration in the absence of the noninformative parameters. Assuming that $\rho_p = 0$ in expression (20), we obtain the variance of the joint efficient estimate of the distance for the a priori unknown acceleration in the absence of the noninformative parameters [4].

When velocity V_0 and acceleration A_0 are a priori unknown, we employ matrix inversion in expression (11) and obtain the variance of the distance estimate

$$D(R|R_0,V_0,A_0,\vec{l}_0) = \frac{c^2(1-r_3^2-\rho_p(r_1^2+r_2^2+2r_1r_2r_3))}{4\alpha Nd(1-\rho_p)}. \tag{22}$$

Here, $d = 1-r_1^2-r_2^2-r_3^2-2r_1r_2r_3$ and $r_3 = r_3(\mu) = -M_3/\sqrt{M_2M_4}$ is the correlation coefficient of the joint efficient estimates of the velocity and acceleration in the absence of the noninformative parameters. Assuming that $\rho_p = 0$ in expression (22), we obtain the variance of the joint efficient estimate of the distance for the a priori unknown velocity and acceleration in the absence of the noninformative parameters [4]:

$$D(R \mid R_0,V_0,A_0) = c^2(1-r_3^2)/4\alpha Nd. \tag{23}$$

Expressions (22) and (23) show that, in the presence of the noninformative parameters, a decrease in the accuracy of the joint efficient estimation of the distance for the unknown velocity and acceleration is characterized by the quantity

$$\begin{aligned} \tilde{\chi}_R &= \frac{D(R \mid R_0,V_0,A_0,\vec{l}_0)}{D(R \mid R_0,V_0,A_0)} \\ &= \frac{1-r_3^2-\rho_p(r_1^2+r_2^2+2r_1r_2r_3)}{(1-\rho_p)(1-r_3^2)}. \end{aligned} \tag{24}$$

The comparison of expressions (15), (16), (18), (20), and (22) makes it possible to determine a decrease in the estimation accuracy of the distance due to a priori unknown velocity and/or acceleration in the presence of noninformative parameters.

Below, we consider the variance of the velocity estimates. When distance R_0 and acceleration A_0 of the target are a priori known, formula (11) yields the following expression for the variance of the efficient estimate of the velocity:

$$D(V \mid V_0, \vec{l}_0) = 1/F_{22} = c^2/4\theta^2\alpha M_2(1-\rho_p r_1^2), \qquad (25)$$

where quantity r_1 is found from expression (19).

Note that the following inequality is always satisfied:

$$|r_1| \le 1. \qquad (26)$$

Indeed, we use the Cauchy–Bunyakovsky inequality in accordance with which the following relationship is valid:

$$\left|\sum_{k=0}^{N-1} x_k y_k\right|^2 \le \left(\sum_{k=0}^{N-1} x_k^2\right)\left(\sum_{k=0}^{N-1} y_k^2\right). \qquad (27)$$

We assume that $x_k = 1$ and $y_k = k - \mu$ and obtain expression $\left|\sum_{k=0}^{N-1}(k-\mu)\right|^2 \le N\sum_{k=0}^{N-1}(k-\mu)^2$, which directly yields inequality (26).

In the absence of the noninformative parameters, the variance of the efficient estimate of the velocity is written as [4]

$$D(V \mid V_0) = c^2/4\theta^2\alpha M_2. \qquad (28)$$

Expressions (25) and (28) show that a decrease in the accuracy of the efficient estimate of the velocity due to the presence of the noninformative parameters is characterized by quantity

$$\chi_V = D(V \mid V_0, \vec{l}_0)/D(V \mid V_0) = (1-\rho_p r_1^2)^{-1}. \qquad (29)$$

The comparison of formulas (17) and (29) shows that a decrease in the estimation accuracy of the velocity with allowance for expression (26) is always no greater than a decrease in the accuracy of the distance estimate.

When sums M_n are calculated using formulas (10), expression (19) is represented as

$$r_1 = \frac{\mu - (N-1)/2}{\sqrt{(\mu-(N-1)/2)^2 + (N^2-1)/12}}. \qquad (30)$$

For parameter μ that is calculated using expression (2) (the arrival time of the optical pulse train is determined relative to the center of the pulse train), formula (30) yields $r_1 = 0$. In accordance with expression (29), a decrease in the accuracy of the efficient estimate of the velocity related to the presence of the noninformative parameters vanishes in this case. Parameter μ determines the point in the optical pulse train that is used to determine the arrival time. Evidently, it is inexpedient to determine the arrival time of the pulse train relative to a point beyond the time interval of the pulse train. Therefore, parameter μ belongs to interval [0, ..., $N-1$]. The analysis of the quantity given by expression (30) versus parameter μ with allowance for the allowed interval of the parameter shows that quantity $r_1^2(\mu)$ reaches maximum at $\mu = 0$ or $N-1$. Thus, we employ the following notation:

$$\psi = \max_{\mu} r_1^2(\mu) = 3(N-1)/2(2N-1). \qquad (31)$$

For the velocity estimation, the optical pulse train with intensity (1) must contain no less than two pulses. For such a pulse train with the minimum length, we have $\psi = 1/2$. For a large number of pulses in the pulse train ($N \gg 1$), expression (31) yields $\psi = 3/4$. Thus, the accuracy of the efficient estimate of the velocity does not decrease in the presence of the noninformative parameters if parameter μ satisfies expression (2) and a maximum decrease in the accuracy corresponds to parameters $\mu = 0$ and $N-1$ at a large number of pulses in the pulse train.

When distance R_0 is a priori unknown and acceleration A_0 is known, the variance of the joint efficient estimate of the velocity coincides with that for the a priori unknown distance in the absence of the noninformative parameters [4]. Hence, the presence of a finite number of arbitrary noninformative parameters does not affect the estimation accuracy of the velocity for unknown distance.

For the a priori unknown acceleration A_0 and a priori known distance R_0, the variance of the joint efficient estimate of the velocity is given by

$$\begin{aligned} D(V \mid V_0, A_0, \vec{l}_0) &= \frac{F_{33}}{F_{22}F_{33} - F_{23}^2} \\ &= \frac{c^2(1-\rho_p r_2^2)}{4\alpha\theta^2 M_2(1 - r_3^2 - \rho_p(r_1^2 + r_3^2 + 2r_1 r_2 r_3))}. \end{aligned} \qquad (32)$$

Assuming that $\rho_p = 0$, we obtain the variance of the joint efficient estimate of the velocity for the unknown acceleration in the absence of the noninformative parameters [4].

If distance R_0 and acceleration A_0 are unknown a priori, the inversion of matrix (11) yields the variance of the joint efficient estimate of the velocity that coincides with the corresponding variance for the a priori unknown distance and acceleration in the absence of the noninformative parameters [4]. Hence, the presence of a finite number of unknown noninformative parameters does not affect the estimation accuracy of the velocity when the distance and acceleration are unknown. The comparison of expressions (25) and (32) and the results of [4] makes it possible to determine a decrease in the estimation accuracy of the velocity due to a priori missing distance and/or acceleration in the presence of the noninformative parameters.

Consider the variance of the acceleration estimates. We assume that distance R_0 and velocity V_0 of the target are known a priori and only the acceleration must be estimated. Then, the variance of the efficient estimate of the acceleration is written as

$$D(A \mid A_0, \vec{l}_0) = 1/F_{33} = c^2/\alpha\theta^4 M_4(1 - \rho_p r_2^2), \quad (33)$$

where quantity r_2 is determined using expression (21).

Note that the following inequality is always valid:

$$|r_2| \le 1. \quad (34)$$

To prove this, we also employ the Cauchy–Bunyakovsky inequality. Assuming that $x_k = 1$ and $y_k = (k - \mu)^2$ in expression (27), we find that relationship (34) is valid.

In the absence of the noninformative parameters, the variance of the efficient estimate of acceleration is written as

$$D(A \mid A_0) = c^2/\theta^4 \alpha M_4. \quad (35)$$

Expressions (33) and (35) show that a decrease in the accuracy of the efficient estimate of acceleration in the presence of the noninformative parameters is characterized by quantity

$$\chi_A = D(A \mid A_0, \vec{l}_0)/D(A \mid A_0) = (1 - \rho_p r_2^2)^{-1}. \quad (36)$$

The comparison of expressions (17) and (36) shows that a decrease in the estimation accuracy of acceleration with allowance for inequality (34) is always no greater than a decreases in the estimation accuracy of the distance.

Calculating sums M_n in expression (10), we represent formula (21) as

$$r_2 = -\frac{(N^2 + 12\kappa^2 - 1)\sqrt{5/3}}{\sqrt{3N^4 - 10N^2 + 240\kappa^4 + 120(N^2 - 1)\kappa^2 + 7}}, \quad (37)$$

where $\kappa = (N-1)/2 - \mu$. The numerical analysis of relationship (36) with allowance for expression (37) makes it possible to assess a decrease in the estimation accuracy of the acceleration due to the presence of noninformative parameters. If distance R_0 is a priori unknown and velocity V_0 is known the variance of the joint efficient estimate of the acceleration coincides with that for the a priori unknown distance in the absence of the noninformative parameters [4]. Consequently, the presence of a finite number of arbitrary parameters does not affect the estimation accuracy of the acceleration when the distance is unknown.

For the a priori unknown velocity V_0 and known distance R_0, the variance of the joint efficient estimate of the acceleration is given by

$$D(A \mid V_0, A_0, \vec{l}_0) = \frac{F_{22}}{F_{22}F_{33} - F_{23}^2} = \frac{c^2(1 - \rho_p r_1^2)}{\alpha\theta^4 M_4(1 - r_3^2 - \rho_p(r_1^2 + r_3^2 + 2r_1r_2r_3))}. \quad (38)$$

Assuming that $\rho_p = 0$, we obtain the variance of the joint efficient estimate of the acceleration for the unknown velocity in the absence of the noninformative parameters [4].

When distance R_0 and velocity V_0 are a priori unknown, the inversion of matrix (11) yields the variance of the joint efficient estimate of the acceleration that coincides with the corresponding variance for the a priori unknown distance and velocity in the absence of the noninformative parameters [4]. Hence, the presence of a finite number of arbitrary noninformative parameters does not affect the estimation accuracy of the acceleration when the distance and velocity are unknown. The comparison of expressions (35), (38), and the results of [4] makes it possible to determine a decrease in the estimation accuracy of the acceleration due to a priori missing distance and/or velocity in the presence of the noninformative parameters.

In the joint estimation of the three parameters (distance, velocity, and acceleration) the noninformative parameters affect only the estimation accuracy of the distance. A decrease in the accuracy of the efficient estimate of the distance can be determined using expression (24).

The above expressions show that a decrease in the accuracy of the efficient estimates of the distance, velocity, and acceleration due to the presence of the noninformative parameters depends on quantity ρ_p given by formula (14). For convenience, we represent quantity ρ_p as

$$\rho_p = \sum_{i=1}^{p}\sum_{j=1}^{p} R_{ti}\tilde{R}_{ij}R_{tj}\kappa_i\kappa_j, \quad (39)$$

where

$$R_{ti} = -\beta_i/\sqrt{\alpha D_{ii}} \quad (40)$$

is the correlation coefficient of the joint efficient estimates of the arrival time and parameter l_i of a single optical pulse when remaining $p - 1$ parameters l_1, ..., and l_{i-1}, l_{i+1}, ..., l_p of this pulse are a priori unknown, $\tilde{R}_{ij} = \Delta_{ij}/\sqrt{\Delta_{ii}\Delta_{jj}}$ is the correlation coefficient of the joint efficient estimates of parameters l_i and l_j in the estimation of p noninformative parameters of a single pulse, and $\kappa_i^2 = = \tilde{\sigma}_i^2/\sigma_i^2 = \Delta_{ii}D_{ii}$ is the ratio of vari-

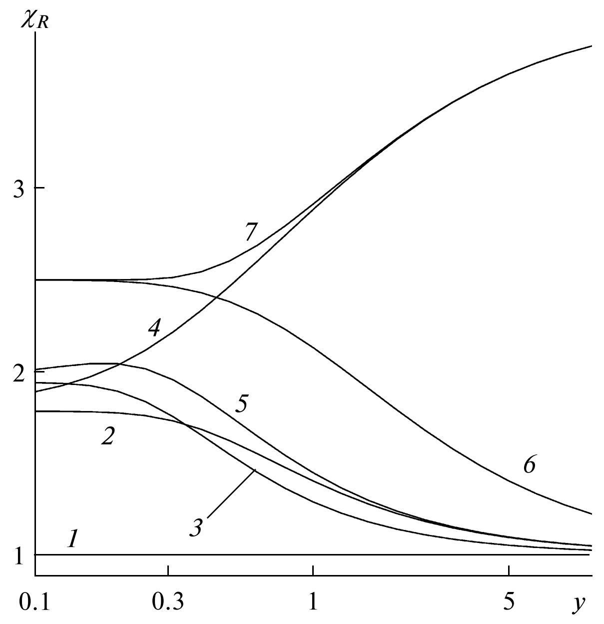

Fig. 1.

ance $\tilde{\sigma}_i^2$ of the joint efficient estimate of parameter l_i in the joint estimation of all noninformative parameters of a single pulse to variance σ_i^2 of the efficient estimate of parameter l_i of a single pulse when the remaining $p-1$ noninformative parameters are a priori unknown. Note that quantity κ_i determines a decrease in the accuracy of the joint efficient estimate of parameter l_i when $p-1$ parameters $l_1, \ldots, l_{i-1}, l_{i+1}, \ldots, l_p$. are unknown.

For most often in practice numbers of parameters $p = 1$, 2, and 3, expression (39) is explicitly represented as

$$\begin{aligned}\rho_1 &= R_{t1}^2, \quad \rho_2 = (R_{t1}^2 + R_{t2}^2 + 2R_{t1}R_{t2}R_{12})/(1 - R_{12}^2),\\ \rho_3 &= (R_{t1}^2(1 - R_{23}^2) + R_{t2}^2(1 - R_{13}^2) + R_{t3}^2(1 - R_{12}^2)\\ &+ 2R_{t1}R_{t2}(R_{12} + R_{13}R_{23}) + 2R_{t1}R_{t3}(R_{13} + R_{12}R_{23})\\ &+ 2R_{t2}R_{t3}(R_{23} + R_{12}R_{13}))/(1 - R_{12}^2 - R_{13}^2\\ &- R_{23}^2 - 2R_{12}R_{13}R_{23}).\end{aligned} \quad (41)$$

where

$$R_{ti} = -\int_0^T \frac{1}{\nu + s(t,\vec{l}_0)} \frac{\partial s(t,\vec{l}_0)}{\partial t} \left(\frac{\partial s(t,\vec{l})}{\partial l_i}\right)_{\vec{l}=\vec{l}_0} dt \times \left(\int_0^T \frac{1}{\nu + s(t,\vec{l}_0)} \left(\frac{\partial s(t,\vec{l}_0)}{\partial t}\right)^2 dt \times \int_0^T \frac{1}{\nu + s(t,\vec{l}_0)} \left(\frac{\partial s(t,\vec{l})}{\partial l_i}\right)^2_{\vec{l}=\vec{l}_0} dt \right)^{-1/2},$$

$$R_{ij} = -\int_0^T \frac{1}{\nu + s(t,\vec{l}_0)} \left(\frac{\partial s(t,\vec{l})}{\partial l_i}\frac{\partial s(t,\vec{l})}{\partial l_j}\right)_{\vec{l}=\vec{l}_0} dt \times \left(\int_0^T \frac{1}{\nu + s(t,\vec{l}_0)} \left(\frac{\partial s(t,\vec{l})}{\partial l_i}\right)^2_{\vec{l}=\vec{l}_0} dt \times \int_0^T \frac{1}{\nu + s(t,\vec{l}_0)} \left(\frac{\partial s(t,\vec{l})}{\partial l_j}\right)^2_{\vec{l}=\vec{l}_0} dt \right)^{-1/2}.$$

Expressions (39) and (41) show that $\rho_p = 0$ if the estimated arrival time of a single optical pulse is not correlated with the estimates of all p noninformative parameters $\vec{l}$, so that $R_{ti} = 0$ $(i = 1, \ldots, p)$ in expression (40). In this case, the accuracy of the efficient estimates of the distance, velocity, and acceleration does not decrease.

The above expressions for the characteristics of the joint efficient estimates can be used for the calculation of the characteristics of asymptotically efficient estimates of the parameters of motion at a relatively high a posteriori accuracy. In particular, they can be used for the calculation of the characteristics of the joint maximum likelihood estimates [1, 3, 6] provided that the signal-to-noise ratio of the pulse train with intensity (3) is relatively high.

We determine a decrease in the accuracy of the joint efficient estimates of the parameters of motion when the intensity of a single optical pulse in pulse train (3) is given by

$$\begin{aligned} s(t,\vec{l}) &= a\left\{\eta(t)\left[1 - \exp\left(-\frac{t}{\delta}\right)\right]\right.\\ &\left.- \eta(t-\tau)\left[1 - \exp\left(-\frac{t-\tau}{\delta}\right)\right]\right\}, \\ \eta(t) &= \begin{cases} 1, & t \ge 0, \\ 0, & t < 0, \end{cases} \quad \vec{l} = \|a, \tau, \delta\|, \end{aligned} \quad (42)$$

where τ is the pulse duration, quantity δ characterizes the duration of the pulse leading edge, and a is the amplitude factor. When the parameters of the motion of a target are estimated using pulses (42), up to three ($p \le 3$) noninformative parameters (a, τ, and δ) are allowed for each pulse.

A short optical pulse that broadens due to propagation in a scattering medium exhibits the intensity distribution that is close to that given by expression (42) [8].

A decrease in the estimation accuracy of the parameters of motion due to the presence of the noninformative parameters is determined by expressions (17), (24), (29), and (36) under different conditions. It can be easily demonstrated that the maximum loss corresponds to the estimation accuracy of the distance (expression (17)).

Figure 1 demonstrates the dependence of a decrease in the estimation accuracy (quantity χ_R given by expression (17)) on parameter $y = \delta/\tau$, which is the

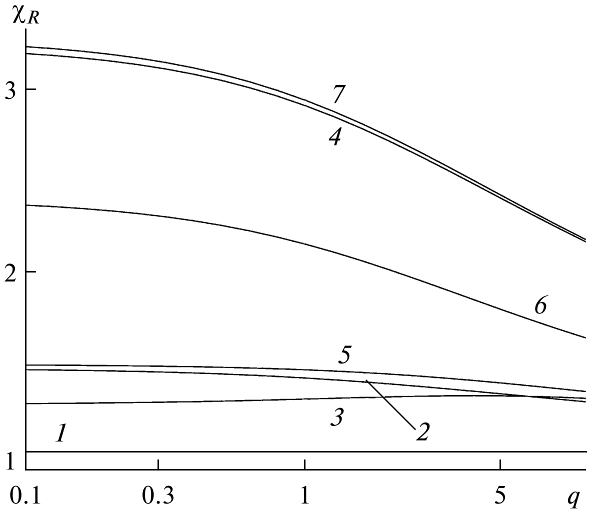

Fig. 2.

ratio of the duration of the leading edge to the pulse duration, for several sets of the noninformative parameters at a signal-to-background ratio of $q = a/\nu = 1$. Curve *1* illustrates a decrease in the estimation accuracy when a serves as the noninformative parameter. Note that the efficient estimation of the parameters of motion does not suffer in the presence of noninformative parameter a. Curves *2* and *3* correspond to single noninformative parameters τ and δ, respectively. Curves *4*–*6* correspond to pairs of the noninformative parameters a and τ, a and δ, and τ and δ, respectively. Curve *7* shows the results for the three noninformative parameters (a, τ, and δ). The comparison of the curves makes it possible to determine the effect of the noninformative parameters of pulse (42) on the accuracy of the efficient estimation of the parameters of motion. Figure 1 shows that an increase in parameter y leads to either a decrease in the loss in the accuracy (curves *2*, *3*, *5*, and *6*) or an increase in the loss to a level of about 4 (curves *4* and *7*).

Figure 2 presents the dependence of a decrease in the estimation accuracy (quantity χ_R given by expression (17)) on parameter q for several sets of the noninformative parameters at $y = 1$. The notation in Fig. 2 is the same as in Fig. 1. It is seen that a loss in the estimation accuracy of the parameters of motion in the presence of the noninformative parameters slightly decreases with an increase in signal-to-background ratio q.

Thus, the above results make it possible to determine a decrease in the estimation accuracy of the distance, velocity, and acceleration in the presence of a finite number of arbitrary unknown noninformative parameters of the optical pulse train.

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