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# Amplitude Estimate of the Radio Signal with Unknown Duration and Initial Phase

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**Abstract**

Quasi-likelihood and maximum likelihood estimate algorithms of the amplitude of a radio signal with free-form envelope and unknown duration and initial phase are synthesized. Characteristics of the synthesized algorithms are found. The comparison of the accuracy of amplitude estimates is carried out.

**Keywords**: radio signal with free-form envelope, estimate of amplitude, unknown duration and phase, amplitude estimate characteristics

## 1 Introduction

The problem of amplitude estimation of the radio signal observed against noise is actual for many practical appendices of information theory and was repeatedly considered in the literature [1-7]. In [1] the determined signal amplitude estimate is considered for case when all other parameters are a priori known, and amplitude estimate characteristics are found. In work [2] the maximum likelihood estimate of amplitude of the signal with unknown nonpower parameters, and also joint estimates of rectangular pulse amplitude and duration are investigated. However in broad range of tasks of the statistical communication theory and signal processing it is often necessary to estimate amplitude of a radio signal with non-rectangular envelope and unknown duration and initial phase. Below estimate algorithms of amplitude of a narrow-band radio signal with unknown duration and initial phase are considered.

## 2 Problem Statement

So, let the additive mix

$$\xi(t) = s(t, \tau_0, a_0, \varphi_0) + n(t) \tag{1}$$

is accessible to be observed during time slice $[0, T]$. Here

$$s(t, \tau_0, a_0, \varphi_0) = \begin{cases} a_0 f(t) \cos(\omega t - \varphi_0), & 0 \le t \le \tau_0, \\ 0, & t < 0, t > \tau_0, \end{cases} \tag{2}$$

is the useful signal and $n(t)$ is Gaussian white noise with one-sided spectral density $N_0$. In Eq. (2) it is designated: $\tau_0$, $a_0$, $\varphi_0$ – unknown duration, amplitude and initial phase of the received signal accordingly, $f(t)$ – a priori known continuous bounded function which describes the form of a narrow-band radio signal envelope. We believe that signal duration possesses the values from prior interval

$$\tau_0 \in [T_1, T_2]. \tag{3}$$

On observable realization $\xi(t)$ it is necessary to generate the amplitude estimate of a useful signal (2) considering its duration and initial phase as spurious parameters in which estimation there is no necessity.

For synthesis of the amplitude estimate algorithm we will use a maximum likelihood (ML) method [2, 3] according to which the amplitude estimate $a_{0m}$ under a priori known duration $\tau_0$ and initial phase $\varphi_0$ is defined as absolute (greatest) maximum position of the logarithm of likelihood ratio functional (LRF)

$$a_{0m} = \arg\sup_a L(\tau_0, a, \varphi_0).$$

Under unknown duration, amplitude and initial phase the logarithm of LRF depends on three unknown parameters [1-3]

$$L(\tau, a, \varphi) = \frac{2a}{N_0} \int_0^{\tau} [\xi(t) - af(t)\cos(\omega t - \varphi)/2] f(t) \cos(\omega t - \varphi) dt . \quad (4)$$

Therefore, there is a prior parametrical uncertainty concerning duration and initial phase. As one of possible overcoming ways of this uncertainty we will use quasi-likelihood (QL) estimate algorithm according to which instead of unknown duration $\tau$ and initial phase $\varphi$ their some expected values $\tau^*$ and $\varphi^*$ accordingly are used in expression (4). Thus, coherent processing is used, but duration and initial phase of reference and received radio signals can not coincide. So, we will name such amplitude estimate algorithm as quasi-coherent QL one. The second way of uncertainty overcoming consists in application incoherent QL estimate algorithm according to which the unknown initial phase is replaced by its ML estimate in expression (4), and some expected value $\tau^*$ instead of unknown duration $\tau$ is used as before. The third way implies application of ML estimation. In this case instead of unknown initial phase and duration their the ML estimates [1-3] are used in expression (4).

## 3 Quasi-coherent quasi-likelihood signal amplitude estimate

In the beginning let us consider quasi-coherent QL algorithm of the amplitude estimation. Quasi-coherent QL estimate represents position of a maximum of solving statistics

$$L^*_{\tau\varphi}(a) = L(\tau^*, a, \varphi^*) = \frac{2a}{N_0} \int_0^{\tau^*} [\xi(t) - af(t)\cos(\omega t - \varphi^*)/2] f(t) \cos(\omega t - \varphi^*) dt , \quad (5)$$

that is

$$\hat{a} = \arg\sup_a L^*_{\tau\varphi}(a). \quad (6)$$

The estimate (6) can be found analytically. For this purpose we will equate a derivative of function (5) on variable a to zero

$$\left.\frac{dL(a, \tau^*, \varphi^*)}{da}\right|_{a=\hat{a}} = \frac{2}{N_0} \int_0^{\tau^*} \xi(t) f(t) \cos(\omega t - \varphi^*) dt - \frac{2\hat{a}}{N_0} \int_0^{\tau^*} f^2(t) \cos^2(\omega t - \varphi^*) dt = 0$$

and will solve the given likelihood equation concerning a. Then, for amplitude estimate we have

$$\hat{a} = \int_0^{\tau^*} \xi(t) f(t) \cos(\omega t - \varphi^*) dt \Bigg/ \int_0^{\tau^*} f^2(t) \cos^2(\omega t - \varphi^*) dt . \quad (7)$$

Eq. (7) defines receiver organization. In Fig. 1 its block diagram is represented, where I is the time interval $t \in [0, \tau^*]$ integrator.

Let us make the analysis of quasi-coherent QL amplitude estimate algorithm.

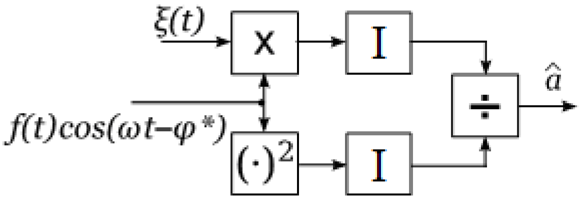


Fig. 1. Block diagram of quasi-coherent quasi-likelihood amplitude measurer of a radio signal with unknown duration and initial phase.

According to Eq. (7) the estimate $\hat{a}$ is Gaussian random value. We substitute realization of the observable data (1) in Eq. (7) and neglect integrals from functions oscillating with the doubled frequency $2\omega$, by virtue of a radio signal bandlimitedness. Then, carrying out averaging operation we find statistical characteristics quasi-coherent QL estimate: bias (systematic error) $B_{\tau\varphi}$ and variance (mean square error) $V_{\tau\varphi}$

$$B_{\tau\varphi} = \langle \hat{a} - a_0 \rangle = -a_0 \left[ 2\sin^2(\Delta\varphi/2) + \delta_\tau \cos(\Delta\varphi)\eta(\tau^* - \tau_0) / (1+\delta_\tau) \right], \quad (8)$$

$$V_{\tau\varphi} = \left\langle (\hat{a} - a_0)^2 \right\rangle = \frac{a_0^2}{z_0^2(1+\delta_\tau)} + a_0^2 \left[ 2\sin^2(\Delta\varphi/2) + \frac{\delta_\tau \cos(\Delta\varphi)\eta(\tau^* - \tau_0)}{1+\delta_\tau} \right]^2, \quad (9)$$

where $z_0^2 = a_0^2 q(\tau_0)$, $\Delta\varphi = \varphi^* - \varphi_0$ is initial phase detuning,

$$q(\tau) = \frac{1}{N_0} \int_0^\tau f^2(t)\,dt \quad (10)$$

is output ML receiver signal-to-noise ratio (SNR) for signal with unit amplitude and duration $\tau$,

$$\eta(x) = \begin{cases} 1, & x \geq 0, \\ 0, & x < 0, \end{cases}$$

is Heaviside function,

$$\delta_\tau = \left[ q(\tau^*) - q(\tau_0) \right] / q(\tau_0) \quad (11)$$

is value characterizing a deviation of expected signal duration from its true value. We will name its as generalized duration detuning. If the initial phase is a priori known, then $\Delta\varphi = 0$ and expressions for bias (8) and variance (9) of the estimate (6) have the appearance

$$B_\tau = -a_0\, \delta_\tau \eta(\tau^* - \tau_0) / (1+\delta_\tau), \quad (12)$$

$$V_\tau = a_0^2 / z_0^2 (1+\delta_\tau) + a_0^2 \delta_\tau^2 \eta(\tau^* - \tau_0) / (1+\delta_\tau)^2. \quad (13)$$

Bias (12) and variance (13) coincide with the similar expressions found in [4] for QL amplitude estimate of a signal without high-frequency filling and with unknown duration. In case of an equality of expected duration and its true value, i.e. $\tau^* = \tau_0$ ($\delta_\tau = 0$), quasi-coherent QL amplitude estimate (6) possesses bias

$$B_\varphi = -2a_0 \sin^2(\Delta\varphi/2) \tag{14}$$

and variance

$$V_\varphi = a_0^2 / z_0^2 + 4a_0^2 \sin^4(\Delta\varphi/2). \tag{15}$$

If expected values of an initial phase and duration are equal to their true values: $\varphi^* = \varphi_0$, $\tau^* = \tau_0$, then quasi-coherent QL estimate $a^*$ (6) coincides with ML amplitude estimate under a priori known initial phase and duration

$$a_{0m} = \int_0^{\tau_0} \xi(t) f(t) \cos(\omega t - \varphi_0)\, dt \Big/ \int_0^{\tau_0} f^2(t) \cos^2(\omega t - \varphi_0)\, dt. \tag{16}$$

Estimate (16) bias and variance have the appearance

$$B_{0m} = 0, \qquad V_{0m} = a_0^2 / z_0^2, \tag{17}$$

and coincide with the similar expressions received in [1].

Let us put into consideration the value $\chi = V_{\tau\varphi}/V_{0m}$ characterizing variance increase of quasi-coherent QL amplitude estimate (6) in case of unknown initial phase and duration in comparison with variance of ML amplitude estimate (16) under known signal initial phase and duration:

$$\chi = V_{\tau\varphi}/V_{0m} = 1/(1+\delta_\tau) + z_0^2 \left[ 2\sin^2(\Delta\varphi/2) + \delta_\tau \cos\Delta\varphi\eta\left(\tau^* - \tau_0\right)/(1+\delta_\tau) \right]^2. \tag{18}$$

As an example, we will consider characteristics of an amplitude estimate of the radio signal, envelope which has the form of a rectangle with bevel top [4, 5]

$$f(t) = [1 + bt/T_{max}] \Big/ \sqrt{1 + b + b^2/3}, \tag{19}$$

where the value b characterizes a bevel top tilt. The multiplier $\left(1 + b + b^2/3\right)^{-1/2}$ is introduced in order to assure the independence of maximum duration signal energy on a bevel pulse top tilt. It gives an opportunity to compare amplitude estimate accuracy of signals with different top tilt, but equal energy. Generalized detuning $\delta_\tau$ (11) for a signal (19) becomes

$$\delta_\tau = (1+\Delta_\tau)\left[1 + b(1+\Delta_\tau)x_0 + b^2(1+\Delta_\tau)^2 x_0^2/3\right] \Big/ \left(1 + bx_0 + b^2 x_0^2/3\right) - 1. \tag{20}$$

Here $x_0 = \tau_0/T_2$, $\Delta_\tau = \left(\tau^* - \tau_0\right)/\tau_0$, $\Delta_\tau \in \left[(T_1 - \tau_0)/\tau_0, (T_2 - \tau_0)/\tau_0\right]$.

Let us put into consideration the value $z_r^2 = a_0^2 T_2 / N_0$ which is SNR for a rectangular radio pulse with amplitude $a_0$ and duration $T_2$. We express SNR $z_0^2$ for the sensed signal through SNR $z_r^2$:

$$z_0^2 = z_r^2 x_0 \left(1 + bx_0 + b^2 x_0^2/3\right) \Big/ \left(1 + b + b^2/3\right), \tag{21}$$

To fix the idea, we choose true value of signal duration in the middle of a prior interval (3): $\tau_0 = (T_1 + T_2)/2$, then $x_0 = (k+1)/2k$, where $k = T_2/T_1$ is a dynamic range of change of unknown duration.

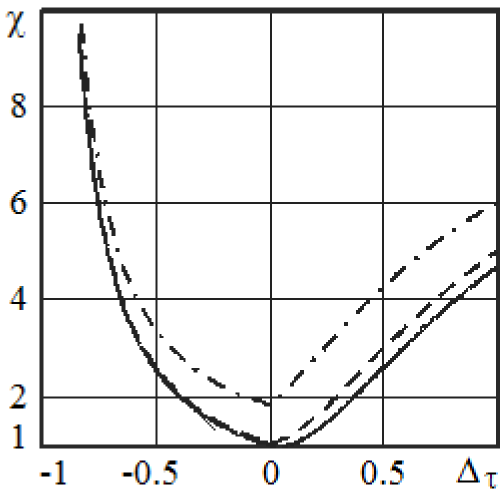


Fig. 2. Loss in accuracy of the quasi-coherent amplitude estimate under various initial phase detunings.

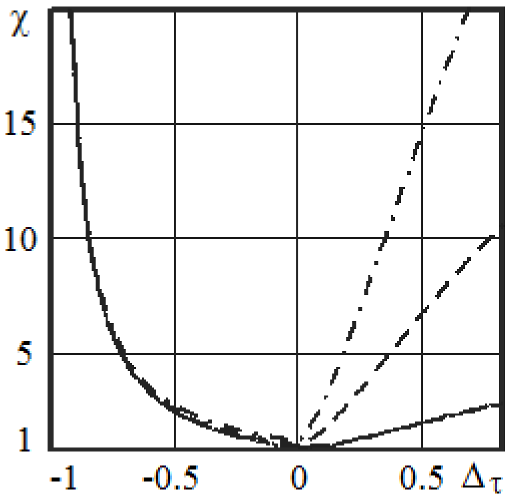


Fig. 3. Loss in accuracy of the quasi-coherent amplitude estimate under various signal-to-noise ratios.

In Figs. 2 and 3 dependences of the loss $\chi$ (18) on value $\Delta_\tau$ for a rectangular radio pulse with bevel top tilt (19) are presented, if $b = 1$, $k = 10$. In Fig. 2 dependences $\chi$ are plotted under SNR $z_r = 5$ and various initial phase detunings. The solid curve corresponds $\Delta\varphi = 0$, dashed – $\Delta\varphi = \pi/8$, dash-dotted – $\Delta\varphi = \pi/4$. In Fig. 3 dependences $\chi$ are calculated in presence of initial phase detuning $\Delta\varphi = \pi/8$ and various SNRs. The solid curve corresponds $z_r = 4$, dashed – $z_r = 8$, dash-dotted – $z_r = 12$. As follows from Figs. 2 and 3, a prior ignorance of signal duration can lead to essential increase in variance of quasi-coherent QL amplitude estimate (6) in comparison with estimate (16), especially under ignorance of a radio signal initial phase.

In Fig. 4 dependences of loss $\chi$ (20) on phase detuning $\Delta\varphi$ are represented under $z_r = 5$, $k = 10$, and various duration detunings $\Delta_\tau$. Here the solid curve corresponds to detuning absence $\Delta_\tau = 0$, dashed – $\Delta_\tau = 0{,}2$, dash-dotted – $\Delta_\tau = 0{,}4$. As follows from Fig. 4, disagreement of expected initial phase value with its true value can lead to increase in variance of quasi-coherent QL estimate up many times. Therefore, in order to raise the accuracy of amplitude estimate we will consider incoherent QL algorithm according to which the unknown initial phase is replaced by its estimate in expression (4), and some expected value $\tau^*$ is used instead of unknown duration $\tau_0$, as before.

## 4 Incoherent quasi-likelihood signal amplitude estimate

Incoherent QL estimate is defined by expression

$$\hat{a}_\tau = \arg\sup_a L^*_\tau(a), \tag{22}$$

where

$$L^*_\tau(a) = \sup_\varphi L\left(a, \tau^*, \varphi\right) = L\left(a, \tau^*, \varphi_m\right), \quad \varphi_m = \arg\sup_\varphi L\left(a, \tau^*, \varphi\right). \tag{23}$$

Maximization of the LRF (4) logarithm on a variable φ can be performed analytically. For this purpose we substitute a signal (2) in expression (4) and present LRF logarithm in a kind

$$L(\tau, a, \varphi) = aX(\tau)\cos\varphi + aY(\tau)\sin\varphi - \frac{a^2}{2N_0}\int_0^\tau f^2(t)\,dt\,. \tag{24}$$

Here integrals from functions oscillating with the doubled frequency are rejected, and it is designated:

$$X(\tau) = \frac{2}{N_0}\int_0^\tau \xi(t) f(t)\cos(\omega t)\,dt\,, \quad Y(\tau) = \frac{2}{N_0}\int_0^\tau \xi(t) f(t)\sin(\omega t)\,dt\,. \tag{25}$$

Carrying out maximization of solving statistics (24) on a variable φ, we receive

$$L(\tau, a) = \sup_\varphi L(a, \tau, \varphi) = a\sqrt{X^2(\tau) + Y^2(\tau)} - \frac{a^2}{2N_0}\int_0^\tau f^2(t)\,dt\,. \tag{26}$$

Then, according to Eq. (23)

$$L^*_\tau(a) = L\left(\tau^*, a\right) = a\sqrt{X^2\left(\tau^*\right) + Y^2\left(\tau^*\right)} - \frac{a^2}{2N_0}\int_0^{\tau^*} f^2(t)\,dt\,. \tag{27}$$

The estimate (22) can be found analytically. For this purpose we equate a derivative of function (27) on variable a to zero

$$\left.\frac{dL^*_\tau(a)}{da}\right|_{a=\hat{a}_\tau} = \sqrt{X^2\left(\tau^*\right) + Y^2\left(\tau^*\right)} - \frac{2\hat{a}_\tau}{N_0}\int_0^{\tau^*} f^2(t)\,dt = 0$$

and solve the obtained likelihood equation relative to $\hat{a}_\tau$

$$\hat{a}_\tau = \sqrt{X^2\left(\tau^*\right) + Y^2\left(\tau^*\right)} \Big/ q\left(\tau^*\right). \tag{28}$$

Here function $q(\tau)$ is defined by Eq. (10). In Fig. 5 the block diagram of incoherent QL amplitude measurer is represented where it is designated: I – time interval $t \in [0, \tau^*]$ integrators.

For the analysis of incoherent QL amplitude estimate algorithm we consider random processes $X(\tau)$ and $Y(\tau)$ (25). Substituting sensed realization (1) in Eqs. (25) we receive following expressions after simple transformations

$$X(\tau) = a_0 G(\tau_0, \tau)\cos\varphi_0 + N_c(\tau), \quad Y(\tau) = a_0 G(\tau_0, \tau)\sin\varphi_0 + N_s(\tau), \quad (29)$$

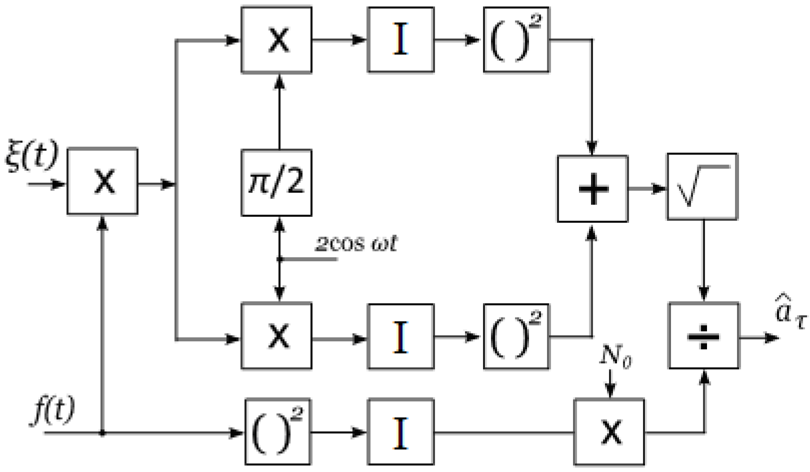


Fig. 5. Block diagram of incoherent quasi-likelihood amplitude measurer of a radio signal with unknown duration and initial phase

where $G(\tau_0, \tau) = q(\min(\tau_0, \tau))$, and

$$N_c(\tau) = \frac{2}{N_0}\int_0^\tau n(t) f(t)\cos(\omega t)\,dt, \quad N_s(\tau) = \frac{2}{N_0}\int_0^\tau n(t) f(t)\sin(\omega t)\,dt.$$

Noise components $N_c(\tau)$ and $N_s(\tau)$ represent linear transformations of Gaussian random process and, therefore, are Gaussian processes also. They have zero mathematical expectations and correlation functions of a kind

$$\langle N_c(\tau_1) N_c(\tau_2)\rangle = \langle N_s(\tau_1) N_s(\tau_2)\rangle = q(\min(\tau_1, \tau_2)), \quad \langle N_c(\tau_1) N_s(\tau_2)\rangle = 0.$$

Let us substitute Eqs. (29) in Eq. (28) and write down for incoherent QL amplitude estimates

$$\hat{a}_\tau = \sqrt{\left[\frac{a_0 G(\tau_0, \tau^*)}{q(\tau^*)}\cos\varphi_0 + \frac{N_c(\tau^*)}{q(\tau^*)}\right]^2 + \left[\frac{a_0 G(\tau_0, \tau^*)}{q(\tau^*)}\sin\varphi_0 + \frac{N_s(\tau^*)}{q(\tau^*)}\right]^2}. \quad (30)$$

We put into consideration normalized random variables $\eta_c = N_c(\tau^*)\big/\sqrt{q(\tau^*)}$, $\eta_s = N_s(\tau^*)\big/\sqrt{q(\tau^*)}$. They represent statistically independent Gaussian random values with zero mathematical expectations and unit dispersions. Using $\eta_c$ and $\eta_s$, we overwrite Eq. (30) as follows

$$\hat{a}_\tau = a_0 \sqrt{\left[\frac{G(\tau_0,\tau^*)}{q(\tau^*)}\cos\varphi_0 + \varepsilon\frac{\eta_c}{\sqrt{1+\delta_\tau}}\right]^2 + \left[\frac{G(\tau_0,\tau^*)}{q(\tau^*)}\sin\varphi_0 + \varepsilon\frac{\eta_s}{\sqrt{1+\delta_\tau}}\right]^2} \,. \quad (31)$$

where $\varepsilon = 1/z_0$. Further, we will consider that SNR $z_0$ is great enough and, therefore, the value ε is small. Developing Eq. (33) as power series in ε and neglecting members of an infinitesimal order $\varepsilon^2$ and less, we receive

$$\hat{a}_\tau = a_0\left[\frac{G(\tau_0,\tau^*)}{q(\tau^*)} + \varepsilon\frac{\eta_c\cos\varphi_0}{\sqrt{1+\delta_\tau}} + \varepsilon\frac{\eta_s\sin\varphi_0}{\sqrt{1+\delta_\tau}}\right], \quad (32)$$

Carrying out averaging, we find asymptotically (with increasing SNR) exact expressions for bias and variance of incoherent QL estimate (28)

$$B_\tau^* = \langle \hat{a}_\tau - a_0 \rangle = -a_0\,\delta_\tau \eta(\tau^* - \tau_0)/(1+\delta_\tau), \quad (33)$$

$$V_\tau^* = \left\langle (\hat{a}_\tau - a_0)^2 \right\rangle = a_0^2/z_0^2(1+\delta_\tau) + a_0^2\delta_\tau^2\eta(\tau^* - \tau_0)/(1+\delta_\tau)^2, \quad (34)$$

which coincide with the similar expressions for QL amplitude estimate of a signal without high-frequency filling and with unknown duration found in [4].

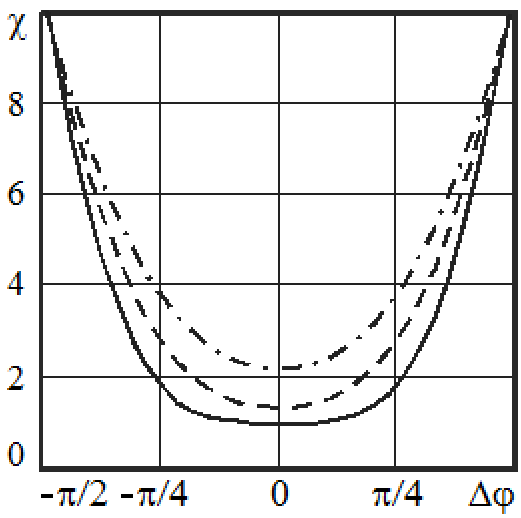


Fig. 4. Loss in accuracy of the quasi-coherent amplitude estimate under various duration detunings.

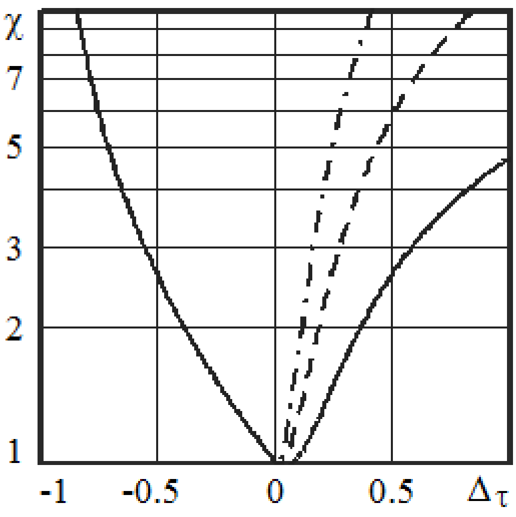


Fig. 6. Loss in accuracy of the incoherent amplitude estimate under various signal-to-noise ratios.

In Fig. 6 dependences of loss value $\chi = V_\tau / V_{0m}$ on $\Delta_\tau$ for a radio pulse with envelope in the form of a rectangular pulse with bevel top tilt (21) are presented under $b = 1$. The solid curve corresponds $z_r = 5$, dashed – $z_r = 8$, dash-dotted – $z_r = 12$. From Fig. 6 it can be seen that under parameter value $\Delta_\tau \le 0$

sacrifices of accuracy for QL estimate do not depend on SNR. For values $\Delta_\tau > 0$ sacrifices of accuracy for QL estimate increase with SNR.

## 5 Maximum likelihood signal amplitude estimate

To improve the accuracy of amplitude estimate, ML algorithm based on searching the position of the absolute maximum of FLR logarithm can be applied:

$$a_m = \arg\sup_a L(a), \quad L(a) = \sup_{\tau,\varphi} L(\tau, a, \varphi) = \sup_\tau L(\tau, a).$$

Here instead of unknown duration and initial phase their maximum likelihood estimates $\tau_m$ and $\varphi_m$ are used that is equivalent to maximization of FLR logarithm on unknown spurious parameters. Carrying out analytical maximizing the FLR logarithm (27) on a variable a we have

$$a_m = a_m(\tau_m) = \sqrt{X^2(\tau_m) + Y^2(\tau_m)} \big/ q(\tau_m), \tag{35}$$

$$\tau_m = \arg\sup_\tau L(\tau), \tag{36}$$

$$L(\tau) = \left[ X^2(\tau) + Y^2(\tau) \right] \big/ 2q(\tau). \tag{37}$$

Eq. (35) defines receiver structure. The receiver should form the random process (37) for all possible values of duration and find ML duration estimate as position of its maximum. Substituting found duration estimate in Eq. (35), we receive required ML amplitude estimate.

In Fig. 7 the block diagram of ML measurer of signal (2) amplitude is represented. Here it is designated: I – time interval $[0, t]$ integrators where $t \in [0, T_2]$, E – retriever carrying out position search of an input signal maximum within time interval $[T_1, T_2]$ (extremator), DL – delay line for time $T_1$, SD – sampling device fixing an input signal value at the point of time $T_1 + \tau_m$.

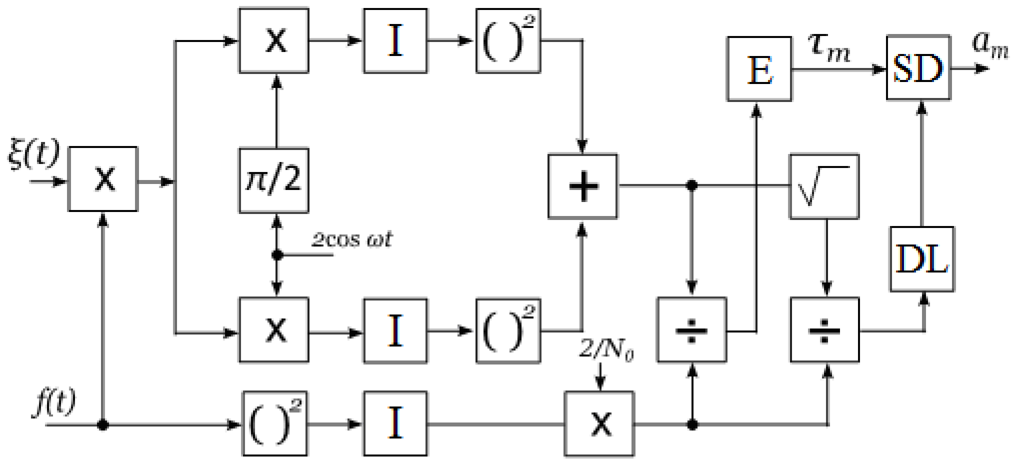


Fig. 7. Block diagram of maximum likelihood amplitude measurer of a radio signal with unknown duration and initial phase

For the analysis of ML amplitude estimate algorithm we consider the FLR logarithm (4). It represents the random field which is differentiable on parameters a, φ and nondifferentiable on a variable τ. Therefore, the amplitude and initial phase are regular (continuous) parameters of a signal (2), and duration is discontinuous parameter [2]. Thus, regularity conditions are partially broken. In work [8] it is shown that accuracy of ML estimates of regular parameters (amplitude and initial phase) does not depend on presence of unknown discontinuous parameter (duration) asymptotically (with increasing SNR). It means that bias and variance of ML amplitude estimate (35) coincide asymptotically with bias and variance (17) of amplitude estimate (16) of radio signal with a priori known duration and initial phase under great SNR. So, the dependences presented in Figs. 2-4 can be interpreted as the functions characterizing a gain in accuracy of ML estimate (35) in comparison with accuracy of quasi-coherent QL estimate (6). Correspondingly, the dependences represented in Fig. 6 show a gain in accuracy of ML estimate (35) in comparison with accuracy incoherent QL estimate (22).

## 6 Conclusion

On the basis of the conducted statistical analysis of processing algorithms of a radio signal with unknown power and nonpower parameters an influence of prior ignorance of a signal initial phase and duration on accuracy of the amplitude estimate can be evaluated. The found theoretical expressions for losses in accuracy of amplitude estimates allow to characterize relative increase in its variance quantitatively. From their analysis, in particular, follows that discrepancy of expected value of an initial phase with its true value can lead to increase in variance of the quasi-coherent quasi-likelihood estimate up many times. Statistical characteristics (bias and variance) of more complex maximum likelihood estimate of amplitude coincide asymptotically (with increasing output signal-to-noise ratio) with bias and variance of the maximum likelihood amplitude estimate of a radio signal with a priori known duration and initial phase.

The received results allow to make a valid choice of amplitude estimate algorithm depending on the available prior information, and also depending on the demands for estimate accuracy and measure of simplicity of algorithm engineering feasibility.

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