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Quasi-Likelihood Estimate of the Arrival Time of Ultrawideband Signal with Unknown Waveform on Exposure to Narrow-Band Interferences[1]

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Abstract—Characteristics of quasi-likelihood estimate of the arrival time of ultrawideband signal with unknown waveform received against the background of narrow-band interferences with unknown parameters and the Gaussian white noise have been investigated.

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In recent years the application of ultrawideband signals (UWBS) has become a new direction in the theory of radioelectronic systems [1–3]. The UWBS spectrum is very wide; therefore its action causes excitation of practically all possible types of natural oscillations of target under investigation that results in highly-informative observed response. An important feature of UWBS is the absence of the carrier frequency proper and, consequently, infeasibility of the classical description of radio signals using complex envelope.

Paper [4] considers the estimation of the arrival time of UWBS against the background of only Gaussian white noise (GWN). In real conditions, both a mixture of signal and white noise and other interfering actions are applied to the receiver input. Paper [3] investigates the estimation algorithms of the arrival time of UWBS against the background of interferences as models using the Gaussian narrow-band process (GNP) [5]. In this case the UWBS waveform was considered to be a priori known. However, in real conditions the waveform of received signal can be unknown, since it changes during the reflection from object (radiolocation), during propagation in different media (navigation and communications), and the signal waveform is always unknown in the case of monitoring.

This paper considers a task of estimating the arrival time of UWBS with unknown waveform against the background of GNP and GWN. In this case characteristics of GNP can be also unknown.

Let us assume that the following realization is observed on time interval $t \in [0, T]$:

$$x(t) = s_0(t - \lambda_0) + n(t) + \xi(t), \qquad (1)$$

where $s_0(t)$ is the useful signal, the waveform of which is unknown (it is only known that this signal is ultrawideband), λ_0 is the unknown time of signal arrival, $n(t)$ is the realization of GWN with single-sided spectral density N_0, $\xi(t)$ is the narrow-band interference.

For a model of narrow-band interference (which is the most universal) [5] we shall use narrow-band stationary Gaussian process $\xi(t)$ with zero mathematical expectation $\langle \xi(t) \rangle = 0$ and correlation function $\langle \xi(t)\xi(t+\Delta) \rangle = B_\xi(\Delta)$. Then the spectral density of GNP can be written in the form

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$$G_\xi(\omega)=\int_{-\infty}^{\infty}B_\xi(\Delta)\exp(-j\omega\Delta)d\Delta=\frac{\gamma_\xi}{2}\left[g_\xi\left(\frac{\omega_{0\xi}-\omega}{\Omega_\xi}\right)+g_\xi\left(\frac{\omega_{0\xi}+\omega}{\Omega_\xi}\right)\right],\tag{2}$$

where $\Omega_\xi=\int_0^\infty G_\xi^2(\omega)d\omega/\max G_\xi^2(\omega)$ is the equivalent frequency band of interference, $\omega_{0\xi}$ is the central frequency.

Since the interference is narrow-band, the following condition $\Omega_\xi \ll \omega_{0\xi}$ is satisfied. Function $g_\xi(x)$ describes the shape of interference spectral density and satisfies the following conditions:

$$g_\xi(x)\ge 0,\quad g_\xi(x)=g_\xi(-x),$$

$$\max g_\xi(x)=\int_{-\infty}^{\infty}g_\xi^2(x)dx=1.$$

We shall assume that processes $n(t)$ and $\xi(t)$ are statistically independent.

In case the waveform of signal received $s_0(t)$ is a priori known, while GNP is not present, the estimation of the arrival time of signal λ_0 can be performed by using the maximum likelihood method [4]. To this end, for estimate we should take the position of the largest maximum of likelihood ratio functional [4]

$$L_F(\lambda)=\frac{2}{N_0}\int_0^T x(t)s_0(t-\lambda)dt.\tag{3}$$

In case that the waveform of signal $s_0(t)$ is known inaccurately, in expression (3) we shall use a certain expected (anticipated) signal $s_1(t)$ as a reference signal.

Thus, we obtain the following expression for the output signal of measuring device (decision making statistic):

$$L(\lambda)=\frac{2}{N_0}\int_0^T x(t)s_1(t-\lambda)dt.\tag{4}$$

For estimate $\hat{\lambda}$ of the unknown time of arrival λ_0 we can assume the value of λ, at which decision making statistic (4) reaches its absolute (largest) maximum. The resultant estimate shall be called quasi-likelihood [6].

Indeed, in case that received signal $s_0(t)$ coincides with expected signal $s_1(t)$ in the absence of GNP, decision-making statistic (4) coincides with the logarithm of likelihood ratio functional (3). Correspondingly, the quasi-likelihood estimate transforms into the maximum likelihood estimate.

The determination of characteristics of quasi-likelihood estimate involves the need of presenting decision-making statistic (4) in the form of a sum of signal function $S_0(\lambda,\lambda_0)$ and noise function $N(\lambda)$ [4]:

$$L(\lambda)=S_0(\lambda,\lambda_0)+N(\lambda),\tag{5}$$

where

$$S_0(\lambda,\lambda_0)=\frac{2}{N_0}\int_0^T s_0(t-\lambda_0)s_1(t-\lambda)dt,\tag{6}$$

$$N(\lambda)=\frac{2}{N_0}\int_0^T s_1(t-\lambda)[n(t)+\xi(t)]dt.\tag{7}$$

Noise function represents a realization of the Gaussian centered random process and has correlation function

$$B_N(\lambda_1,\lambda_2) = \langle N(\lambda_1)N(\lambda_2)\rangle = S_1(\lambda_1,\lambda_2) + B_1(\lambda_1,\lambda_2), \tag{8}$$

$$S_1(\lambda_1,\lambda_2) = \frac{2}{N_0}\int_0^T s_1(t-\lambda_1)s_1(t-\lambda_2)\mathrm{d}t,$$

$$B_1(\lambda_1,\lambda_2) = \frac{4}{N_0^2}\int_0^T\int_0^T B_\xi(t_2-t_1)s_1(t-\lambda_1)s_1(t-\lambda_2)\mathrm{d}t_1\mathrm{d}t_2.$$

Let us assume that signal function $S_0(\lambda,\lambda_0)$ (6) at fixed value λ_0 reaches its largest value at point $\tilde{\lambda}$ and has only one pronounced maximum. Then the signal-to-noise ratio (SNR) at the output of quasi-likelihood receiver can be written in the form [4]:

$$z^2 = S_0^2(\tilde{\lambda},\lambda_0)\,/\,B_N(\tilde{\lambda},\tilde{\lambda}). \tag{9}$$

In what follows we assume that SNR is sufficiently large, so that the quasi-likelihood estimate features high a posteriori accuracy [4]. Then quasi-likelihood estimate $\hat{\lambda}$ of arrival time λ_0 can be found by solving the following equation

$$\left[\frac{\mathrm{d}L(\lambda)}{\mathrm{d}\lambda}\right]_{\hat{\lambda}} = 0. \tag{10}$$

The approximate solution of this equation can be obtained by the method of small parameter [4]; for a small parameter we shall use the quantity inverse to SNR (9).

Limiting our consideration with the first approximation we obtain the following expression for the bias of quasi-likelihood estimate

$$b_1(\hat{\lambda}|\lambda_0) = \tilde{\lambda} - \lambda_0 = \Delta\lambda. \tag{11}$$

Thus, in the general case the quasi-likelihood estimate is not consistent. Its application leads to a systematic error $\Delta\lambda$ (11) that is not dependent on SNR and is determined by the waveforms of received and expected signals.

According to [4] the dispersion of quasi-likelihood estimate has the form

$$D_1(\hat{\lambda}|\lambda_0) = \langle(\hat{\lambda} - \langle\hat{\lambda}\rangle)^2\rangle = \left\{\frac{\partial^2 B_N(\lambda_1,\lambda_2)}{\partial\lambda_1\partial\lambda_2}\left[\frac{\mathrm{d}^2 S_0(\lambda,\lambda_0)}{\mathrm{d}\lambda^2}\right]^2\right\}_{\lambda_1=\lambda_2=\lambda=\tilde{\lambda}}. \tag{12}$$

Substituting the signal function and correlation function of the noise function into expression (12) after differentiation we find an expression for dispersion:

$$D_1(\hat{\lambda}|\lambda_0) = \left\{\frac{N_0}{2}\int_0^T\left[\frac{\mathrm{d}s_1(t)}{\mathrm{d}t}\right]^2\mathrm{d}t + \int_0^T\int_0^T B_\xi(t_2-t_1)\frac{\mathrm{d}s_1(t_1)}{\mathrm{d}t_1}\frac{\mathrm{d}s_1(t_2)}{\mathrm{d}t_2}\mathrm{d}t_1\mathrm{d}t_2\right\}$$

$$\times\left[\int_0^T\frac{\mathrm{d}s_0(t)}{\mathrm{d}t}\frac{\mathrm{d}s_1(t-\Delta\lambda)}{\mathrm{d}t}\mathrm{d}t\right]^{-2}. \tag{13}$$

It should be noted that with a rise of SNR (9) the estimate dispersion (13) tends to zero. The estimate accuracy can be also characterized by the value of scattering (average squared error) [4]

$$V_1(\hat{\lambda}|\lambda_0) = \langle(\hat{\lambda} - \lambda_0)^2\rangle = b_1^2(\hat{\lambda}|\lambda_0) + D_1(\hat{\lambda}|\lambda_0). \tag{14}$$

If the estimate is inconsistent ($\Delta l \neq 0$), with a rise of SNR its scattering tends to quantity Δl^2. For a consistent estimate ($\Delta l = 0$) the scattering of estimate with rising SNR tends to zero.

Let us consider certain particular cases of expressions (11) and (13). If GNP is not present and UWBS waveform is a priori known, it is possible to select expected signal $s_1(t) = s_0(t)$. In this case quasi-likelihood estimate $\hat{\lambda}$ transforms into maximum likelihood estimate λ_m. This estimate of the arrival time of UWBS with known waveform against the background of GWN has the bias and dispersion:

$$b_0(\lambda_m|\lambda_0) = 0,$$

$$D_0(\lambda_m|\lambda_0) = \left[\frac{2}{N_0}\int_0^T\left[\frac{\mathrm{d}s_0(t)}{\mathrm{d}t}\right]^2 \mathrm{d}t\right]^{-1}. \tag{15}$$

Since the maximum likelihood estimate is unbiased, its scattering coincides with dispersion

$$V_0(\lambda_m|\lambda_0) = D_0(\lambda_m|\lambda_0). \tag{16}$$

Comparing expressions (13) and (14) with (15) and (16) it is possible to determine losses in accuracy of the estimate of UWBS arrival time due to a priori lack of knowledge of UWBS waveform and GNP action.

In particular, from expressions (13) and (15) it follows that the quasi-likelihood estimate has the dispersion, which exceeds the dispersion of maximum likelihood estimate by a factor of ρ_1, in this case

$$\rho_1 = D_1(\hat{\lambda}|\lambda_0) / D_0(\lambda_m|\lambda_0) = \chi_1 / R^2(\Delta\lambda), \tag{17}$$

where $\chi_1 = 1 + \dfrac{\dfrac{2}{N_0}\displaystyle\int_0^T\int_0^T B_\xi(t_2 - t_1)\frac{\mathrm{d}s_1(t_1)}{\mathrm{d}t_1}\frac{\mathrm{d}s_1(t_2)}{\mathrm{d}t_2}\mathrm{d}t_1\mathrm{d}t_2}{\displaystyle\int_0^T\left[\frac{\mathrm{d}s_1(t)}{\mathrm{d}t}\right]^2\mathrm{d}t}$ and characterizes the impact of GNP on dispersion of quasi-likelihood estimate, while

$$R(\Delta l) = \frac{\displaystyle\int_0^T \frac{\mathrm{d}s_0(t)}{\mathrm{d}t}\frac{\mathrm{d}s_1(t - \Delta l)}{\mathrm{d}t}\mathrm{d}t}{\sqrt{\displaystyle\int_0^T\left[\frac{\mathrm{d}s_0(t)}{\mathrm{d}t}\right]^2\mathrm{d}t\int_0^T\left[\frac{\mathrm{d}s_1(t)}{\mathrm{d}t}\right]^2\mathrm{d}t}} \tag{18}$$

is the coefficient of mutual correlation between the derivative of signal received and derivative of expected signal delayed by the value of systematic error (11) of quasi-likelihood estimate.

It is obvious that quantity (18) characterizes the impact of difference between the waveforms of received and expected signals on the dispersion of quasi-likelihood estimate. It should be noted that a rise of estimate dispersion (17) does not depend on the amplitudes of the received and expected signals.

For a number of tasks the estimate scattering is a more complete characteristic of accuracy as compared with the estimate dispersion. From expression (14) and (16) it follows that the quasi-likelihood estimate has the scattering that exceeds the scattering of maximum likelihood estimate by a factor of κ_1, in this case

$$\kappa_1 = \frac{V_1(\hat{\lambda}|\lambda_0)}{V_0(\hat{\lambda}_m|\lambda_0)} = \frac{2\Delta\lambda^2}{N_0}\int_0^T \left[\frac{\mathrm{d}s_0(t)}{\mathrm{d}t}\right]^2 \mathrm{d}t + \rho_1. \tag{19}$$

It now follows that the loss in accuracy of inconsistent quasi-likelihood estimate as compared with the accuracy of maximum likelihood estimate increases with a rise of SNR. Indeed, as the power of received signal increases and the spectral density of GWN decreases, the first term in the right-hand side of expression (19) increases.

For a number of waveforms of received and expected signals the quasi-likelihood estimate can be consistent. In particular, it will be consistent if the received and expected signals are even

$$s_0(t) = s_0(-t), \quad s_1(t) = s_2(-t) \tag{20}$$

or odd

$$s_0(t) = -s_0(-t), \quad s_1(t) = -s_1(-t) \tag{21}$$

functions of time.

In this case, the position of the largest maximum of signal function (6) coincides with the true value of the arrival time ($\tilde{\lambda} = \lambda_0$) and in expression (11)

$$\Delta\lambda = 0. \tag{22}$$

Then dispersion (13) and scattering (14) of quasi-likelihood estimate coincide, and the loss in accuracy of quasi-likelihood estimate as compared to the accuracy of maximum likelihood estimate is characterized by the following quantity:

$$\kappa_{01} = \rho_{01} = \frac{V_1(\hat{\lambda}|\lambda_0)}{V_0(\hat{\lambda}|\lambda_0)} = \frac{D_1(\hat{\lambda}|\lambda_0)}{D_0(\lambda_m|\lambda_0)} = \frac{\chi_1}{R^2(0)}. \tag{23}$$

As a particular case we shall determine the loss in accuracy of the estimate of arrival time in the absence of GNP. Indeed, assuming $B_\xi(\Delta) \equiv 0$, from expressions (17) and (23) we obtain

$$\kappa_{01} = R^{-2}(0). \tag{24}$$

If waveforms of the received and expected signals coincide, the loss in the estimate accuracy due to impact of GNP has the form:

$$\kappa_{01} = \chi_0, \tag{25}$$

where $\chi_0 = 1 + \dfrac{\dfrac{2}{N_0}\displaystyle\int_0^T\int_0^T B_\xi(t_2 - t_1)\frac{\mathrm{d}s_0(t_1)}{\mathrm{d}t_1}\frac{\mathrm{d}s_0(t_2)}{\mathrm{d}t_2}\mathrm{d}t_1\mathrm{d}t_2}{\displaystyle\int_0^T\left[\frac{\mathrm{d}s_0(t)}{\mathrm{d}t}\right]^2\mathrm{d}t}$.

Indeed, if the waveforms of the received and expected signals coincide, the cross-correlation coefficient of their derivatives $R(0) = 1$ and from expression (23) we obtain expression (25).

The calculation of determined characteristics of quasi-likelihood estimates of the arrival time of UWBS with unknown waveform can be simpler and more convenient in case of using spectral characteristics of signals and interference.

Let us designate the spectra of received and expected signals as follows:

$$S_i(\mathrm{j}\omega) = \int_{-\infty}^{\infty} s_i(t)\exp(-\mathrm{j}\omega t)\mathrm{d}t, \quad i = 0, 1.$$

Then from expressions (15), (17) and (18) we have

$$D_0(\lambda_m|\lambda_0) = \left[\frac{1}{2\pi N_0}\int_{-\infty}^{\infty}\omega^2 |S_0(\mathrm{j}\omega)|^2 \mathrm{d}\omega\right]^{-1}, \tag{26}$$

$$\chi_i = 1 + \frac{\dfrac{2}{N_0}\displaystyle\int_{-\infty}^{\infty}\omega^2 G_\xi(\omega)|S_i(\mathrm{j}\omega)|^2\mathrm{d}\omega}{\displaystyle\int_{-\infty}^{\infty}\omega^2|S_i(\mathrm{j}\omega)|^2\mathrm{d}\omega}, \quad i = 0, 1. \tag{27}$$

$$R(\Delta\lambda) = \frac{\displaystyle\int_{-\infty}^{\infty}\omega^2 S_0(\mathrm{j}\omega)S_1^*(\mathrm{j}\omega)\exp(\mathrm{j}\omega\Delta\lambda)\mathrm{d}\omega}{\sqrt{\displaystyle\int_{-\infty}^{\infty}\omega^2|S_0(\mathrm{j}\omega)|^2\mathrm{d}\omega\int_{-\infty}^{\infty}\omega^2|S_1(\mathrm{j}\omega)|^2\mathrm{d}\omega}}. \tag{28}$$

Let us consider the impact of GNP with square shape of the spectral density on the accuracy of quasi-likelihood estimate of the time of arrival. To this end, we shall assume in expression (2) that $g_\xi(x) = 1$ at $|x| < 1/2$ and $g_\xi(x) = 0$ at $|x| > 1/2$.

Next substituting expression (2) into (27) we obtain the loss in accuracy of the estimate due to the impact of GNP in the form:

$$\chi_i = 1 + \varepsilon_i q, \tag{29}$$

where $q = \gamma_\xi / N_0$ is the ratio of spectral densities of GNP and GWN, while

$$\varepsilon_i = \int_{\omega_{0\xi}-\Omega_\xi/2}^{\omega_{0\xi}+\Omega_\xi/2}\omega^2|S_i(\mathrm{j}\omega)|^2\mathrm{d}\omega \Big/ \int_{0}^{\infty}\omega^2|S_i(\mathrm{j}\omega)|^2\mathrm{d}\omega, \tag{30}$$

ε_i is the relative fraction of the power of signal derivative $S_i(t)$, $i = 0, 1$ in the frequency range, where GNP is present.

From expression (29) it follows that the loss in accuracy of the quasi-likelihood estimate increases with a rise of GNP frequency band and its intensity.

The Gaussian $s_\mathrm{G}(t,\tau)$ and Lorentz $s_\mathrm{L}(t,\tau)$ monocycles [3] can serve as examples of the received and expected UWBS, for which the quasi-likelihood estimate of arrival time will be consistent:

$$s_\mathrm{G}(t,\tau) = \frac{\mathrm{d}}{\mathrm{d}t}f_\mathrm{G}(t/\tau) = a_1 t\exp(-\pi t^2/2\tau^2), \tag{31}$$

$$s_\mathrm{L}(t,\tau) = \frac{\mathrm{d}}{\mathrm{d}t}f_\mathrm{L}(t/\tau) = a_2 t[1 + (\pi t/2\tau)^2]^{-2}, \tag{32}$$

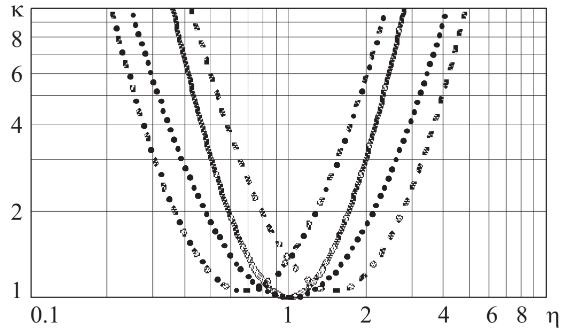

Fig. 1.

where $f_G(x)$ and $f_L(x)$ are the Gaussian and Lorentz monocycles:

$$f_G(x) = \exp(-\pi x^2/2), \tag{33}$$

$$f_L(x) = [1 + (\pi x/2)^2]^{-1}, \tag{34}$$

a_i, $i = 1, 2$ are the amplitude factors, while parameter τ characterizes the duration of signals. Signals (31) and (32) satisfy condition (21) of the consistency of quasi-likelihood estimate of arrival time.

According to (24) the loss in accuracy of the consistent quasi-likelihood estimate caused by a priori lack of knowledge of the UWBS waveform is determined by the value of correlation coefficient (18), (28) of derivatives of the received and expected signals in the absence of systematic error ($\Delta\lambda \equiv 0$).

Using these formulas for signals (31) and (32) we calculated the loss κ (24) in accuracy of quasi-likelihood estimate for different values of duration τ_0 of the received signal and duration τ of the expected signal. Figure 1 presents the $\kappa(\eta)$ relationships of the loss in accuracy of quasi-likelihood estimate as compared to the accuracy of maximum likelihood estimate as a function of ratio $\eta = \tau/\tau_0$. Solid line in Fig. 1 shows the loss for the case, where the received and expected signals are Gaussian monocycles (31) with different durations. Dotted line indicates the loss in case that the received and expected signals are Lorentz monocycles (32) with different durations.

The comparison of solid and dotted curves indicates that the application of Gaussian monocycle leads to more significant losses caused by the duration difference of the received and expected signals than that of the Lorentz monocycle. For both kinds of signal at $\eta = 1$ (durations coincide) the losses in accuracy of the estimate are equal to zero.

Dashed line in Fig. 1 shows the loss in estimate accuracy when the Gaussian monocycle (31) with duration τ_0 is received, while the expected signal is the Lorentz monocycle (32) of duration τ. Due to the waveform difference of received and expected signals, the minimum loss exceeding unity is achieved at $\eta > 1$. That is why, when the interval of possible duration values of received signal is known, it is necessary to select the duration of expected signal with exceedence of the possible duration of received signal.

Dash-and-dot line in Fig. 1 shows the loss in accuracy of quasi-likelihood estimate as compared with the maximum likelihood estimate when the Lorentz monocycle (32) of duration τ_0 is received, while the expected signal is the Gaussian monocycle (31) with duration τ. In this case, the minimum loss exceeding unity is achieved at $\eta < 1$. That is why it is expedient to choose the duration of expected signal less than the possible value of the received signal duration.

The determined characteristics of quasi-likelihood estimate make it possible to perform a sound selection of the waveform and parameters of expected signal depending on the available a priori information and admissible loss in accuracy of the arrival time of UWBS.

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