ISSN 0735-2727, Radioelectronics and Communications Systems, 2014, Vol. 57, No. 1, pp. 39–48. © Allerton Press, Inc., 2014.
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Threshold Characteristics of Quazi-Likelihood Estimates of Motion Parameters during the Target Probing with a Sequence of Optical Pulses[1]

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Received in final form June 12, 2013

Abstract—Characteristics of quasi-likelihood estimates of the target range, velocity, and acceleration have been found with due regard for anomalous errors. The losses in reliability of quasi-likelihood estimates as compared with the reliability of maximum likelihood estimates were also found.

DOI: 10.3103/S0735272714010038

Sequences of optical pulses are widely used in optical detection-and-ranging systems [1–5], etc. The characteristics of jointly effective range, velocity and acceleration estimates were determined in [3], while the threshold effects occurring due to the possible appearance of anomalous errors were investigated in [4]. In this case it was assumed that the intensity waveform of the pulse sequence scattered by target was a priori known.

However, fluctuations of reflections from the target in real conditions and also physical effects accompanying the light scattering and propagation result in distortion of the signal intensity waveform. If the intensity waveform of signal scattered by target is known inexactly, the quasi-likelihood estimation can be applied for measuring the range, velocity and acceleration [5].

The expressions obtained in [5] for the characteristics of reliable, quasi-likelihood estimates can be used only under conditions of high a posteriori accuracy when anomalous errors are not present [6]. Next we shall investigate the threshold characteristics of quasi-likelihood estimates of motion parameters with due regard for anomalous errors.

Let us assume that sequence of optical pulses is radiated with the following intensity:

$$\hat{s}_N(t) = \sum_{k=0}^{N-1} \hat{s}(t-(k-\mu)\theta-\lambda), \tag{1}$$

where $\hat{s}(t)$ is the function describing the intensity of individual optical pulse, θ is the pulse period, λ is the time position of sequence. Parameter μ determines the point of sequence (1) related to its time position λ. Hence, at $\mu = 0$ quantity λ represents the time position of the first pulse, at $\mu = (N-1)/2$ it represents the time position of the middle of sequence (1), while at $\mu = N-1$ it is the time position of the last pulse.

As a result of scattering of probing sequence (1) by target, the intensity of signal received will have the form [1, 3]:

$$s_N(t, R_0, V_0, A_0) = \sum_{k=0}^{N-1} s(t-2R_0/c-(k-\mu)$$

[1] The study was carried out with support of RFFI (Projects Nos. 13-01-97504 and 13-08-00735).

$$\times\theta(1+2V_0/c)-A_0(k-\mu)^2\theta^2/c), \quad (2)$$

where function $s(t)$ describes the intensity waveform of one scattered optical pulse of the sequence, and in the general case it is different from $\hat{s}(t)$ in expression (1), while c is the velocity of light. The following designations are used in (2): R_0 is the range, V_0 is the velocity, and A_0 is the target acceleration; in this case it is assumed that the unknown range, unknown velocity, and unknown acceleration assume the values from a priori intervals $[R_{min}, R_{max}]$, $[V_{min}, V_{max}]$, and $[A_{min}, A_{max}]$, respectively. We assume that under the terrestrial conditions the following inequalities are satisfied: $|V_0| << c$ and $N\theta|A_0| << c$. The true values of unknown parameters R, V and A of the received sequence of optical pulses (2) are marked with zero subscript.

Let the signal with intensity (2) be observed on the time interval $[0, T]$ against the background of optical noise representing a stationary Poisson process with intensity $\nu > 0$. In this case signal $\pi(t)$ accessible for processing represents a Poisson process with intensity

$$\beta(t, R_0, V_0, A_0) = s_N(t, R_0, V_0, A_0) + \nu,$$

where parameters R_0, V_0, and A_0 are subject to estimation.

Since the intensity waveform $s(t)$ and noise intensity ν can be unknown, the synthesis of estimation algorithm is performed for the signal with intensity

$$\beta_1(t, R_0, V_0, A_0) = s_{1N}(t, R_0, V_0, A_0) + \nu_1,$$

where ν_1 is the expected intensity of optical noise, while

$$s_{1N}(t, R, V, A) = \sum_{k=0}^{N-1} s_1(t-2R/c-(k-\mu)\theta(1+2V/c)-A(k-\mu)^2\theta^2/c), \quad (3)$$

where s_{1N} is the intensity of the sequence with the expected intensity waveform of one pulse $s_1(t)$.

If the intensity waveform of received signal (2) and the intensity of optical noise ν are a priori known, the estimation of motion parameters (R, V, A) can be performed by using the maximum likelihood method [6]. To this end, it is necessary to use the position of the absolute (largest) maximum of the logarithm of likelihood ratio functional [3, 4, 8]:

$$L_F(R, V, A) = \int_0^T \ln(1+s_N(t, R, V, A)/\nu)\mathrm{d}\pi(t) - \int_0^T s_N(t, R, V, A)\mathrm{d}t. \quad (4)$$

If the intensity waveform of signal (2) and the noise intensity are known inexactly, the signal intensity $s_N(t, R, V, A)$ and noise intensity ν in formula (4) should be replaced with the expected signal intensity $s_{1N}(t, R, V, A)$ (3) and noise intensity ν_1. Thus, we obtain the following expression for decision making statistic [5]:

$$L(R, V, A) = \int_0^T \ln(1+s_{1N}(t, R, V, A)/\nu_1)\mathrm{d}\pi(t) - \int_0^T s_{1N}(t, R, V, A)\mathrm{d}t. \quad (5)$$

Therefore, quasi-likelihood estimates $(\hat{R}, \hat{V}, \hat{A})$ of motion parameters represent the position of absolute maximum of random field (5):

$$(\hat{R}, \hat{V}, \hat{A}) = \mathrm{argsup} L(R, V, A), \quad (R, V, A) \in \mathbf{W}, \quad (6)$$

where

$$\mathbf{W} = \{[R_{\min}, R_{\max}], [V_{\min}, V_{\max}], [A_{\min}, A_{\max}]\}, \tag{7}$$

where **W** is the a priori domain of possible values of the range, velocity and acceleration.

Estimate (6) will be called quasi-likelihood [5]. Indeed, in case the intensities of received signal $s_N(t,R,V,A)$ (2) and expected signal $s_{1N}(t,R,V,A)$ (3) coincide and the true and expected intensities of optical noise (ν and ν_1) also coincide, the decision making statistic (5) coincides with the logarithm of likelihood ratio functional (4). Hence, the quasi-likelihood estimate (6) transforms into the maximum likelihood estimate.

In order to determine the threshold characteristics of quasi-likelihood estimate (6), we shall present the decision making statistic (5) in the form of a sum of the signal and noise functions [5, 6]:

$$L(R,V,A) = S(R_0,V_0,A_0,R,V,A) + N(R,V,A) + C, \tag{8}$$

where signal function

$$S(R_0,V_0,A_0;R,V,A) = \langle L(R,V,A)\rangle - C$$

$$= \sum_{k=0}^{N} \int_0^T s(t + 2(R_0 - R)/c + 2(k-\mu)\theta(V_0 - V)/c$$

$$+ (k-\mu)^2\theta^2(A_0 - A)/c)\ln[1 + s_1(t)/\nu_1]\mathrm{d}t, \tag{9}$$

while

$$C = N\int_0^T [\nu\ln(1 + s_1(t)/\nu_1) - s_1(t)]\mathrm{d}t,$$

where C is an inessential constant that can be neglected in what follows. Hereafter the angular brackets designate the operation of averaging over a set of realizations.

Noise function

$$N(R,V,A) = L(R,V,A) - \langle L(R,V,A)\rangle \tag{10}$$

is the realization of random field, moreover

$$\langle N(R,V,A)\rangle = 0,$$

$$B(R_1,V_1,A_1,R_2,V_2,A_2) = \langle N(R_1,V_1,A_1)N(R_2,V_2,A_2)\rangle$$

$$= \sum_{k=0}^{N-1} \int_0^T [\nu + s(t - 2R_0/c - (k-\mu)\theta(1 + 2V_0/c)$$

$$- A_0(k-\mu)^2\theta^2/c)]\ln[1 + s_1(t - 2R_1/c$$

$$- (k-\mu)\theta(1 + 2V_1/c) - A_1(k-\mu)^2\theta^2/c)/\nu_1]$$

$$\times\ln[1 + s_1(t - 2R_2/c - (k-\mu)\theta(1 + 2V_2/c) - A_2(k-\mu)^2\theta^2/c)/\nu_1]\mathrm{d}t. \tag{11}$$

The derivation of signal function (9) and correlation function (11) of noise function (10) was performed under an assumption that

$$\max\{|R-R_0|,|R_1-R_2|,|R_1-R_0|,|R_2-R_0|\} \le c\theta/2,$$

$$\max\{|V-V_0|,|V_1-V_2|,|V_1-V_0|,|V_2-V_0|\} << c,$$

$$N\theta \max\{|A-A_0|,|A_1-A_2|,|A_1-A_0|,|A_2-A_0|\} << c,$$

hence formulas (9) and (11) describe central peaks of appropriate functions.

Let signal function (9) at the fixed values (R_0, V_0, A_0) reach its largest value at point (R^*, V^*, A^*) and have only one pronounced maximum. Then the signal-to-noise ratio z at the output of quasi-likelihood meter can be written as follows:

$$z_1^2 = \frac{S^2(R_0,V_0,A_0,R^*,V^*,A^*)}{B(R^*,V^*,A^*,R^*,V^*,A^*)} = \frac{\left[\sum_{k=0}^{N-1}\int_0^T s(t+\Delta_k)\ln(1+s_1(t)/\nu_1)\mathrm{d}t\right]^2}{\nu N\int_0^T \ln^2(1+s_1(t)/\nu_1)\mathrm{d}t}, \tag{12}$$

where

$$\Delta_k = 2(R^*-R_0)/c + 2(k-\mu)\theta(V^*-V_0)/c + (k-\mu)^2\theta^2(A^*-A_0)/c.$$

If the signal-to-noise ratio (12) is sufficiently large, quasi-likelihood estimates are reliable [6] and their characteristics can be found by using the results of paper [5]. However, if the signal-to-noise ratio (12) is not large enough, while the size of a priori domain (7) of possible values of motion parameters is substantially larger than the duration of the central peak of signal function (9), it may lead to the appearance of anomalous errors [6]. Consequently, the threshold effects arise that result in a significant degradation of the accuracy of quasi-likelihood estimates.

Let us assume that ΔR, ΔV, and ΔA are the durations of signal function (9) in terms of appropriate arguments. Then, it is evident that

$$S(R_0,V_0,A_0,R^* \pm \Delta R,V^*,A^*)$$

$$\simeq S(R_0,V_0,A_0,R^*,V^* \pm \Delta V,A^*)$$

$$\simeq S(R_0,V_0,A_0,R^*,V^*,A^* \pm \Delta A) \simeq 0.$$

Let us introduce the following designation:

$$\mathbf{W}_S = \{[R^*-\Delta R, R^*+\Delta R],[V^*-\Delta V, V^*+\Delta V],[A^*-\Delta A, A^*+\Delta A]\},$$

where $\mathbf{W}_S$ is the subdomain of a priori domain $\mathbf{W}$ (7) of possible values of the range, velocity, and acceleration, where the central peak of signal function (9) is essentially different from zero. Then, we can introduce into consideration the probability of reliable estimate [6]:

$$P_{01} = P[(\hat{R},\hat{V},\hat{A}) \in \mathbf{W}_S], \tag{13}$$

that can be used for the description of threshold properties of quasi-likelihood estimate (6).

An approximate value of the probability of reliable estimate (13) can be determined if the Gaussian approximation of the distribution of decision making statistic (5) is admissible. The distribution of decision making statistic (5) can be approximated with the Gaussian distribution if the following condition is fulfilled [7]:

$$N\nu \min(\tau, \tau_1) >> 1, \tag{14}$$

where τ and τ_1 are the durations of received $s(t)$ and expected $s_1(t)$ signals, respectively.

In addition, let a priori domain (7) of possible values of the range, velocity and acceleration contain sufficiently many resolution cells so that at least one of the following inequalities is satisfied:

$$(R_{\max} - R_{\min}) / \Delta R >> 1,$$

$$(V_{\max} - V_{\min}) / \Delta V >> 1,$$

$$(A_{\max} - A_{\min}) / \Delta A >> 1. \tag{15}$$

By virtue of the definition of quasi-likelihood estimate (6) relationship (13) can be rewritten in the form:

$$P_{01} = P(H_S > H_N), \tag{16}$$

where

$$H_S = \sup L(R, V, A), \quad (R, V, A) \in \mathbf{W}_S,$$

$$H_N = \sup L(R, V, A), \quad (R, V, A) \in \mathbf{W}_N, \tag{17}$$

while $\mathbf{W}_N$ is the complement $\mathbf{W}_S$ to $\mathbf{W}$ (7) so that $\mathbf{W} = \mathbf{W}_S \cup \mathbf{W}_N$. Provided conditions (15) are satisfied, random quantities H_S and H_N are roughly statistically independent [8] and expression (16) assumes the form:

$$P_{01} = \int_{-\infty}^{\infty} F_N(H) \mathrm{d} F_S(H), \tag{18}$$

where $F_N(H)$ is the distribution function of random quantity H_N, $F_S(H)$ is the distribution function of random quantity H_S.

Since subdomain $\mathbf{W}_S$ roughly coincides with the domain of high correlation of random field (5), given the sufficiently large values of signal-to-noise ratio (12), in accordance with expression (17) we can approximately assume that $H_S = L(\hat{R}, \hat{V}, \hat{A}) \approx L(R^*, V^*, A^*)$. Therefore, provided condition (14) is satisfied and $z_1 >> 1$ (12), random quantity H_S has a roughly Gaussian distribution with mathematical expectation $m_S = S(R_0, V_0, A_0, R^*, V^*, A^*)$ (9) and dispersion $\sigma_S^2 = B(R^*, V^*, A^*, R^*, V^*, A^*)$ (11). Correspondingly, the distribution of random quantity H_S (17) can be approximated with expression

$$F_S(H) \approx \Phi\,[(H - m_S) / \sigma_S], \tag{19}$$

where $\Phi(x) = \int_{-\infty}^{x} \exp(-t^2/2) \mathrm{d}t / \sqrt{2\pi}$ is the probability integral.

When $(R, V, A) \in \mathbf{W}_N$, term $S(R_0, V_0, A_0, R, V, A) \approx 0$ in (8), while the expression for correlation function (11) of noise function (10) assumes the form

$$B(R_1, R_2, V_1, V_2, A_1, A_2) = B_N(R_1, R_2, V_1, V_2, A_1, A_2)$$

$$= \nu \sum_{k=0}^{N-1} \int_0^T \ln\{1 + s_1[t - 2(R_1 - R_2 + (k-\mu)$$

$$\times \theta (V_1 - V_2) + (k-\mu)^2 \theta^2 (A_1 - A_2)/2)/c]/\nu_1\} \ln(1 + s_1(t)/\nu_1) \mathrm{d}t. \quad (20)$$

Correspondingly, now random quantity H_N (17) can be presented as follows:

$$H_N = \sup N(R, V, A), \quad (R, V, A) \in \mathbf{W}_N . \quad (21)$$

In accordance with expression (20), provided condition (14) is satisfied, noise function $N(R, V, A)$ in expression (21) approximately represents a Gaussian uniform field. Therefore, $F_N(H)$ is a distribution function of the largest maximum of the Gaussian uniform random field in subdomain $\mathbf{W}_N$.

Since the duration of each pulse of the expected sequence (3) is limited, correlation function (20) $B_N(R_1, V_1, A_1, R_2, V_2, A_2)$ tends to zero at $|R_1 - R_2| \to \infty$, $|V_1 - V_2| \to \infty$, and $|A_1 - A_2| \to \infty$. That is why it can be assumed that with the rise of H the distribution of the number of surges of field realization $N(R, V, A)$ in subdomain $\mathbf{W}_N$ over level H converges to the Poisson law [9]. Therefore, for large but finite values of H we can write [8, 9]

$$F_N(H) \approx \exp[-\Pi(H)],$$

where $\Pi(H)$ is the average number of surges of the realization of Gaussian uniform random field with correlation function (20) over level H in domain $\mathbf{W}_N$.

If condition (15) is satisfied, approximation $\Pi(H)$ represents the average number of surges over the entire a priori domain (7) of possible values of the range, velocity, and acceleration.

Using [9] for the average number of field surges $N(R, V, A)$ with correlation function (20) in domain $\mathbf{W}$ (7), we obtain the following expression:

$$\Pi(H) = \frac{\xi_1 H^2}{4\pi^2 \sigma_N^2} \exp\left(-\frac{H^2}{2\sigma_N^2}\right), \quad (22)$$

where

$$\sigma_N^2 = B_N(R, V, A, R, V, A),$$

$$\xi_1 = (R_{\max} - R_{\min})(V_{\max} - V_{\min})(A_{\max} - A_{\min})\sqrt{\Omega} / \sigma_N^3 . \quad (23)$$

Here

$$\Omega = \begin{vmatrix} \dfrac{\partial^2 B_N}{\partial R_1 \partial R_2} & \dfrac{\partial^2 B_N}{\partial R_1 \partial V_2} & \dfrac{\partial^2 B_N}{\partial R_1 \partial A_2} \\ \dfrac{\partial^2 B_N}{\partial V_1 \partial R_2} & \dfrac{\partial^2 B_N}{\partial V_1 \partial V_2} & \dfrac{\partial^2 B_N}{\partial V_1 \partial A_2} \\ \dfrac{\partial^2 B_N}{\partial A_1 \partial R_2} & \dfrac{\partial^2 B_N}{\partial A_1 \partial V_2} & \dfrac{\partial^2 B_N}{\partial A_1 \partial A_2} \end{vmatrix}, \quad (24)$$

where Ω is the determinant, in which the derivatives of correlation function (20) are calculated at $R_1 = R_2$, $V_1 = V_2$, and $A_1 = A_2$.

Differentiating in expression (24) and substituting the result of determinant calculation into expression (23) we obtain

$$\xi_1 = \frac{Q\theta^3}{\sigma_N^3 c^3 \nu_1^3} \frac{N(N^2-1)}{12} \sqrt{\frac{N(N^2-4)}{15}} \left[\nu \int_0^T \left[\frac{ds_1(t)/dt}{1+s_1(t)/\nu_1} \right]^2 dt \right]^{3/2},$$

where $Q = (R_{\max} - R_{\min})(V_{\max} - V_{\min})(A_{\max} - A_{\min})$ is the Euclidean volume of a priori domain **W** (7) of possible values of unknown motion parameters.

Quantity ξ_1 represents a reduced volume [8] of possible values of unknown range, velocity and acceleration that determines the number of uncorrelated values of random field $N(R, V, A)$ in domain **W** (7).

Using expression (22) for approximating the distribution function of the largest maximum of noise function (21), where condition (15) is satisfied, we obtain the following expression [8]:

$$F_N(H) = \begin{cases} \exp\left[-\frac{\xi_1 H^2}{4\pi^2 \sigma_N^2} \exp\left(-\frac{H^2}{2\sigma_N^2} \right) \right], & H \geq \sigma_N \sqrt{2}, \\ 0, & H < \sigma_N \sqrt{2}. \end{cases} \quad (25)$$

Substituting expressions (19) and (25) into (18) and making the change of integration variable $H = x\sigma_N$, we determine an approximate expression for the probability of reliable quasi-likelihood estimate of the range, velocity and acceleration:

$$P_{01} \approx \frac{1}{\sqrt{2\pi(1+\kappa_1^2)}} \int_{\sqrt{2}}^{\infty} \exp\left[-\frac{(x-z_1)^2}{2(1+\kappa_1^2)} - \frac{\xi_1 x^2}{4\pi^2} \exp\left(-\frac{x^2}{2} \right) \right] dx, \quad (26)$$

where

$$\kappa_1^2 = \frac{\sum_{k=0}^{N-1} \int_0^T \ln^2(1+s_1(t)/\nu_1) s(t+\Delta_k) dt}{\nu N \int_0^T \ln^2(1+s_1(t)/\nu_1) dt},$$

while z_1 is determined by formula (12). The accuracy of approximate formula (26) improves with the rise of ξ_1 and z_1 [8].

Formula (26) for the probability of reliable estimate is pretty cumbersome; hence, it can be used for calculations only by applying numerical methods. That is why we shall find a simple upper bound for the probability of anomalous errors $P_{a1} = 1 - P_{01}$.

Having used inequality $1 - \exp(-x) \leq x$ at $x > 0$, from expression (26) we get by analogy with [8]:

$$P_{a1} \leq P_{a1}^* \approx \frac{\xi_1 \left[z_1^2 + (1+\kappa_1^2)(2+\kappa_1^2) \right]}{4\pi^2 (2+\kappa_1^2)^{5/2}} \exp\left[-\frac{z_1^2}{2(2+\kappa_1^2)} \right]. \quad (27)$$

In a particular case, when the intensity waveforms of the received and expected signals coincide ($s_1(t) \equiv s(t)$), as well as optical noise intensities ($\nu_1 \equiv \nu$), expressions (26) and (27) transform into similar expressions for the probabilities of reliable estimate and anomalous errors of the maximum likelihood estimate [4]. To this end, it is only required to replace quantities z_1, κ_1, and ξ_1 in expressions (26) and (27) with the following respective quantities:

$$z^2 = \frac{N\left[\int_0^T \ln(1+s(t)/\nu)s(t)\mathrm{d}t\right]^2}{\nu\int_0^T \ln^2(1+s(t)/\nu)\mathrm{d}t}, \tag{28}$$

$$\kappa^2 = \frac{\int_0^T \ln^2(1+s(t)/\nu)s(t)\mathrm{d}t}{\nu\int_0^T \ln^2(1+s(t)/\nu)\mathrm{d}t}, \tag{29}$$

$$\xi = \frac{Q\theta^3}{c^3\nu^3}\frac{N^2-1}{12}\sqrt{\frac{N^2-4}{15}}\left[\frac{\int_0^T \left[\frac{\mathrm{d}s(t)/\mathrm{d}t}{1+s(t)/\nu}\right]^2\mathrm{d}t}{\int_0^T \ln^2(1+s(t)/\nu)\mathrm{d}t}\right]^{3/2}. \tag{30}$$

The substitution of expressions (28)–(30) into (26), (27) results in obtaining the known expressions [4] for the maximum likelihood estimation. In particular, the probability of anomalous error P_{a} for maximum likelihood estimation can be written by analogy with expression (27) in the form:

$$P_{\mathrm{a}}^* = \frac{\xi[z^2+(1+\kappa^2)(2+\kappa^2)]}{4\pi^2(2+\kappa^2)^{5/2}}\exp\left[-\frac{z^2}{2(2+\kappa^2)}\right]. \tag{31}$$

Let us compare the probabilities of anomalous errors of quasi-likelihood estimate and maximum likelihood estimate. Comparing expressions (27) and (31) we obtain

$$\chi = \frac{P_{\mathrm{a}1}^*}{P_{\mathrm{a}}^*} = \frac{\xi_1[z_1^2+(1+\kappa_1^2)(2+\kappa_1^2)]}{\xi[z^2+(1+\kappa^2)(2+\kappa^2)]}\left(\frac{2+\kappa^2}{2+\kappa_1^2}\right)^{5/2}\exp\left[\frac{z^2}{2(2+\kappa^2)}-\frac{z_1^2}{2(2+\kappa_1^2)}\right]. \tag{32}$$

This quantity shows how many times the probability of anomalous error increases due to the deviation of the intensity waveform of received signal $s(t)$ from the intensity waveform of expected signal $s_1(t)$, for which the algorithm of quasi-likelihood estimation was synthesized.

Let us render concrete the above derived general expressions for a particular case when the intensity of optical noise is a priori known ($\nu_1 = \nu$), while the intensities of received and expected signals differ only by their duration so that

$$s(t) = af(t/\tau), \quad s_1(t) = af(t/\tau_1).$$

Here function $f(x)$ is normalized in such way that

$$\max f(x) = \int_{-\infty}^{+\infty} f^2(x)\mathrm{d}x = 1,$$

while quantities τ and τ_1 are the equivalent durations of the received and expected signals, respectively.

We shall restrict ourselves to the situation where optical signals are weak ($q = a / \nu << 1$). Then expressions for parameters determining the probability of anomalous error assume the form:

$$\kappa \approx \kappa_1 \approx 0, \quad z^2 = \nu N q^2 \tau, \quad z_1^2 = z^2 \rho^2, \tag{33}$$

$$\xi = \frac{Q\theta^2}{c^3 \tau^3} \frac{N^2 - 1}{12} \sqrt{\frac{N^2 - 4}{15}} \left[\int_{-\infty}^{\infty} \left(\frac{\mathrm{d}f(x)}{\mathrm{d}x} \right)^2 \mathrm{d}x \right]^{3/2},$$

$$\xi_1 = \xi / \eta^3, \tag{34}$$

where $\eta = \tau_1 / \tau$, while

$$\rho = \rho(\eta) = \frac{\int_{-\infty}^{\infty} s(t) s_1(t) \mathrm{d}t}{\sqrt{\int_{-\infty}^{\infty} s^2(t) \mathrm{d}t \int_{-\infty}^{\infty} s_1^2(t) \mathrm{d}t}} = \sqrt{\eta} \int_{-\infty}^{\infty} f(x) f(\eta x) \mathrm{d}x, \tag{35}$$

where ρ is the correlation coefficient of received and expected signals.

Substituting expressions (33)–(35) into (32) we obtain an expression for the loss in the value of the anomalous error probability:

$$\chi = \frac{z^2 \rho^2 + 2}{\eta^3 (z^2 + 2)} \exp\left[\frac{z^2 (1 - \rho^2)}{4} \right]. \tag{36}$$

This quantity indicates to what extent the anomalous error probability changes due to the difference between the duration τ of received signal and the duration τ_1 of expected signal, for which the algorithm of quasi-likelihood estimation was synthesized.

Let the intensity of optical pulse be described by the Gaussian curve so that

$$f_G(x) = \exp(-\pi x^2 / 2). \tag{37}$$

Then correlation coefficient (35) assumes the form:

$$\rho_G = \sqrt{2\eta / (1 + \eta^2)}.$$

If the intensity of optical pulse is described by the Lorentz curve:

$$f_L(x) = \left[1 + \left(\frac{\pi x}{2} \right)^2 \right]^{-1}, \tag{38}$$

then correlation coefficient (35) assumes the form:

$$\rho_L = 2\sqrt{\eta} / (1 + \eta).$$

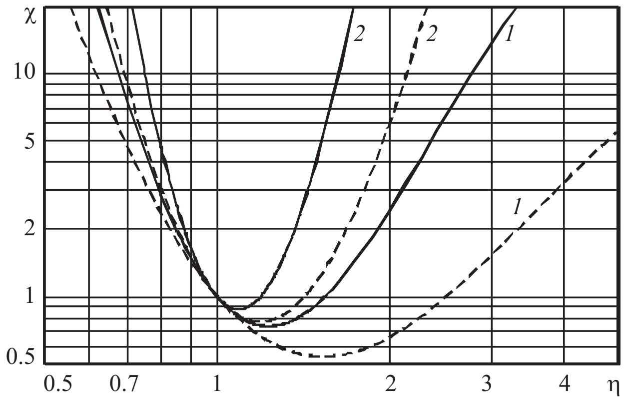

Fig. 1.

Solid lines in Fig. 1 indicate the $\chi(\eta)$ relationships (36) for the intensity of optical pulse described by the Gaussian curve (37), while dashed lines indicate the same relationships for the Lorentz curve (38) at the following signal-to-noise ratio values: $z = 8$ (curves *1*) and $z = 12$ (curves *2*). From Fig. 1 it follows that for the Gaussian curve the loss in the value of anomalous error probability is larger than for the Lorentz curve. In addition, this loss increases with the rise of the signal-to-noise ratio and with the growing deviation of η from unity.

The determined threshold characteristics of quasi-likelihood estimates allow us to make a reasonable choice of the algorithm of estimation of the intensity and waveform of expected signal depending on the available a priori information and the admissible rise of the anomalous error probability.

REFERENCES

1. V. I. Vorob'ev, *Optical Detection and Ranging for Radio Engineers* (Radio i Svyaz', Moscow, 1983) [in Russian].
2. N. A. Dolinin and A. F. Terpugov, *Statistical Methods in Optical Detection and Ranging* (TGU, Tomsk, 1982) [in Russian].
3. A. P. Trifonov, M. B. Bespalova, and M. V. Maksimov, "Estimation of the range, speed and acceleration during the probing with optical pulse sequence," Radiotekhnika, No. 4, 99 (2001).
4. A. P. Trifonov, M. B. Bespalova, and M. V. Maksimov, "Threshold characteristics of range, speed, and acceleration estimates when sounding by a sequence of optical pulses," Izv. Vyssh. Uchebn. Zaved., Radioelektron. **45** (6), 3 (2002) [Radioelectron. Commun. Syst. **45** (6), 1 (2002)].
5. A. P. Trifonov, M. B. Bespalova, A. V. Kurbatov, "Quazi-likelihood estimation of motion parameters during the target probing with a sequence of optical pulses," Izv. Vyssh. Uchebn. Zaved., Radioelektron. **56** (1), 24 (2013) [Radioelectron. Commun. Syst. **56** (1), 20 (2013)]. DOI: 10.3103/S0735272713010020.
6. E. I. Kulikov and A. P. Trifonov, *Estimation of Signal Parameters against the Background of Interferences* (Sov. Radio, Moscow, 1978) [in Russian].
7. I. A. Bol'shakov and V. S. Rakoshits, *Applied Theory of Random Streams* (Sov. Radio, Moscow, 1978) [in Russian].
8. A. P. Trifonov and Yu. S. Shinakov, *Joint Discrimination of Signals and Estimation of Signal Parameters against the Background of Interferences* (Radio i Svyaz', Moscow, 1986) [in Russian].
9. V. I. Piterbarg, *Asymptotic Methods in the Theory of Gaussian Random Processes and Fields* (MGU, Moscow, 1988) [in Russian].
10. P. S. Akimov, P. A. Bakut, V. A. Bogdanovich, et al., *Theory of Signal Detection* (Radio i Svyaz', Moscow, 1984) [in Russian, ed. by P. A. Bakut].