Quasioptimal Estimation of Motion Parameters Based on Laser Ranging

Article *in* Radiophysics and Quantum Electronics · September 2014

DOI: 10.1007/s11141-014-9513-x

Alexandr Kurbatov
Voronezh State University

DOI 10.1007/s11141-014-9513-x
Radiophysics and Quantum Electronics, Vol. 57, No. 4, September, 2014
(Russian Original Vol. 57, No. 4, April, 2014)

QUASIOPTIMAL ESTIMATION OF MOTION PARAMETERS BASED ON LASER RANGING

A. P. Trifonov* and **A. V. Kurbatov** UDC 621.391

An algorithm for estimation of range, velocity, and acceleration based on measurements of range in each repetition period of the probing sequence of optical pulses is synthesized. Characteristics of estimates with allowance for abnormal errors are found. It is shown that the proposed estimates are consistent, asymptotic (the signal-to-noise ratio increases for each pulse), unbiased, and effective.

1. INTRODUCTION

In many problems of optical detection and ranging [1–5], besides the range to a target, estimation of its radial velocity and radial acceleration is also of interest. Sequences of optical pulses are widely used to determine the target motion parameters. This makes it topical to study the accuracy of the methods based on their use.

Estimation of the maximum likelihood of the target range, velocity, and acceleration by the probing sequence of optical pulses were discussed earlier in [3, 4], where potential characteristics of the estimates of range, velocity, and acceleration were found. However, the hardware implementation of maximum-likelihood algorithms for estimation of all three motion parameters encounters significant difficulties. The procedure for estimation of range, velocity, and acceleration can be simplified by a quasioptimal algorithm. This algorithm is based on determining the range, velocity, and acceleration of a target from the results of laser measurements of range in each repetition period of the sequence of optical pulses.

2. CHARACTERISTICS OF THE MAXIMUM-LIKELIHOOD ESTIMATES OF MOTION PARAMETERS

The intensity of the probing sequence of optical pulses can be written according to [3–5] in the form

$$s_N(t) = \sum_{k=0}^{N-1} \hat{s}[t - (k - \mu)\,\theta - \lambda], \tag{1}$$

where the function $\hat{s}(t)$ describes the intensity of an individual pulse, λ is the temporal position of the sequence, and θ is the pulse repetition period. The parameter μ determines the sequence point with which the temporal position λ of the sequence is related. For example, the quantity λ is the arrival time of the first pulse for $\mu = 0$, the arrival time of the middle of sequence (1) for $\mu = (N-1)/2$, and the arrival time of the last pulse of the sequence for $\mu = N - 1$.

* trifonov@phys.vsu.ru

State University of Voronezh, Voronezh, Russia. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Radiofizika, Vol. 57, No. 4, pp. 319–332, April 2014. Original article submitted October 14, 2013; accepted February 28, 2014.

Assume that the probing sequence (1) is scattered by a target with range R_0, velocity V_0, and acceleration A_0, and the possible values of the vector (R, V, A) lie in the *a priori* region $\mathbf{W} = \{[R_{\min}, R_{\max}], [V_{\min}, V_{\max}], [A_{\min}, A_{\max}]\}$, where $|V_{\max}| \ll c$, $N\theta\,|A_{\max}| \ll c$, and c is the speed of light. Then, following [1–5], the received-signal intensity can be written in the form

$$s_N(t, R_0, V_0, A_0) = \sum_{k=0}^{N-1} s\big[t - 2R_0/c - (k-\mu)\,(1 + 2V_0/c)\,\theta - A_0(k-\mu)^2\,\theta^2/c\big]. \tag{2}$$

Here, the function $s(t)$ describes the temporal profile of the intensity of one received optical pulse of the sequence and can generally differ from $\hat{s}(t)$ in the sum given by Eq. (1). In Eq. (2) and below, the zero index denotes the true (unknown) values of the parameters R, V, or A of the received sequence.

Assume that a signal with intensity (2) is observed in the time interval $[0, T]$ against the background of optical noise that is a stationary Poisson process with the intensity $\nu > 0$. In this case, the signal $\pi(t)$ that is accessible for processing is a Poisson process with the intensity $\beta(t, R, V, A) = s_N(t, R, V, A) + \nu$, where the values of the parameters R, V, and A are subject to estimation. To estimate the target motion parameters R_0, V_0, and A_0 with the maximum likelihood, a logarithm of the likelihood function (log-likelihood) should be formed according to [3]:

$$L(R, V, A) = \int_0^T \ln\big[1 + s_N(t, R, V, A)/\nu\big]\,\mathrm{d}\pi(t) - \int_0^T s_N(t, R, V, A)\,\mathrm{d}t. \tag{3}$$

As an estimate with the maximum likelihood, we take the value of the vector $(\tilde{R}, \tilde{V}, \tilde{A})$ from the region $\mathbf{W}$, which corresponds to the largest maximum of the log-likelihood (3):

$$(\tilde{R}, \tilde{V}, \tilde{A}) \in \mathbf{W} \colon L(\tilde{R}, \tilde{V}, \tilde{A}) = \sup L(R, V, A). \tag{4}$$

Unconditional characteristics of estimates (4) were found in [3, 4] assuming that the values of R_0, V_0, and A_0 are uniformly distributed in the region $\mathbf{W}$. For the unconditional biases, we obtained the expressions

$$b(\tilde{R}) = \langle \tilde{R} - R_0 \rangle = 0, \qquad b(\tilde{V}) = \langle \tilde{V} - V_0 \rangle = 0, \qquad b(\tilde{A}) = \langle \tilde{A} - A_0 \rangle = 0 \tag{5}$$

and for the conditional biases we obtained

$$\begin{aligned} B(\tilde{R}) &= \langle (\tilde{R} - R_0)^2 \rangle = P_0 D_0(\tilde{R}) + (1 - P_0)\,\Delta R_{\mathrm{pr}}^2/6, \\ B(\tilde{V}) &= \langle (\tilde{V} - V_0)^2 \rangle = P_0 D_0(\tilde{V}) + (1 - P_0)\,\Delta V_{\mathrm{pr}}^2/6, \\ B(\tilde{A}) &= \langle (\tilde{A} - A_0)^2 \rangle = P_0 D_0(\tilde{A}) + (1 - P_0)\,\Delta A_{\mathrm{pr}}^2/6, \end{aligned} \tag{6}$$

where

$$\Delta R_{\mathrm{pr}} = R_{\max} - R_{\min}, \qquad \Delta V_{\mathrm{pr}} = V_{\max} - V_{\min}, \qquad \Delta A_{\mathrm{pr}} = A_{\max} - A_{\min}.$$

In Eqs. (5) and (6), the averaging (denoted by angle brackets) is performed over realizations of the Poisson process $\pi(t)$ and over the true values of estimated parameters R_0, V_0, and A_0, while the averaging of $D_0(\tilde{R})$, $D_0(\tilde{V})$, and $D_0(\tilde{A})$ is performed over the variance of reliable estimates (4), according to [6, 7], i.e., in the absence of abnormal errors. These variances coincide with those of the mutually effective estimates and are determined by the expressions from [3]

$$\begin{aligned} D_0(\tilde{R}) &= \frac{c^2}{4\alpha}\,\frac{M_2 M_4 - M_3^2}{(2M_1M_3 + M_0M_4)\,M_2 - M_2^3 - M_0M_3^2 - M_1^2M_4}, \\ D_0(\tilde{V}) &= \frac{c^2}{4\vartheta^2\alpha}\,\frac{M_0 M_4 - M_2^2}{(2M_1M_3 + M_0M_4)\,M_2 - M_2^3 - M_0M_3^2 - M_1^2M_4}, \end{aligned}$$

$$D_0(\tilde{A}) = \frac{c^2}{\vartheta^4 \alpha} \frac{M_0 M_2 - M_1^2}{(2M_1 M_3 + M_0 M_4) M_2 - M_2^3 - M_0 M_3^2 - M_1^2 M_4}, \tag{7}$$

where

$$\alpha = \int_0^T \frac{1}{\nu + s(t)} \left[\frac{\mathrm{d}s(t)}{\mathrm{d}t}\right]^2 \mathrm{d}t, \qquad M_n = \sum_{k=0}^{N-1} (k - \mu)^n. \tag{8}$$

The quantity P_0 in Eqs. (6) is the reliable-estimates probability taken from [4, 6]:

$$P_0 = \frac{1}{\sqrt{2\pi\,(1+\kappa^2)}} \int_{\sqrt{2}}^{\infty} \exp\left[-\frac{(x-\eta)^2}{2\,(1+\kappa^2)} - \frac{\xi x^2}{(2\pi)^2} \exp\Bigl(-\frac{x^2}{2}\Bigr)\right] \mathrm{d}x,$$

where

$$\xi = \frac{Q\theta^3}{\sigma_N^3 c^3} \frac{N\,(N^2-1)}{12} \sqrt{\frac{N\,(N^2-4)}{15}} \left[\frac{1}{\nu} \int_0^T \left(\frac{\mathrm{d}s(t)/\mathrm{d}t}{1 + s(t)/\nu}\right)^2 \mathrm{d}t\right]^{3/2},$$

$$\kappa^2 = \int_0^T \ln^2\bigl[1 + s(t)/\nu\bigr] s(t)/\nu \,\mathrm{d}t \Bigg/ \int_0^T \ln^2\bigl[1 + s(t)/\nu\bigr]\,\mathrm{d}t,$$

$$\eta = \sqrt{N} \int_0^T \ln\bigl[1 + s(t)/\nu\bigr] s(t)/\,\mathrm{d}t \Bigg/ \sqrt{\nu \int_0^T \ln^2\bigl[1 + s(t)/\nu\bigr]\,\mathrm{d}t},$$

$$\sigma_N^2 = \nu \int_0^T \ln^2\bigl[1 + s(t)/\nu\bigr]\,\mathrm{d}t,$$

and $Q = (R_{\max} - R_{\min})\,(V_{\max} - V_{\min})\,(A_{\max} - A_{\min})$ is the Eucleadian volume of the *a priori* region $\mathbf{W}$ of possible values of unknown parameters. The quantity ξ has the meaning of the reduced volume, taken from [7], of the *a priori* region of possible values of unknown range, velocity, and acceleration, which determines the number of discernible values of range, velocity, and acceleration (4) in the region $\mathbf{W}$.

The signal-to-noise ratio (SNR) for the receiver with the maximum likelihood (3), according to [4, 6], has the form

$$z_N^2 = \frac{N}{\nu} \left[\int_{-\infty}^{+\infty} \ln[1 + s(t)/\nu] s(t)\,\mathrm{d}t\right]^2 \Bigg/ \int_{-\infty}^{+\infty} \ln^2[1 + s(t)/\nu]\,[1 + s(t)/\nu]\,\mathrm{d}t. \tag{9}$$

The estimate of the maximum likelihood (4) for a large signal-to-noise ratio (9) is asymptotically effective, as well as unbiased and consistent.

Hardware implementation of estimate (4) is quite difficult since, as a rule, for the creation of a gauge with the maximum likelihood, one has to use a multichannel (velocity and acceleration) circuit. In this case, the gauge contains parallel channels, and each channel generates an algorithm for the log-likelihoods in one of some set of points in the *a priori* region of possible values of the velocity and acceleration. Each channel of the gauge contains a matched filter for one pulse and a perfect comb filter. Howerer, the technical implementation of a comb filter for a large number of pulses and a large *a priori* variation range of the velocity and acceleration is difficult because of the rigid requirements to stability of the delay-line parameters and high accuracy of location of taps for provision of synchronous accumulation of pulses.

3. ESTIMATES OF THE TEMPORAL POSITION OF A PULSE

In order to simplify the implementation of an optical device for measurement of range, velocity, and acceleration, we consider the possibility of determining them by the results of laser measurements of the range R_k in each repetition period of the optical-pulse sequence. Let us introduce the designations

$$\lambda_k = 2R/c + (k-\mu)\,(1+2V/c)\,\theta + A\,(k-\mu)^2\,\theta^2/c,$$
$$\lambda_{0k} = 2R_0/c + (k-\mu)\,(1+2V_0/c)\,\theta + A_0\,(k-\mu)^2\,\theta^2/c. \tag{10}$$

Here, the variables λ_k and λ_{0k} have the meaning of temporal positions of the kth pulse. In designations (10), the received-signal intensity (2) can be written as

$$s_N(t, R_0, V_0, A_0) = \sum_{k=0}^{N-1} s(t-\lambda_{0k}).$$

Here, the temporal position λ_k of the kth pulse, which determines the target range $R_k = c\lambda_k/2$ in each repetition period $[t_k, t_{k+1}]$, takes the values from the *a priori* interval

$$\mathbf{\Lambda}_k = [\Lambda_{k\,\min}, \Lambda_{k\,\max}], \tag{11}$$

$$\Lambda_{k\,\min} = \begin{cases} 2R_{\min}/c + (k-\mu)\,(1+2V_{\min}/c)\,\theta + A_{\min}\,(k-\mu)^2\,\theta^2/c, & k>\mu; \\ 2R_{\min}/c + (k-\mu)\,(1+2V_{\max}/c)\,\theta + A_{\min}\,(k-\mu)^2\,\theta^2/c, & k\le\mu, \end{cases}$$

$$\Lambda_{k\,\max} = \begin{cases} 2R_{\max}/c + (k-\mu)\,(1+2V_{\max}/c)\,\theta + A_{\max}\,(k-\mu)^2\,\theta^2/c, & k>\mu; \\ 2R_{\max}/c + (k-\mu)\,(1+2V_{\min}/c)\,\theta + A_{\max}\,(k-\mu)^2\,\theta^2/c, & k\le\mu. \end{cases}$$

The boundaries $\Lambda_{k\,\min}$ and $\Lambda_{k\,\max}$ are determined differently depending on the ratio between the parameters k and μ. The middles $\Lambda_{\mathrm{pr}k} = (\Lambda_{k\,\min} + \Lambda_{k\,\max})/2$ of the intervals $\mathbf{\Lambda}_k$ can be found from the formulas

$$\Lambda_{\mathrm{pr}k} = 2R_{\mathrm{pr}}/c + (k-\mu)\,(1+2V_{\mathrm{pr}}/c)\,\theta + A_{\mathrm{pr}}\,(k-\mu)^2\,\theta^2/c,$$

where

$$R_{\mathrm{pr}} = \frac{R_{\max}+R_{\min}}{2}, \qquad V_{\mathrm{pr}} = \frac{V_{\max}+V_{\min}}{2}, \qquad A_{\mathrm{pr}} = \frac{A_{\max}+A_{\min}}{2}.$$

With allowance for definitions (10), the estimates of range, velocity, and acceleration can be formed from the estimates $\hat\lambda_k$ of the parameters λ_{0k}. We will seek the estimates $\hat\lambda_k$ of the arrival times λ_{0k} of the sequence pulses separately in each repetition period by the maximum likelihood technique. Consider a log-likelihood for one pulse

$$L_k(\lambda_k) = \int_{t_k}^{t_{k+1}} \ln\bigl[1 + s(t-\lambda_k)/\nu\bigr]\,\mathrm{d}\pi(t) - \int_{t_k}^{t_{k+1}} s(t-\lambda_k)\,\mathrm{d}t. \tag{12}$$

As the estimate $\hat\lambda_k$ we take the point of the greatest maximum of the log-likelihood (12)

$$\hat\lambda_k\colon L_k(\hat\lambda_k) = \sup L_k(\lambda_k), \qquad \lambda_k \in \mathbf{\Lambda}_k. \tag{13}$$

In order to calculate statistical characteristics of the estimate $\hat\lambda_k$ of the parameter λ_{0k}, we represent logarithm (12) as the sum of the signal and noise functions according to [6]:

$$L_k(\lambda_k) = S_k(\lambda_{0k}, \lambda_k) + N_k(\lambda_k) + C. \tag{14}$$

The signal function is determined from the relation

$$S_k(\lambda_{0k}, \lambda_k) = \langle L_k(\lambda_k)\rangle - C,$$

where the angle brackets denote the conditional (for a fixed λ_{0k}) mathematical expectation assuming that the received signal $\pi(t)$ is a Poisson process with the intensity $\beta_k(t) = s(t-\lambda_k)+\nu$ and corresponds to the true value λ_{0k} of the parameter λ_k. The quantity C is determined by the expression

$$C = \nu \int\limits_{t_k}^{t_{k+1}} \ln\bigl[1 + s(t-\lambda_k)/\nu\bigr]\,\mathrm{d}t - \int\limits_{t_k}^{t_{k+1}} s(t-\lambda_k)\,\mathrm{d}t. \tag{15}$$

Since the parameter λ_k is independent of the signal energy, the quantity C is independent of the estimated parameter λ_k. Moreover, since the pulse energy is independent of the pulse number k, the quantity C is independent of the number k. Thus, for the signal function we have a representation

$$S_k(\lambda_{0k}, \lambda_k) = \int\limits_{t_k}^{t_{k+1}} \ln\bigl[1 + s(t-\lambda_k)/\nu\bigr]\,s(t-\lambda_{0k})\,\mathrm{d}t. \tag{16}$$

Let us introduce the following designation for the maximum of the signal function:

$$m_S = S_k(\lambda_{0k}, \lambda_{0k}) = \int\limits_{t_k}^{t_{k+1}} \ln\bigl[1 + s(t)/\nu\bigr]\,s(t)\,\mathrm{d}t. \tag{17}$$

The point at which the signal function (16) reaches the greatest maximum is the true value λ_{0k} of the parameter λ_k, such that $S_k(\lambda_{0k}, \lambda_{0k}) = \sup S_k(\lambda_{0k}, \lambda_k)$ according to [6]. We define the noise function by the expression $N_k(\lambda_k) = L_k(\lambda_k) - \langle L_k(\lambda_k)\rangle$. The mathematical expectation of the noise function is equal to zero, and the correlation function for arbitrary positions λ_{1k} and $\lambda_{2k} \in \mathbf{\Lambda}_k$ has the form

$$\begin{aligned} K_N(\lambda_{1k}, \lambda_{2k}) &= \langle N_k(\lambda_{1k}) N_k(\lambda_{2k})\rangle \\ &= \nu \int\limits_{t_k}^{t_{k+1}} \ln[1 + s(t-\lambda_{1k})/\nu]\ln[1 + s(t-\lambda_{2k})/\nu]\,\bigl[1 + s(t-\lambda_{0k})/\nu\bigr]\,\mathrm{d}t, \end{aligned} \tag{18}$$

where $\lambda_{1k} = 2R_1/c + (k-\mu)\,(1+2V_1/c)\,\theta + A_1\,(k-\mu)^2\,\theta^2/c$, $\lambda_{2k} = 2R_2/c + (k-\mu)\,(1+2V_2/c)\,\theta + A_2\,(k-\mu)^2\,\theta^2/c$. We now write an expression for the noise-function variance at the maximum λ_{0k} of the signal function:

$$\sigma_{SN}^2 = K_N(\lambda_{0k}, \lambda_{0k}) = \nu \int\limits_{t_k}^{t_{k+1}} \ln^2[1 + s(t)/\nu]\,[1 + s(t)/\nu]\,\mathrm{d}t. \tag{19}$$

From Eqs. (17) and (19) it follows that the signal-to-noise ratio in the case of reception of one pulse of the sequence (2), according to [6], has the form

$$z_1^2 = \frac{m_S^2}{\sigma_{SN}^2} = \frac{1}{\nu}\left[\int\limits_{-\infty}^{+\infty} \ln\bigl[1 + s(t)/\nu\bigr]\,s(t)\,\mathrm{d}t\right]^2 \Bigg/ \int\limits_{-\infty}^{+\infty} \ln^2[1 + s(t)/\nu]\,[1 + s(t)/\nu]\,\mathrm{d}t. \tag{20}$$

We now determine the duration $\Delta\lambda$ of signal functions (16) by the relation $S_k(\lambda_{0k}, \lambda_{0k} \pm \Delta\lambda) \approx 0$.

The durations $\Delta\lambda$ of signal functions are independent of the number k since it is assumed that the shapes of all pulses are identical in one sequence. We designate by $\mathbf{\Lambda}_{kS} = [\lambda_{0k} - \Delta\lambda; \lambda_{0k} + \Delta\lambda]$ the subrange of *a priori* range (11), in which the central peak of signal function (16) is significantly different from zero, and by $\mathbf{\Lambda}_{kN}$ we designate the completion of the interval $\mathbf{\Lambda}_{kS}$ to $\mathbf{\Lambda}_k$. We call the interval $\mathbf{\Lambda}_{kS}$ a signal region and $\mathbf{\Lambda}_{kN}$, a noise region. By definition, the signal function $S_k(\lambda_{0k}, \lambda_k) \approx 0$ for $\lambda_k \in \mathbf{\Lambda}_{kN}$.

Consider at first the case where the estimate $\hat{\lambda}_k$ enters the signal region $\mathbf{\Lambda}_{kS}$. Such an estimate is called reliable according to [6].

The characteristics of a reliable estimate can be found from the solution of the likelihood equation given in [6]. We will solve this equation by the method [6] of a small parameter, as which we use the quantity $\varepsilon = 1/z_1$, which is inverse of the signal-to-noise ratio (20). Confining ourselves to the first approximation, we find the mathematical expectation of a reliable estimate

$$m_{0k}(\hat{\lambda}_k \mid R_0, V_0, A_0) = \langle \hat{\lambda}_k \rangle = \lambda_{0k} = 2R_0/c + (k - \mu)\,(1 + 2V_0/c)\,\theta + A_0\,(k - \mu)^2\,\theta^2/c \tag{21}$$

and its variance according to [8]:

$$\sigma_0^2 = \left\langle (\hat{\lambda}_k - \lambda_{0k})^2 \right\rangle = 1/\alpha. \tag{22}$$

Here, α was determined by Eq. (8). If, besides the requirement

$$z_1 \gg 1,$$

the condition from [9] is fulfilled,

$$\int\limits_{-\infty}^{+\infty} s(t)\,\mathrm{d}t + \nu\tau \gg 1, \tag{23}$$

then the reliable estimate is roughly a Gaussian random value. In inequality (23), by τ we designate the duration of one optical pulse with intensity $s(t)$.

If the signal-to-noise ratio (20) is not too large and for the interval of possible values of the temporal positions λ_k the inequality

$$\Delta\Lambda_{\mathrm{pr}k}/\Delta\lambda \gg 1, \tag{24}$$

where

$$\Delta\Lambda_{\mathrm{pr}k} = \Lambda_{k\,\max} - \Lambda_{k\,\min} = 2\,\Delta R_{\mathrm{pr}}/c + 2\,\Delta V_{\mathrm{pr}}\,|k - \mu|\,\theta/c + \Delta A_{\mathrm{pr}}\,(k - \mu)^2\,\theta^2/c, \tag{25}$$

is fulfilled, then, according to [6], abnormal errors may appear [6]. To determine the impact of abnormal errors, it is necessary to find, following [6], the reliable-estimate probability

$$P_{0\lambda k} = P[\hat{\lambda}_k \in \mathbf{\Lambda}_{k\,S}]. \tag{26}$$

The approximate reliable-estimate probability $P_{0\lambda k}$ can be found when the Gaussian approximation of the decision statistics (12) is acceptable. In the signal region $\mathbf{\Lambda}_{kS}$, the distribution of the log-likelihood is roughly Gaussian if condition (23) is fulfilled. In the noise region $\mathbf{\Lambda}_{kN}$, the distribution of the log-likelihood can be approximated by a Gaussian one if, along with inequality (23), the condition from [9] is fulfilled:

$$\nu\tau \gg 1. \tag{27}$$

Obviously, if relation (27) is fullfilled, then inequality (23) is always fulfilled.

By virtue of definition (13) for the estimate of the position $\hat{\lambda}_k$ with the maximum likelihood, the probability (26) of a reliable estimate can be represented as

$$P_{0\lambda k} = P(H_S > H_N), \tag{28}$$

where

$$H_S = \sup_{\lambda_k \in \mathbf{\Lambda}_{kS}} L_k(\lambda_k), \qquad H_N = \sup_{\lambda_k \in \mathbf{\Lambda}_{kN}} L_k(\lambda_k).$$

If condition (24) is fulfilled, then the random quantities H_S and H_N are roughly statistically independent according to [6]. Therefore, Eq. (28) takes the form

$$P_{0\lambda k} = \int_{-\infty}^{+\infty} F_N(H)\,\mathrm{d}F_S(H), \tag{29}$$

where $F_N(H)$ and $F_S(H)$ are the distribution functions of the random quantities H_N and H_S, respectively.

Let us find the distribution function $F_S(H)$. Since the subintervals $\mathbf{\Lambda}_{kS}$ roughly coincide with the region of high correlation of random process (14), then for a sufficiently large signal-to-noise ratio (20), the approximation

$$L_k(\hat{\lambda}_k) \approx m_S + N_k(\lambda_{0k}). \tag{30}$$

is valid. Hereafter, insignificant constant (15) is omitted. Since the random quantity H_S is roughly Gaussian with mathematical expectation (17) and variance (19), then it can be written that

$$F_S(H) = \Phi\big[(H - m_S)/\sigma_{SN}\big], \tag{31}$$

where $\Phi(x) = (2\pi)^{-1/2} \int_{-\infty}^{x} \exp(-t^2/2)\,\mathrm{d}t$ is the probability integral.

In the region of abnormal errors $\mathbf{\Lambda}_{kN}$, the decision statistics $L_k(\lambda_k)$ has also a roughly Gaussian distribution. The correlation function of the noise function in the noise region is somewhat different from expression (18). Namely, for the positions $\lambda_{1k}, \lambda_{2k} \in \mathbf{\Lambda}_{kN}$ the function

$$K_N(\lambda_{2k}, \lambda_{1k}) = \nu \int_{t_k}^{t_{k+1}} \ln[1 + s(t - \lambda_{1k})/\nu] \ln[1 + s(t - \lambda_{2k})/\nu]\,\mathrm{d}t,$$

so that in the noise region the log-likelihood is a stationary random process with the variance

$$\sigma_N^2 = K_N(\lambda_k, \lambda_k) = \nu \int_{t_k}^{t_{k+1}} \ln^2[1 + s(t)/\nu]\,\mathrm{d}t.$$

Approximation for the distribution function of the greatest maximum of the noise component H_N with the inequality (24) fulfilled and for a large ratio H/σ_N, according to [6], has the form

$$F_N(H) = \begin{cases} \exp\left[-\dfrac{\xi_k}{2\pi} \exp\left(-\dfrac{H^2}{2\sigma_N^2}\right)\right], & H \geq 0, \\ 0, & H < 0. \end{cases} \tag{32}$$

Here, the variable

$$\xi_k = \Delta\Lambda_{\mathrm{pr}k} \sqrt{\frac{1}{\sigma_N^2} \frac{\partial^2 K_N(\lambda_{1k}, \lambda_{2k})}{\partial\lambda_{1k}\,\partial\lambda_{2k}}\bigg|_{\lambda_{1k}=\lambda_{2k}}} = \frac{\Delta\Lambda_{\mathrm{pr}k}}{\nu} \sqrt{\int_{-\infty}^{+\infty} \frac{\big[\mathrm{d}s/\mathrm{d}t(t)\big]^2}{\big[1 + s(t)/\nu\big]^2}\,\mathrm{d}t} \bigg/ \sqrt{\int_{-\infty}^{+\infty} \ln^2\big[1 + s(t)/\nu\big]\,\mathrm{d}t},$$

and the parameter $\Delta\Lambda_{\mathrm{pr}k}$ was determined by Eq. (25). The quantities ξ_k have the meaning of the reduced lengths taken from [7] for *a priori* intervals of possible values of the temporal positions of the pulses and determine the number of discernible values of the temporal positions in the interval $\mathbf{\Lambda}_k$.

Substituting expressions (31) and (32) into Eq. (29), we obtain an approximate reliable-estimate probability according to [8]

$$P_{0\lambda k} = \frac{1}{\sqrt{2\pi\,(1+\kappa_1^2)}} \int\limits_0^\infty \exp\left[-\frac{(x-\eta_1)^2}{2\,(1+\kappa_1^2)} - \frac{\xi_k}{2\pi}\exp\Bigl(-\frac{x^2}{2}\Bigr)\right] \mathrm{d}x. \tag{33}$$

Here, the coefficients

$$\kappa_1^2 = \int\limits_{-\infty}^{+\infty} \ln^2\bigl[1+s(t)/\nu\bigr]\, s(t)/\nu \,\mathrm{d}t \Bigg/ \int\limits_{-\infty}^{+\infty} \ln^2\bigl[1+s(t)/\nu\bigr]\,\mathrm{d}t,$$

$$\eta_1 = \int\limits_{-\infty}^{+\infty} \ln\bigl[1+s(t)/\nu\bigr]\, s(t)\,\mathrm{d}t \Bigg/ \sqrt{\nu \int\limits_{-\infty}^{+\infty} \ln^2[1+s(t)/\nu]\,\mathrm{d}t}.$$

The accuracy of Eq. (33) increases with increasing variables ξ_k and signal-to-noise ratio (20).

For the possible presence of abnormal errors, estimates (13) are statistically independent random quantities with the probability density from [6]

$$W_k(\hat\lambda_k) = P_{0\lambda k} W_0(\hat\lambda_k) + P_{\mathrm{a}\lambda k} W_{\mathrm{a}}(\hat\lambda_k). \tag{34}$$

Here, $W_0(\hat\lambda_k)$ is the Gaussian probability density of the reliable estimate $\hat\lambda_k$, which has momenta (21) and (22). By virtue of the stationarity of logarithm (12) in the noise region, the probability density $W_{\mathrm{a}}(\hat\lambda_k)$ of an abnormal error is constant in the *a priori* interval (11) according to [6]. Therefore, the first two conditional (with respect to R_0, V_0, and A_0) momenta of estimate (13) are determined by the expressions

$$\begin{aligned} m_k = m_k(\hat\lambda_k \mid R_0, V_0, A_0) = \langle \hat\lambda_k \rangle &= P_{0\lambda k}\lambda_{0k} + (1-P_{0\lambda k})\,\Lambda_{\mathrm{pr}k} \\ &= P_{0\lambda k}\bigl[2R_0/c + (k-\mu)\,(1+2V_0/c)\,\theta + A_0\,(k-\mu)^2\,\theta^2/c\bigr] \\ &\quad + (1-P_{0\lambda k})\,\bigl[2R_{\mathrm{pr}}/c + (k-\mu)\,(1+2V_{\mathrm{pr}}/c)\,\theta + A_{\mathrm{pr}}\,(k-\mu)^2\,\theta^2/c\bigr], \end{aligned} \tag{35}$$

$$\begin{aligned} \sigma_k^2 = \sigma_k^2(\hat\lambda_k \mid R_0, V_0, A_0) = \bigl\langle (\hat\lambda_k - \langle\hat\lambda_k\rangle)^2 \bigr\rangle &= \frac{P_{0\lambda k}}{\alpha} + \frac{1-P_{0\lambda k}}{12}\,\Delta\Lambda_{\mathrm{pr}k}^2 + P_{0\lambda k}\,(1-P_{0\lambda k})\,(\Lambda_{\mathrm{pr}k} - \lambda_{0k})^2 \\ &= \frac{P_{0\lambda k}}{\alpha} + \frac{1-P_{0\lambda k}}{12c^2}\bigl[2\,\Delta R_{\mathrm{pr}} + 2\,\Delta V_{\mathrm{pr}}\,|k-\mu|\,\theta + \Delta A_{\mathrm{pr}}\,(k-\mu)^2\,\theta^2\bigr]^2 \\ &\quad + \frac{P_{0\lambda k}\,(1-P_{0\lambda k})}{c^2}\bigl[2\,(R_{\mathrm{pr}}-R_0) + 2\,(V_{\mathrm{pr}}-V_0)\,(k-\mu)\,\theta + (A_{\mathrm{pr}}-A_0)\,(k-\mu)^2\,\theta^2\bigr]^2. \end{aligned} \tag{36}$$

Correspondingly, the conditional bias and conditional spread of estimate (13) can be written as

$$\begin{aligned} b_k = b_k(\hat\lambda_k \mid R_0, V_0, A_0) = \langle \hat\lambda_k - \lambda_{0k} \rangle &= (1-P_{0\lambda k})\,(\Lambda_{\mathrm{pr}k} - \lambda_{0k}) \\ &= 2\,(R_{\mathrm{pr}}-R_0)/c + 2\,(k-\mu)\,(V_{\mathrm{pr}}-V_0)\,\theta/c + (A_{\mathrm{pr}}-A_0)\,(k-\mu)^2\,\theta^2/c, \end{aligned}$$

$$\begin{aligned} B_k = B_k(\hat\lambda_k \mid R_0, V_0, A_0) = \bigl\langle (\hat\lambda_k - \lambda_{0k})^2 \bigr\rangle &= \frac{P_{0\lambda k}}{\alpha} + (1-P_{0\lambda k})\left[\frac{\Delta\Lambda_{\mathrm{pr}k}^2}{12} + \bigl(\Lambda_{\mathrm{pr}k} - \lambda_{0k}\bigr)^2\right] \\ &= \frac{P_{0\lambda k}}{\alpha} + \frac{1-P_{0\lambda k}}{c^2}\Bigl\{\frac{1}{12}\bigl[2\,\Delta R_{\mathrm{pr}}\,c + 2\,\Delta V_{\mathrm{pr}}\,|k-\mu|\,\theta + \Delta A_{\mathrm{pr}}\,(k-\mu)^2\,\theta^2\bigr]^2 \\ &\quad + \bigl[2\,(R_{\mathrm{pr}}-R_0) + 2\,(V_{\mathrm{pr}}-V_0)\,(k-\mu)\,\theta + (A_{\mathrm{pr}}-A_0)\,(k-\mu)^2\,\theta^2\bigr]^2\Bigr\}. \end{aligned}$$

4. QUASIOPTIMAL ESTIMATES OF MOTION PARAMETERS

Consider the possibility of using estimates (13) with the maximum likelihood to obtain the estimates of range, velocity, and acceleration. It is seen from Eq. (34) that the distribution of estimates (13) in the presence of abnormal errors is non-Gaussian. Due to the relatively complex shape of this non-Gaussian distribution, the synthesis of a quasioptimal estimate is difficult. Therefore, when seeking the quasioptimal estimate we will confine ourselves to the Gaussian approximation of the distribution of estimates (13), which is valid for the values $P_{0\lambda k}$ that are close to unity. Then for the conditional probability density of random quantity (13) in the case of a reliable estimate, one can write an approximate formula

$$W_k(\hat{\lambda}_k \mid R, V, A) = \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left\{-\frac{[\hat{\lambda}_k - m_{0k}(\hat{\lambda}_k \mid R, V, A)]^2}{2\sigma_0^2}\right\}, \qquad (37)$$

where the mathematical expectation $m_{0k}(R, V, A)$ was determined by expression (21), and variance σ_0^2, by equality (22).

We will use a set of N independent random quantities (13) as initial statistics for obtaining quasioptimal estimates of the range $\hat{R}$, velocity $\hat{V}$, and acceleration $\hat{A}$. The corresponding likelihood function has the form

$$W(\hat{\lambda}_0, \ldots, \hat{\lambda}_{N-1} \mid R, V, A) = \prod_{k=0}^{N-1} W_k(\hat{\lambda}_k \mid R, V, A). \qquad (38)$$

Substituting formula (37) into equality (38) and omitting insignificant terms, for the log-likelihood function we obtain the expression

$$\ln W(\hat{\lambda}_0, \ldots, \hat{\lambda}_{N-1} \mid R, V, A) = -\frac{1}{2\sigma_0^2} \sum_{k=0}^{N-1} \left[\hat{\lambda}_k - 2R/c - (k-\mu)(1 + 2V/c)\,\theta - A\,(k-\mu)^2\,\theta^2/c\right]^2. \qquad (39)$$

According to the maximum-likelihood technique, as the estimates $\hat{R}$, $\hat{V}$, and $\hat{A}$ one should take the values for which the function (39) is the maximum. As is well known, the conditions

$$\frac{\partial \ln W}{\partial R} = 0, \qquad \frac{\partial \ln W}{\partial V} = 0, \qquad \frac{\partial \ln W}{\partial A} = 0$$

should be fulfilled at the maximum. In detailed form, these equations are given by

$$\frac{1}{c}\begin{pmatrix} 4M_0 & 4\theta M_1 & 2\theta^2 M_2 \\ 4\theta M_1 & 4\theta^2 M_2 & 2\theta^3 M_3 \\ 2\theta^2 M_2 & 2\theta^3 M_3 & \theta^4 M_4 \end{pmatrix} \begin{pmatrix} \hat{R} \\ \hat{V} \\ \hat{A} \end{pmatrix} = \begin{pmatrix} 2\hat{M}_0 - 2\theta M_1 \\ 2\theta \hat{M}_1 - 2\theta^2 M_2 \\ \theta^2 \hat{M}_2 - \theta^3 M_3 \end{pmatrix}, \qquad (40)$$

where the coefficients M_n are defined by formula (8), and the quantities

$$\hat{M}_n = \sum_{k=0}^{N-1} \left[(k-\mu)^n\,\hat{\lambda}_k\right].$$

Solving Eq. (40), we obtain the vector

$$\begin{pmatrix} \hat{R} \\ \hat{V} \\ \hat{A} \end{pmatrix} = \sum_{k=0}^{N-1} \begin{pmatrix} \delta_{R\,k} \\ \delta_{V\,k} \\ \delta_{A\,k} \end{pmatrix} \hat{\lambda}_k - \begin{pmatrix} 0 \\ c/2 \\ 0 \end{pmatrix}, \qquad (41)$$

where the elements

$$\delta_{Rk} = \frac{c\left[(M_2M_4 - M_3^2) + (k-\mu)(M_2M_3 - M_1M_4) + (k-\mu)^2(M_1M_3 - M_2^2)\right]}{2\left[(2M_1M_3 + M_0M_4)M_2 - M_2^3 - M_0M_3^2 - M_1^2M_4\right]},$$
$$\delta_{Vk} = \frac{c\left[(M_2M_3 - M_1M_4) + (k-\mu)(M_0M_4 - M_2^2) + (k-\mu)^2(M_1M_2 - M_0M_3)\right]}{2\theta\left[(2M_1M_3 + M_0M_4)M_2 - M_2^3 - M_0M_3^2 - M_1^2M_4\right]},$$
$$\delta_{Ak} = \frac{c\left[(M_1M_3 - M_2^2) + (k-\mu)(M_1M_2 - M_0M_3) + (k-\mu)^2(M_0M_2 - M_1^2)\right]}{\theta^2\left[(2M_1M_3 + M_0M_4)M_2 - M_2^3 - M_0M_3^2 - M_1^2M_4\right]}.$$

The elements δ_{Rk}, δ_{Vk}, and δ_{Ak} depend only on the number k and the quantity μ, and it is therefore sufficient to calculate them one time.

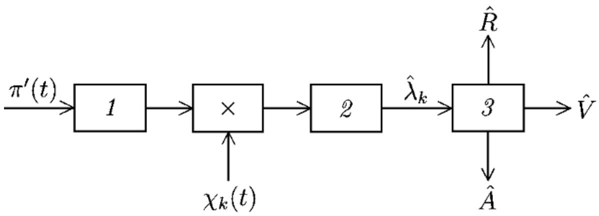

Fig. 1. Block diagram of a quasioptimal gauge.

Since the estimates (41) were obtained on the basis of information only about the estimates $\hat{\lambda}_k$, but not full information about the received sample, these estimates are not the maximum-likelihood estimates.

The described quasioptimal algorithm can be implemented using the block diagram shown in Fig. 1. A flow of short pulses, which is the derivative of the realization of the Poisson random process $\pi'(t)$, is fed to the receiver input. The pulse flow passes through the filter (unit *1*) with the impulse response $h(t) = a\ln\left[1 + s(t^* - t)/\nu\right]$, where a is the filter gain and t^* is the delay, which should be $t^* > \tau$ if inequality (23) is fulfilled. After the filter, the signal is multiplied by the function

$$\chi_k(t) = \begin{cases} 1, & t \in \mathbf{\Lambda}_k, \\ 0, & t \notin \mathbf{\Lambda}_k. \end{cases}$$

successively for all numbers $k = 0, \ldots, N-1$. Unit *2* (extremator) determines the temporal position of the greatest maximum of the signal fed to the input and generates a sequence of estimates $\hat{\lambda}_k$. Unit *3* is the computing device that calculates the estimates of range, velocity, and acceleration using Eq. (41).

Let us find the characteristics of quasioptimal estimates (41). In order to obtain mathematical expectations of estimates (41), it is needed that instead of the estimates $\hat{\lambda}_k$, their mathematical expectations be substituted into them. As a result, we obtain

$$\begin{pmatrix} \langle \hat{R} \mid R_0, V_0, A_0 \rangle \\ \langle \hat{V} \mid R_0, V_0, A_0 \rangle \\ \langle \hat{A} \mid R_0, V_0, A_0 \rangle \end{pmatrix} = \sum_{k=0}^{N-1} \begin{pmatrix} \delta_{Rk} \\ \delta_{Vk} \\ \delta_{Ak} \end{pmatrix} \langle \hat{\lambda}_k \rangle - \begin{pmatrix} 0 \\ c/2 \\ 0 \end{pmatrix}. \qquad (42)$$

Subtracting from Eq. (42) the identity

$$\begin{pmatrix} R_0 \\ V_0 \\ A_0 \end{pmatrix} = \sum_{k=0}^{N-1} \begin{pmatrix} \delta_{Rk} \\ \delta_{Vk} \\ \delta_{Ak} \end{pmatrix} \lambda_{0k} - \begin{pmatrix} 0 \\ c/2 \\ 0 \end{pmatrix}$$

and using Eq. (35), we obtain an expression for the conditional biases of quasioptimal estimates:

$$b(\hat{l} \mid R_0, V_0, A_0) = \frac{2}{c}\varphi_{l0}(R_{\rm pr} - R_0) + \frac{2\theta}{c}\varphi_{l1}(V_{\rm pr} - V_0) + \frac{\theta^2}{c}\varphi_{l2}(A_{\rm pr} - A_0). \qquad (43)$$

Here, l is one of the motion parameters to estimate, namely, R, V, or A, the coefficients

$$\varphi_{lm} = \sum_{k=0}^{N-1} \delta_{lk} P_{\mathrm{a}\lambda k}\,(k-\mu)^m, \qquad m = 0, 1, 2, \tag{44}$$

and $P_{\mathrm{a}\lambda k} = 1 - P_{0\lambda k}$ is the abnormal-error probability [6].

According to Eq. (43), quasioptimal estimates (41) in the presence of abnormal estimates are conditionally biased. Using the non-dependence of random quantities (13), from equalities (36), (41), and (43) we obtain an expression for the conditional spread of quasioptimal estimates:

$$B(\hat{l} \mid R_0, V_0, A_0) = \sum_{k=0}^{N-1} \delta_{lk}^2 \sigma_k^2(\hat{\lambda}_k \mid R_0, V_0, A_0) + b^2(\hat{l} \mid R_0, V_0, A_0).$$

Substituting expressions (36) and (43) into this formula, we obtain

$$\begin{aligned} B(\hat{l} \mid R_0, V_0, A_0) = \frac{1}{\alpha} \sum_{k=0}^{N-1} \delta_{lk}^2 P_{0\lambda k} + \frac{1}{c^2} \Big[\frac{1}{12} \big(& 4\chi_{l0}\,\Delta R_{\mathrm{pr}}^2 + 4\theta^2 \chi_{l2}\,\Delta V_{\mathrm{pr}}^2 + \theta^4 \chi_{l4}\,\Delta A_{\mathrm{pr}}^2 \\ & + 8\theta\chi_{l1}\,\Delta R_{\mathrm{pr}}\,\Delta V_{\mathrm{pr}} + 4\theta^2\chi_{l2}\,\Delta R_{\mathrm{pr}}\,\Delta A_{\mathrm{pr}} + 4\theta^3\chi_{l3}\,\Delta V_{\mathrm{pr}}\,\Delta A_{\mathrm{pr}} \big) \\ + 4\,(\psi_{l0} + \varphi_{l0}^2)\,(R_{\mathrm{pr}} - R_0)^2 & + 4\theta^2\,(\psi_{l2} + \varphi_{l1}^2)\,(V_{\mathrm{pr}} - V_0)^2 + \theta^4\,(\psi_{l4} + \varphi_{l2}^2)\,(A_{\mathrm{pr}} - A_0)^2 \\ + 8\theta\,(\psi_{l1} + \varphi_{l0}\varphi_{l1})\,(R_{\mathrm{pr}} - R_0)\,& (V_{\mathrm{pr}} - V_0) + 4\theta^3\,(\psi_{l2} + \varphi_{l0}\varphi_{l2})\,(R_{\mathrm{pr}} - R_0)\,(A_{\mathrm{pr}} - A_0) \\ & + 4\theta^3\,(\psi_{l3} + \varphi_{l1}\varphi_{l2})\,(V_{\mathrm{pr}} - V_0)\,(A_{\mathrm{pr}} - A_0) \Big], \end{aligned} \tag{45}$$

where

$$\chi_{lm} = \sum_{k=0}^{N-1} \delta_{lk}^2\, P_{\mathrm{a}\lambda k}\, |k-\mu|^m, \qquad \psi_{lm} = \sum_{k=0}^{N-1} \delta_{lk}^2\, P_{0\lambda k} P_{\mathrm{a}\lambda k}\,(k-\mu)^m, \tag{46}$$

and the coefficients φ_{lm} are defined by expression (44).

As the signal-to-noise ratio (20) increases for one pulse of sequence (2), the reliable-error probability of the temporal position of optical pulse (33) also increases, i. e., $P_{0\lambda k} \to 1$, and $P_{\mathrm{a}\lambda k} \to 0$ for $z_1 \to \infty$. Accordingly, from Eqs. (43) and (45) we have

$$b(\hat{R} \mid R_0, V_0, A_0) \to 0, \qquad b(\hat{V} \mid R_0, V_0, A_0) \to 0, \qquad b(\hat{A} \mid R_0, V_0, A_0) \to 0 \tag{47}$$

and

$$B(\hat{l} \mid R_0, V_0, A_0) \to \frac{1}{\alpha} \sum_{k=0}^{N-1} \delta_{lk}^2. \tag{48}$$

Performing the summation, for a large signal-to-noise ratio (20) we obtain

$$B(\hat{R} \mid R_0, V_0, A_0) = D_0(\tilde{R}), \qquad B(\hat{V} \mid R_0, V_0, A_0) = D_0(\tilde{V}), \qquad B(\hat{A} \mid R_0, V_0, A_0) = D_0(\tilde{A}). \tag{49}$$

Here, the quantities $D_0(\tilde{R})$, $D_0(\tilde{V})$, and $D_0(\tilde{A})$ are defined by expressions (7) and represent the variances of the mutually effective estimates of the motion parameters. Consequently, quasioptimal estimates (41) are asymptotically (with increasing signal-to-noise ratio (20)) conditionally unbiased according to limits (47) and are mutually effective according to the limit (49).

We then find the unconditional characteristics of quasioptimal estimates (41). We assume that the true values of R_0, V_0, and A_0 are uniformly distributed in the *a priori* region $\mathbf{W}$. Averaging expression (43) over the uniformly distributed values of R_0, V_0, and A_0, for the unconditional biases we obtain

$$b(\hat{R}) = \langle B(\hat{R} \mid R_0, V_0, A_0)\rangle = 0, \qquad b(\hat{V}) = \langle B(\hat{V} \mid R_0, V_0, A_0)\rangle = 0,$$
$$b(\hat{A}) = \langle B(\hat{A} \mid R_0, V_0, A_0)\rangle = 0.$$

Consequently, quasioptimal estimates (41) are unconditionally unbiased.

We now turn to finding the unconditional spreads. Averaging expression (45) over uniformly distributed values of R_0, V_0, and A_0, for the unconditional spreads of quasioptimal estimates (41) we find

$$B(\hat{l}) = \frac{1}{\alpha}\sum_{k=0}^{N-1} \delta_{lk}^2\, P_{0\lambda k} + \frac{1}{12c^2}\Big[4\,(\chi_{l0} + \varphi_{l0} + \psi_{l0}^2)\,\Delta R_{\mathrm{pr}}^2 + 4\theta^2\,(\chi_{l2} + \varphi_{l2} + \psi_{l2}^2)\,\Delta V_{\mathrm{pr}}^2$$
$$+ \theta^4\,(\chi_{l4} + \varphi_{l4} + \psi_{l4}^2)\,\Delta A_{\mathrm{pr}}^2 + 8\theta\chi_{l1}\,\Delta R_{\mathrm{pr}}\,\Delta V_{\mathrm{pr}} + 4\theta^2\chi_{\mathrm{pr}}\,\Delta R_{\mathrm{pr}}\,\Delta A_{\mathrm{pr}} + 4\theta^3\chi_{l3}\,\Delta V_{\mathrm{pr}}\,A_{\mathrm{pr}}\Big]. \quad (50)$$

Comparing expressions (6) and (50), one can find the losses in accuracy of quasioptimal estimates (41) compared with the accuracy of the maximum-likelihood estimates (4).

With increasing signal-to-noise ratio (20), the reliable-estimate probability $P_{0\lambda k} \to 1$ and $P_{\mathrm{a}\lambda k} \to 0$, see Eq. (33). Accordingly, from expression (50) we have

$$B(\hat{l}) \to \frac{1}{\alpha}\sum_{k=0}^{N-1} \delta_{lk}^2,$$

which is similar to the limit (48). Consequently, as the signal-to-noise ratio (20) increases, unconditional spreads of quasioptimal estimates (41) asymptotically coincide with the unconditional spreads of the maximum-likelihood estimates (4).

5. CONCLUSIONS

The proposed quasioptimal estimates (41), as well as the maximum-likelihood estimates (4), are asymptotically effective with increasing signal-to-noise ratio. To make the maximum-likelihood estimates (4) close to effective, the signal-to-noise ratio (9) should be high for the whole observed sequence of optical pulses with intensity (2). To make the quasioptimal estimates close to effective, the signal-to-noise ratio (20) should be high for each pulse of the observed sequence of optical pulses. Therefore, the provision of a high *a posteriori* accuracy of quasioptimal estimates requires a significantly higher energy of the useful signal.

Thus, if the conditions for a high *a posteriori* accuracy of the estimates of temporal positions for each pulse are fulfilled, then instead of the maximum-likelihood algorithm (4), which is difficult to implement, one can use, actually without losses in accuracy, the quasioptimal algorithm (41), which is much easier to implement. Moreover, algorithm (41) can be used for processing of the measurement results in the existing high-accuracy laser range meters to obtain additional information on velocity and acceleration.

This work was supported in part by the Russian Foundation for Basic Research (project Nos. 13–01–97504 and 13–08–00735), the Russian Science Foundation (project No. 14–29–00208), and Ministry of Edication and Science of the Russian Federation (project No. 978).

REFERENCES

1. V. I. Vorob'yov, *Optical Detection and Ranging for Radioengineers* [in Russian], Radio i Svyaz', Moscow (1983).
2. N. A. Dolinin and A. F. Terpugov, *Statistical Methods in Optical Radar* [in Russian], Tomsk State Univ., Tomsk (1982).

3. A. P. Trifonov, M. B. Bespalova, and M. V. Maksimov, *Radiotekhnika*, No. 4, 99 (2001).
4. A. P. Trifonov, M. B. Bespalova, and M. V. Maksimov, *Radiotekhnika*, **45**, No. 6, 3 (2002).
5. A. P. Trifonov, M. B. Bespalova, and A. V. Kurbatov, *Izv. Vyssh. Uchebn. Zaved., Radioelektron.*, **56**, No. 1, 24 (2013).
6. E. I. Kulikov and A. P. Trifonov, *Estimation of the Signal Parameters Against the Noise Background* [in Russian], Sovetskoe Radio, Moscow (1978).
7. A. P. Trifonov and Yu. S. Shinakov, *Joint Distinction of Singals and Estimates of Their Parameters Against the Noise Background* [in Russian], Radio i Svyaz', Moscow (1986).
8. A. P. Trifonov and Yu. V. Nevezhin, *Radiophys. Quantum Electron.*, **42**, No. 12, 1059 (1999).
9. I. A. Bol'shakov and V. S. Rakoshits, *Applied Theory of Random Flows* [in Russian], Sovetskoe Radio, Moscow (1978).