ISSN 1064-2269, Journal of Communications Technology and Electronics, 2015, Vol. 60, No. 4, pp. 375–385. © Pleiades Publishing, Inc., 2015.
Original Russian Text © A.P. Trifonov, Yu.E. Korchagin, O.V. Chernoyarov, B.I. Shakhtarin, 2015, published in Radiotekhnika i Elektronika, 2015, Vol. 60, No. 4, pp. 399–410.

THEORY AND METHODS OF SIGNAL PROCESSING

Detection of Radio Signals That Appear and Disappear at Unknown Moments

A. P. Trifonov[a], Yu. E. Korchagin[a], O. V. Chernoyarov[b], and B. I. Shakhtarin[c]

[a] *Voronezh State University, Universitetskaya pl. 1, Voronezh, 394007 Russia*

[b] *Moscow Power Engineering Institute (National Research University), Krasnokazarmennaya ul. 14, Moscow, 111250 Russia*

[c] *Bauman State Technical University, Vtoraya Baumanskaya ul. 5, str. 1, Moscow, 105005 Russia*

e-mail: trifonov@phys.vsu.ru

Received October 8, 2014

Abstract—The most plausible algorithm for the detection of a radio signal with unknown amplitude, phase and the moments of appearance and disappearance is synthesized. A two-channel quasi-optimal detector of radio signal the structure of which is simpler than the structure of the optimal detector is proposed. Asymptotic characteristics of the quasi-optimal detector are determined.

DOI: 10.1134/S1064226915040130

INTRODUCTION

The detection of a radio signal that appears and disappears at unknown moments is an important problem for radar and sonar technologies, radio communications, seismology, telemetry, etc. An algorithm of [1, 2] can be employed in the detection of a signal that appears and disappears at unknown moments using a discrete data sample. Several optimal algorithms for the detection of signals in the absence of the high-frequency carrier were synthesized and the corresponding asymptotic characteristic were determined in [3]. The exact characteristics of the most plausible (MP) algorithm for the detection of signals with unknown moments of appearance and disappearance and a priori unknown amplitudes were determined in [4]. It is expedient to use the algorithm of [5] for the detection of signals with unknown moments of appearance and disappearance and the unknown power.

Several practical applications employ signals with high-frequency carriers (radio signals) that have unknown amplitudes and initial phases owing to specific propagation conditions. In this work, we consider the algorithm for the detection of a radio signal with unknown moments of appearance and disappearance, amplitude, and initial phase.

We consider the problem of the detection of the signal represented as

$$s(t,a,\varphi,\theta_1,\theta_2)=\begin{cases}af(t)\cos(\omega t-\varphi), & \theta_1\le t\le\theta_2,\\ 0, & t<\theta_1, t>\theta_2.\end{cases}\quad(1)$$

Gaussian white noise $n(t)$ with single-sided spectral density N_0 is added to the signal. Here, unknown continuous function $f(t)$ describes the envelope of the radio signal, a is the amplitude parameter, $\varphi\in[0,2\pi]$ is the initial phase, and θ_1 and θ_2 are the moments of appearance and disappearance, respectively, that belong to the a priori intervals:

$$\theta_1\in[\theta_{1\min},\theta_{1\max}],\quad \theta_2\in[\theta_{2\min},\theta_{2\max}].\quad(2)$$

The moment at which the signal appears precedes the moment of disappearance, so that $\theta_{1\max}\le\theta_{2\min}$. At $t\in[\theta_{1\min},\theta_{2\max}]$, function $f(t)$ can be zero only at null-measure intervals. Function $f(t)$ is not zero at moments of appearance θ_1 and disappearance θ_2 of signal (1): $f(\theta_i)\ne 0$, $i=1,2$ (signal (1) is discontinuous) [6, 7].

The data samples can be represented as

$$\xi(t)=\gamma_0 s(t,a_0,\varphi_0,\theta_{01},\theta_{02})+n(t),\quad(3)$$

where a_0, φ_0, θ_{01}, and θ_{02} are the true amplitude, initial phase, and the moments of appearance and disappearance, respectively, and γ_0 is the discrete parameter. Two values of the parameter are possible: $\gamma_0=0$ if the signal is absent in the sample and $\gamma_0=1$ if the sample contains the signal. Using data sample $\xi(t)$, we must determine parameter γ.

1. THE MOST PLAUSIBLE ALGORITHM FOR THE DETECTION

For the synthesis of the detection algorithm, we use the maximum-likelihood method [6, 8, 9], which employs the logarithm of the likelihood-ratio functional (LRF) of sample $\xi(t)$ (3) and compares the maximum value of this logarithm with a threshold. When the maximum value is greater (less) than the threshold, we conclude that signal $s(t, a_0, \varphi_0, \theta_{01}, \theta_{02})$ is (not) contained in sample $\xi(t)$ (3).

In the problem under study, the logarithm of LRF

$$L(\gamma, a, \varphi, \theta_1, \theta_2) = \frac{2a\gamma}{N_0}\int_{\theta_1}^{\theta_2} f(t)\cos(\omega t - \varphi)[\xi(t) - af(t)\cos(\omega t - \varphi)/2]\,dt$$

depends on five unknown parameters: amplitude a, initial phase φ, moments of appearance and disappearance θ_1 and θ_2, and discrete parameter γ, which characterizes the presence or absence of the signal in the experimental sample [6, 8, 9].

The detection problem can be interpreted as the estimation of discrete parameter γ. To solve the detection problem, we use the generalized MP algorithm [2, 8] that is based on the comparison of the absolute maximum logarithm of LRF and threshold h. In accordance with the algorithm, estimation γ_m of parameter γ_0 is written as

$$\gamma_m = \begin{cases} 1, & L > h, \\ 0, & L \le h, \end{cases} \quad L = \sup_{a,\varphi,\theta_1,\theta_2} L(a, \varphi, \theta_1, \theta_2), \tag{4}$$

where

$$L(a, \varphi, \theta_1, \theta_2) = L(\gamma = 1, a, \varphi, \theta_1, \theta_2). \tag{5}$$

Logarithm of LRF (5) can be analytically maximized with respect to variable φ. For this purpose, we substitute signal (1) in expression (5) and represent the logarithm of LRF as

$$L(a, \varphi, \theta_1, \theta_2) = aX(\theta_1, \theta_2)\cos\varphi + aY(\theta_1, \theta_2)\sin\varphi - \frac{a^2}{2N_0}\int_{\theta_1}^{\theta_2} f^2(t)\,dt, \tag{6}$$

where we use the notation

$$X(\theta_1, \theta_2) = \frac{2}{N_0}\int_{\theta_1}^{\theta_2} \xi(t) f(t)\cos(\omega t)\,dt,$$

$$Y(\theta_1, \theta_2) = \frac{2}{N_0}\int_{\theta_1}^{\theta_2} \xi(t) f(t)\sin(\omega t)\,dt$$

and disregard the integrals of the terms that oscillate at the doubled frequency. We assume that the derivative of function (6) with respect to variable φ is zero:

$$\frac{dL(a, \varphi, \theta_1, \theta_2)}{d\varphi} = -aX(\theta_1, \theta_2)\sin\varphi + aY(\theta_1, \theta_2)\cos\varphi = 0.$$

This likelihood equation is solved to find parameter φ:

$$\varphi = \arctan(Y(\theta_1, \theta_2)/X(\theta_1, \theta_2)).$$

Substituting this solution in expression (6), we obtain

$$L(a, \theta_1, \theta_2) = \sup_{\varphi} L(a, \varphi, \theta_1, \theta_2) = a\sqrt{X^2(\theta_1, \theta_2) + Y^2(\theta_1, \theta_2)} - \frac{a^2}{2N_0}\int_{\theta_1}^{\theta_2} f^2(t)\,dt. \tag{7}$$

Then, logarithm of LRF (7) is maximized with respect to the amplitude. We assume that the derivative of function (7) with respect to variable a is zero

$$\frac{dL(a, \theta_1, \theta_2)}{da} = \sqrt{X^2(\theta_1, \theta_2) + Y^2(\theta_1, \theta_2)} - \frac{a}{N_0}\int_{\theta_1}^{\theta_2} f^2(t)\,dt = 0$$

and solve this likelihood equation to obtain amplitude a:

$$a = \frac{N_0\sqrt{X^2(\theta_1, \theta_2) + Y^2(\theta_1, \theta_2)}}{\int_{\theta_1}^{\theta_2} f^2(t)\,dt}. \tag{8}$$

Substituting solution (8) for a priori unknown amplitude a in expression (7), we obtain

$$L(\theta_1, \theta_2) = \sup_{a} L(a, \theta_1, \theta_2) = \frac{N_0\left(X^2(\theta_1, \theta_2) + Y^2(\theta_1, \theta_2)\right)}{2\int_{\theta_1}^{\theta_2} f^2(t)\,dt}. \tag{9}$$

Decision statistics L in expression (4) is written as

$$L = \sup_{\theta_1, \theta_2} L(\theta_1, \theta_2).$$

Using expression (9), we can construct the structure of the receiver. Function (9) cannot be represented as a continuous function of moments θ_1 and θ_2. Thus, the receiver must generate samples $L_{mg} = L(\theta_{1m}, \theta_{2g}) = L(m\Delta\theta_1, g\Delta\theta_2)$ of random field (9) for each discrete value of the moments of appearance $m\Delta\theta_1$ and disappearance $g\Delta\theta_2$ and determine the absolute maximum. Here, $\Delta\theta_1$ and $\Delta\theta_2$ are sampling steps for the moments of appearance and disappearance, respectively ($m = 1, 2, \ldots, n_1$ and $g = 1, 2, \ldots, n_2$). The accuracy for expression (9) increases with a decrease in steps $\Delta\theta_1$ and $\Delta\theta_2$ and an increase in number of channels n_1 and n_2 of the receiver. Therefore, an increase in the number of channels leads to an increase in the accuracy with which the multichannel detector implements the MP algorithm for the detection. The formation of a 2D random field causes difficulties in the technical implementation of the receiver, since a multichannel ($n_1 \times n_2$) structure must be used.

Figure 1 presents the block diagram of one channel of the MP detector that generates logarithm of LRF (9) for fixed moments of appearance $m\Delta\theta_1$ and disappearance $g\Delta\theta_2$ (I are integrators that work at time interval $[m\Delta\theta_1, g\Delta\theta_2]$). Using a set of values $L(m\Delta\theta_1, g\Delta\theta_2)$ that are formed by $n_1 \times n_2$ channels (Fig. 1), we find the maximum value and compare it with threshold h. When the maximum value is greater (less) than the threshold, we conclude that sample $\xi(t)$ (3) contains (does not contain) desired signal $s(t, a_0, \varphi_0, \theta_{01}, \theta_{02})$.

The analysis of the characteristics of the MP algorithm is a time-consuming problem, and the multichannel character of the receiver leads to the problems in the practical implementation, since $n_1 \times n_2$ channels are needed (Fig. 1 shows the structure of one channel).

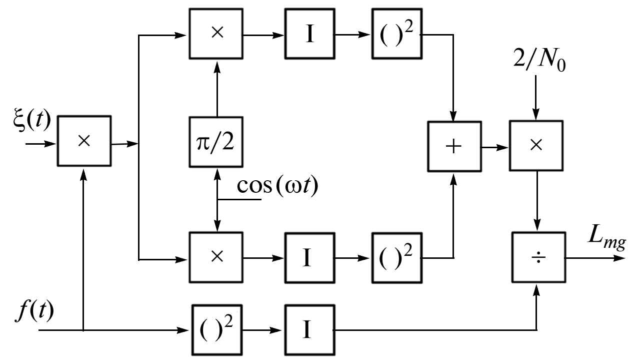

Fig. 1. Block diagram of one channel for the MP detector of radio signal.

2. QUASI-OPTIMAL DETECTION ALGORITHM

It is expedient to employ a quasi-optimal (QO) detection algorithm owing to the complexity of the analysis and implementation of the MP algorithm. The logarithm of the LRF is represented as

$$L(a, \varphi, \theta_1, \theta_2) = L_1(a, \varphi, \theta_1) + L_2(a, \varphi, \theta_2),$$

where

$$L_1(a, \varphi, \theta_1) = \frac{2a}{N_0} \times \int_{\theta_1}^{\theta} f(t)\cos(\omega t - \varphi)[\xi(t) - af(t)\cos(\omega t - \varphi)/2]\,dt, \quad (10)$$

$$L_2(a, \varphi, \theta_2) = \frac{2a}{N_0} \times \int_{\theta}^{\theta_2} f(t)\cos(\omega t - \varphi)[\xi(t) - af(t)\cos(\omega t - \varphi)/2]\,dt, \quad (11)$$

and $\theta \in (\theta_{1\max}, \theta_{2\min})$. To obtain an expression that is similar to expression (9), we analytically maximize functions (10) and (11) with respect to the initial phase and amplitude. Thus, we have

$$L_{1a\varphi}(\theta_1) = \sup_{a,\varphi} L_1(a, \varphi, \theta_1) = \frac{N_0\left(X_1^2(\theta_1) + Y_1^2(\theta_1)\right)}{2\int_{\theta_1}^{\theta} f^2(t)\,dt}, \quad (12)$$

$$L_{2a\varphi}(\theta_2) = \sup_{a,\varphi} L(a, \varphi, \theta_2) = \frac{N_0\left(X_2^2(\theta_2) + Y_2^2(\theta_2)\right)}{2\int_{\theta}^{\theta_2} f^2(t)\,dt}, \quad (13)$$

where we use the notation

$$X_1(\theta_1) = \frac{2}{N_0}\int_{\theta_1}^{\theta} \xi(t) f(t)\cos(\omega t)\,dt,$$
$$Y_1(\theta_1) = \frac{2}{N_0}\int_{\theta_1}^{\theta} \xi(t) f(t)\sin(\omega t)\,dt, \quad (14)$$

$$X_2(\theta_2) = \frac{2}{N_0}\int_{\theta}^{\theta_2} \xi(t) f(t)\cos(\omega t)\,dt,$$
$$Y_2(\theta_2) = \frac{2}{N_0}\int_{\theta}^{\theta_2} \xi(t) f(t)\sin(\omega t)\,dt. \quad (15)$$

In the QO algorithm for the detection of signal $s(t, a_0, \varphi_0, \theta_{01}, \theta_{02})$, we compare threshold h and sum L_q of the absolute maxima of processes $L_{1a\varphi}(\theta_1)$ and $L_{2a\varphi}(\theta_2)$

$$L_q = \sup L_{1a\varphi}(\theta_1) + \sup L_{2a\varphi}(\theta_2) \quad (16)$$

rather than absolute maximum L (4) of the sum of processes $L_1(a, \varphi, \theta_1)$ and $L_2(a, \varphi, \theta_2)$.

Using QO estimation γ_q of discrete parameter γ, we represent the detection algorithm as

$$\gamma_q = \begin{cases} 1, & L_q > h, \\ 0, & L_q \le h. \end{cases} \quad (17)$$

Expressions (16) and (17) show that the QO detection algorithm can be implemented as a two-channel device.

Figure 2 shows the block diagram of the device that implements QO detection algorithm (17) with the aid of expressions (12)–(16). Here, I1 and I2 are the integrators that work at time intervals $[\theta_{1\min}, \theta]$ and $[\theta, \theta_{2\max}]$, respectively; DL is the delay line for delay time $\theta - \theta_{1\min}$; PD1 and PD2 are the peak detectors that measure the absolute maxima of signals at time

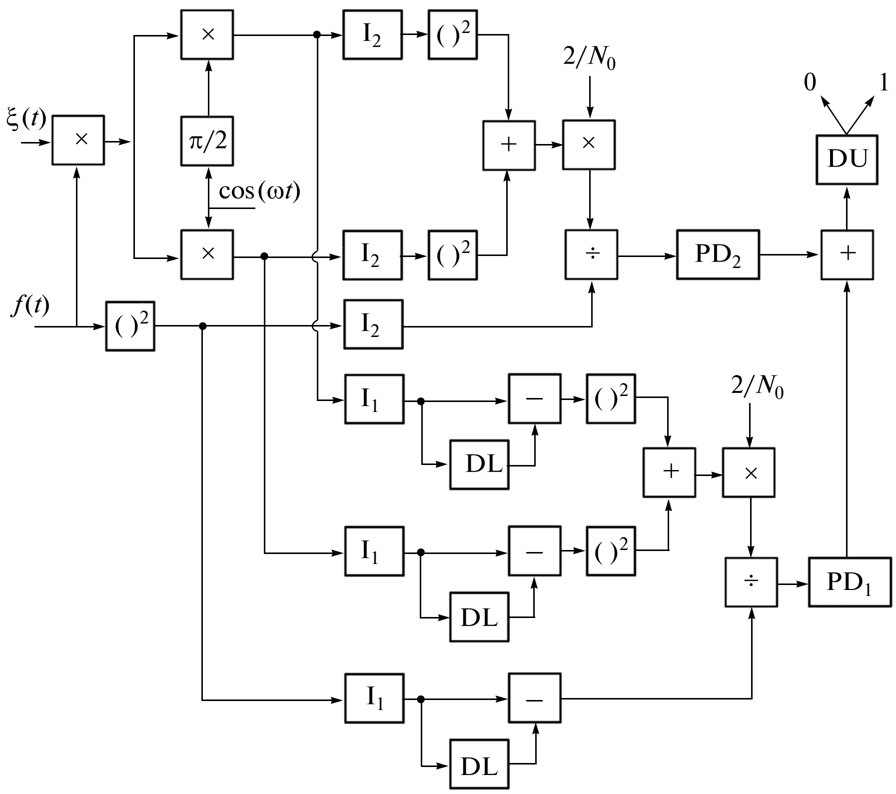

Fig. 2. Block diagram of the QO detector of radio signal.

intervals $[\theta, 2\theta - \theta_{1\min}]$ and $[\theta, \theta_{2\max}]$, respectively; DU is the decision unit that compares the input signal and threshold h and generates the decision on the presence ($\gamma_q = 1$) or absence ($\gamma_q = 0$) of the desired signal. Thus, QO detection algorithm (17) makes it possible to substantially simplify the technical implementation of the detector. Indeed, the two-channel system (Fig. 2) is sufficient for the implementation of the proposed detection algorithm whereas $n_1 \times n_2$ channels (Fig. 1) are needed for the implementation of the MP detection algorithm.

3. PROPERTIES OF DECISION STATISTICS

We study random processes (12) and (13) that are formed by the QO detection algorithm. Substituting experimental sample (3) in expressions (14) and (15) and expressions (14) and (15) in formulas (12) and (13), respectively, we obtain

$$L_{ia\varphi}(\theta_i) = \frac{[\gamma_0 G_i(\theta_{0i}, \theta_i)\cos\varphi_0 + N_{ic}(\theta_i)]^2 + [\gamma_0 G_i(\theta_{0i}, \theta_i)\sin\varphi_0 + N_{is}(\theta_i)]^2}{2(-1)^i q(\theta, \theta_i)}, \tag{18}$$

where

$$G_i(\theta_{0i}, \theta_i) = (-1)^i q(\theta, \min(\theta_{0i}, \theta_i)), \quad i = 1; 2.$$

For the signal with moments of appearance x and disappearance y, noise components $N_{ic}(\theta_i)$ and $N_{is}(\theta_i)$ in expression (18) and the signal-to-noise ratio (SNR) of the output signal of the MP detector are given by

$$N_{ic}(\theta_i) = (-1)^i \frac{2a_0}{N_0} \int_\theta^{\theta_i} n(t) f(t) \cos(\omega t)\, dt, \tag{19}$$

$$N_{is}(\theta_i) = (-1)^i \frac{2a_0}{N_0} \int_\theta^{\theta_i} n(t) f(t) \sin(\omega t)\, dt, \tag{20}$$

$$q(x, y) = \frac{a_0^2}{N_0} \int_x^y f^2(t)\, dt. \tag{21}$$

Noise components $N_{ic}(\theta_i)$ and $N_{is}(\theta_i)$ represent Gaussian functions, since they result from linear transformations of the Gaussian random process.

These components exhibit zero mean values and correlation functions

$$\langle N_{ic}(\theta_{i1})N_{ic}(\theta_{i2})\rangle = \langle N_{is}(\theta_{i1})N_{is}(\theta_{i2})\rangle = (-1)^i q(\theta, \min(\theta_{i1}, \theta_{i2})),$$
$$\langle N_{ic}(\theta_{i1})N_{is}(\theta_{i2})\rangle = 0, \quad i = 1;2.$$

Expression (19) shows that processes $N_{1c}(\theta_1)$ and $N_{2c}(\theta_2)$ contain integrals of the white noise over nonoverlapped intervals and, hence, the processes are statistically independent. Similarly, random processes $N_{1s}(\theta_1)$ and $N_{2s}(\theta_2)$ are statistically independent (20).

Function $f(t)$ may be zero only at a part of interval $[\theta_{1\min}, \theta_{2\max}]$. Thus, $q(\theta_1, \theta)$ is a monotonically decreasing function of argument θ_1, $q(\theta, \theta_2)$ is a monotonically increasing function of argument θ_2, and the following equalities are satisfied: $q(x, \theta) = -q(\theta, x)$,

$$q[\max(x, y), \theta] = \min[q(x, \theta), q(y, \theta)],$$
$$q[\theta, \min(x, y)] = \min[q(\theta, x), q(\theta, y)].$$

In expression (18), we introduce variables $\lambda_i = (-1)^i q(\theta, \theta_i)$, $\lambda_i \in [\Lambda_{i\min}, \Lambda_{i\max}]$, $\Lambda_{1\min} = q(\theta_{1\max}, \theta)$, $\Lambda_{1\max} = q(\theta_{1\min}, \theta)$, $\Lambda_{2\min} = q(\theta, \theta_{2\min})$, and $\Lambda_{2\max} = q(\theta, \theta_{2\max})$, $i = 1;2$. Then, the following expression is valid for random processes (18) as functions of variables λ_i:

$$L_{ia\varphi}(\lambda_i) = \frac{\gamma_0 G^2(\lambda_{0i}, \lambda_i) + 2\gamma_0 G(\lambda_{0i}, \lambda_i) N_{i1}(\lambda_i) + N_{ic}^2(\lambda_i) + N_{is}^2(\lambda_i)}{2\lambda_i}, \tag{22}$$

where

$$G(\lambda_{0i}, \lambda_i) = \min(\lambda_{0i}, \lambda_i), \quad \lambda_{0i} = (-1)^i q(\theta, \theta_{0i}),$$

and functions $N_{ic}(\lambda_i)$, $N_{is}(\lambda_i)$

and

$$N_{i1}(\lambda_i) = N_{ic}(\lambda_i)\cos\varphi_0 + N_{is}(\lambda_i)\sin\varphi_0, \quad i = 1;2$$

are the Gaussian random processes with zero mean values and correlation functions

$$\langle N_{ic}(\lambda_{i1})N_{ic}(\lambda_{i2})\rangle = \langle N_{is}(\lambda_{i1})N_{is}(\lambda_{i2})\rangle = \langle N_{i1}(\lambda_{i1})N_{i1}(\lambda_{i2})\rangle = \min(\lambda_{i1}, \lambda_{i2}).$$

4. FALSE-ALARM PROBABILITY

We consider the characteristics of the QO detection algorithm. The main characteristics of the detection quality are the probabilities of the first-kind (false alarm) and second-kind (signal omission) errors [8]. The probability of the false alarm is given by (17)

$$\alpha = P[\gamma_q = 1 | \gamma_0 = 0] = P[L_q > h | \gamma_0 = 0] = 1 - P_0(h), \tag{23}$$

where $P_0(h)$ is the distribution function of quantity L_q (16), which is the sum of absolute maxima of processes $L_{1a\varphi}(\theta_1)$ (12) and $L_{2a\varphi}(\theta_2)$ (13) in the absence of signal in sample $\xi(t)$ (3) ($\gamma_0 = 0$). Thus, we have

$$P_0(h) = P[L_q < h | \gamma_0 = 0] = P[\sup L_{10}(\theta_1) + \sup L_{20}(\theta_2) < h], \tag{24}$$

where $L_{i0}(\theta_i) = L_{ia\phi}(\theta_i | \gamma_0 = 0)$, $i = 1;2$. Hence, we need the distribution function of the absolute maxima

$$F_{i0}(u) = P[\sup L_{i0}(\theta_i) < u, \theta_{i\min} \le \theta_i \le \theta_{i\max}] = P[\sup L_{i0}(\lambda_i) < u, \Lambda_{i\min} \le \lambda_i \le \Lambda_{i\max}]$$

of random processes

$$L_{i0}(\lambda_i) = \frac{N_{ic}^2(\lambda_i) + N_{is}^2(\lambda_i)}{2\lambda_i}, \quad i = 1;2 \tag{25}$$

to determine the probability of the false alarm for the QO detector.

First, we consider random process $L_{20}(\lambda_2)$ using the notation

$$X_{2c}(\lambda_2) = \frac{N_{2c}(\lambda_2)}{\sqrt{\lambda_2}}, \quad X_{2s}(\lambda_2) = \frac{N_{2s}(\lambda_2)}{\sqrt{\lambda_2}} \tag{26}$$

and represent expression (25) at $i = 2$ as

$$L_{20}(\lambda_2) = \left[X_{2c}^2(\lambda_2) + X_{2s}^2(\lambda_2)\right]/2. \tag{27}$$

Independent random processes (26) are Gaussian processes with zero mean values, unity variances, and correlation coefficients

$$\langle X_{2c}(\lambda_{21})X_{2c}(\lambda_{22})\rangle = \langle X_{2s}(\lambda_{21})X_{2s}(\lambda_{22})\rangle = \frac{\min(\lambda_{21}, \lambda_{22})}{\sqrt{\lambda_{21}\lambda_{22}}}.$$

In expressions (26) and (27), we use the change of variables: $m_2 = \ln(\lambda_2/\Lambda_{2\min})$, $m_2 \in [0, \tilde{m}_2]$, $\tilde{m}_2 = \ln(\Lambda_{2\max}/\Lambda_{2\min})$, and $\lambda_2 = \Lambda_{2\min}\exp(m_2)$. In this case, random processes $X_{2c}(\lambda_2)$ and $X_{2s}(\lambda_2)$ as functions of variable m_2 have correlation functions

$$\langle X_{2c}(m_{21})X_{2c}(m_{22})\rangle = \langle X_{2s}(m_{21})X_{2s}(m_{22})\rangle = \frac{\min(\exp(m_{21}), \exp(m_{22}))}{\exp((m_{21} + m_{22})/2)} = \exp[-|m_{22} - m_{21}|/2].$$

Therefore, random processes $X_{2c}(m_2)$ and $X_{2s}(m_2)$ are statistically independent stationary Gaussian Markovian processes [10]. Then, random process (27) as a function of variable m_2 has the mean value $\langle L_{20}(m_2)\rangle = 1$, correlation function $K(m_{21}, m_{22}) = \exp(-|m_{22} - m_{21}|)$, and 1D probability density $W_2(x) = \exp(-x)$. The results of [8] show that process $L_{20}(m_2)$ represents a Markovian process with the drift and diffusion coefficients $k_1 = 1 - L_{20}$ and $k_2 = 2L_{20}$, respectively. An approximate expression for the distribution of the absolute maximum of random process $L_{20}(m_2)$ is written as (p. 85 in [8]):

$$P\{\sup L_{20}(m_2) < x\} \approx \exp[-\tilde{m}_2 x \exp(-x)] H(x-1),$$

where

$$H(x) = \begin{cases} 1, & x \ge 0, \\ 0, & x < 0 \end{cases}$$

is the Heaviside function. Using variable λ_2, we obtain the following expression for relatively high threshold h and ratio $\Lambda_{2\max}/\Lambda_{2\min}$:

$$F_{20}(h) = P\{\sup L_{20}(\lambda_2) < h\} \approx \left(\frac{\Lambda_{2\max}}{\Lambda_{2\min}}\right)^{-h\exp(-h)} H(h-1). \quad (28)$$

A similar procedure for random process $L_{10}(\lambda_1)$ yields an approximate expression for the distribution function of the absolute maximum that is valid for relatively high threshold h and ratio $\Lambda_{1\max}/\Lambda_{1\min}$:

$$F_{10}(h) = P\{\sup L_{10}(\lambda_1) < h\} \approx \left(\frac{\Lambda_{1\max}}{\Lambda_{1\min}}\right)^{-h\exp(-h)} H(h-1). \quad (29)$$

For distribution function $P_0(h)$ (expression (24)) of the sum of independent random quantities $\sup L_{10}(\lambda_1)$ and $\sup L_{20}(\lambda_2)$ with distribution functions $F_{10}(h)$ and $F_{20}(h)$ (expressions (29) and (28), respectively), we have

$$P_0(h) = \int_{-\infty}^{\infty} F_{20}(h-x)\,dF_{10}(x) = \int_{-\infty}^{\infty} F_{10}(h-x)\,dF_{20}(x). \quad (30)$$

Substituting distribution functions (28) and (29) in formula (30), we obtain the probability with which the sum of independent random quantities $\sup L_{10}(\lambda_1)$ and $\sup L_{20}(\lambda_2)$ does not reach the threshold:

$$P_0(h) = H(h-2)\Bigg[\exp(-\tilde{m}_1\exp(-1) - \tilde{m}_2(h-1)\exp(h-1)) + \tilde{m}_1\int_1^{h-1}(x-1) \times \exp[-\tilde{m}_2(h-x)\exp(h-x) - \tilde{m}_1 x\exp(-x) - x]\,dx\Bigg].$$

Then, probability of the false alarm (23) is written as

$$\alpha = 1 - H(h-2)\Bigg[\exp(-\tilde{m}_1\exp(-1) - (\tilde{m}_2(h-1)\exp(h-1)) + \tilde{m}_1\int_1^{h-1}(x-1) \times \exp[-\tilde{m}_2(h-x)\exp(h-x) - \tilde{m}_1 x\exp(-x) - x]\,dx\Bigg]. \quad (31)$$

This formula is an approximate formula the accuracy of which increases with increasing threshold h and parameters $\tilde{m}_1 = \ln(\Lambda_{1\max}/\Lambda_{1\min})$ and $\tilde{m}_2 = \ln(\Lambda_{2\max}/\Lambda_{2\min})$.

5. PROBABILITY OF SIGNAL OMISSION

Expression (17) shows that the probability of signal omission is given by

$$\beta(\theta_{01}, \theta_{02}) = P[\gamma_q = 0 | \gamma_0 = 1] = P[L_q < h | \gamma_0 = 1] = P_1(h), \quad (32)$$

where $P_1(h)$ is the distribution function of quantity L_q (16), which is the sum of the absolute maxima of processes $L_{1a\varphi}(\theta_1)$ (12) and $L_{2a\varphi}(\theta_2)$ (13) in the presence of the signal in sample $\xi(t)$ (3) ($\gamma_0 = 1$):

$$P_1(h) = P[L_q < h | \gamma_0 = 1] = P[\sup L_{11}(\theta_1) + \sup L_{21}(\theta_2) < h]. \quad (33)$$

Here, we have $L_{i1}(\theta_i) = L_{ia\varphi}(\theta_i | \gamma_0 = 1)$, $i = 1; 2$. In the presence of the signal in the experimental sample ($\gamma_0 = 1$), expression (22) yields

$$L_{i1}(\lambda_i) = \frac{\min^2(\lambda_{0i}, \lambda_i)}{2\lambda_i} + \frac{\min(\lambda_{0i}, \lambda_i) N_{1i}(\lambda_i)}{\lambda_i} + \frac{N_{ic}^2(\lambda_i) + N_{is}^2(\lambda_i)}{2\lambda_i}.$$

In this expression, we use variables $l_i = \lambda_i/\lambda_{0i}$, $l_i \in \left[\Lambda_{i\min}/z_{0i}^2, \Lambda_{i\max}/z_{0i}^2\right]$, $i = 1;2$ and obtain

$$L_{i1}(l_i) = z_{0i}^2 \frac{\min^2(1,l_i)}{2l_i} + z_{0i}\frac{\min(1,l_i)N_{1i}(l_i)}{l_i} + \frac{N_{ic}^2(l_i)+N_{is}^2(l_i)}{2l_i}. \quad (34)$$

Quantity $z_{01}^2 = \lambda_{01} = \frac{a_0^2}{N_0}\int_{\theta_{01}}^{\theta} f^2(t)\,dt$ is the SNR for the radio signal with amplitude a_0 and moments of appearance θ_{01} and disappearance θ. Quantity $z_{02}^2 = \lambda_{02} = \frac{a_0^2}{N_0}\int_{\theta}^{\theta_{02}} f^2(t)\,dt$ is the SNR for the radio signal with amplitude a_0 and moments of appearance θ and disappearance θ_{02}. For the received signal, the SNR is written as

$$z_0^2 = z_{01}^2 + z_{02}^2 = \frac{a_0^2}{N_0}\int_{\theta_{01}}^{\theta_{02}} f^2(t)\,dt. \quad (35)$$

At relatively high SNRs $z_{01}^2 \gg 1$ and $z_{02}^2 \gg 1$, the last term in expression (34) can be disregarded and the following approximate expression is valid in terms of variables λ_i:

$$L_{i1}(\lambda_i) \approx \frac{\min^2(\lambda_{0i},\lambda_i)}{2\lambda_i} + \frac{\min(\lambda_{0i},\lambda_i)}{\lambda_i}N_{1i}(\lambda_i), \quad i = 1;2. \quad (36)$$

These functions represent Gaussian random processes with mean values

$$S_{i1}(\lambda_i) = \frac{\min^2(\lambda_{0i},\lambda_i)}{2\lambda_i} \quad (37)$$

and correlation functions

$$K_{i1}(\lambda_{i1},\lambda_{i2}) = \frac{\min(\lambda_{i1},\lambda_{0i})\min(\lambda_{i2},\lambda_{0i})}{\lambda_{i1}\lambda_{i2}}\min(\lambda_{i1},\lambda_{i2}).$$

The correlation functions of random processes $L_{i1}(\lambda_i)$ (36) are represented as

$$R_{i1}(\lambda_{i1},\lambda_{i2}) = \min(\lambda_{i1},\lambda_{i2})/\sqrt{\lambda_{i1}\lambda_{i2}}$$

and satisfy the condition [10, 11]

$$R_{i1}(x,y) = R_{i1}(x,t)R_{i1}(t,y), \quad x > t > y.$$

Hence, random processes $L_{i1}(\lambda_i)$ are Markovian random processes with the drift and diffusion coefficients given by

$$k_{1i}(\lambda_i) = \frac{1}{2}\begin{cases}1, & \lambda_i < \lambda_{0i},\\ -\lambda_{0i}^2/\lambda_i^2, & \lambda_i \ge \lambda_{0i},\end{cases}$$

$$k_{2i}(\lambda_i) = \begin{cases}1, & \lambda_i < \lambda_{0i},\\ \lambda_{0i}^2/\lambda_i^2, & \lambda_i \ge \lambda_{0i}.\end{cases}$$

At relatively high SNR, the greatest maxima of decision statistics (36) are located in small neighborhoods of the maxima of their mean values [7]. Mean values (37) reach maximum levels at $\lambda_i = \lambda_{0i}$. We introduce quantities $\varepsilon_i = (\lambda_i - \lambda_{0i})/\lambda_{0i}$, the absolute values of which decrease with an increase in SNR z_0^2 (35), and represent the drift and diffusion coefficients as

$$k_{1i}(\lambda_i) = \frac{1}{2}\begin{cases}1, & \lambda_i < \lambda_{0i},\\ -(1+\varepsilon_i)^{-2}, & \lambda_i \ge \lambda_{0i},\end{cases}$$

$$k_{2i}(\lambda_i) = \begin{cases}1, & \lambda_i < \lambda_{0i},\\ (1+\varepsilon_i)^{-2}, & \lambda_i \ge \lambda_{0i}.\end{cases}$$

For $z_0^2 \to \infty$, we have $\varepsilon_i \to 0$ in mean square, so that random processes $L_{i1}(\lambda_i)$ (36) at relatively high SNRs can be approximated in the vicinity of points $\lambda_i = \lambda_{0i}$ using Gaussian Markovian processes $\mu_{i1}(\lambda_i)$, $i = 1;2$, with the drift and diffusion coefficients

$$k_{1i}(\lambda_i) = \frac{1}{2}\begin{cases}1, & \lambda_i < \lambda_{0i},\\ -1, & \lambda_i \ge \lambda_{0i},\end{cases} \quad k_{2i}(\lambda_i) = 1. \quad (38)$$

We use this approximation at the entire interval of the possible values of parameter $\lambda_i \in [\Lambda_{i\min}, \Lambda_{i\max}]$. With allowance for formula (33), we derive the following expressions for omission probability (32) of QO detection algorithm (17):

$$\beta(\lambda_{01},\lambda_{02}) = P[\sup\mu_{11}(\lambda_1) + \sup\mu_{21}(\lambda_2) < h] = \int_{-\infty}^{\infty} F_{21}(h-x)\,dF_{11}(x). \quad (39)$$

Here,

$$F_{i1}(u) = P[\sup\mu_{i1}(\lambda_i) < u, \Lambda_{i\min} \le \lambda_i \le \Lambda_{i\max}] \quad (40)$$

is the distribution function of the absolute maxima of Gaussian Markovian processes $\mu_{i1}(\lambda_i)$ with drift $k_{1i}(\lambda_i)$ and diffusion $k_{2i}(\lambda_i)$ coefficients (38).

First, we determine distribution function $F_{21}(u)$. In accordance with definition (40), this distribution function is equal to the probability with which random process $\mu_{21}(\lambda_2)$ does not reach threshold u at $\lambda_2 \in [\Lambda_{2\min}, \Lambda_{2\max}]$. To find function $F_{21}(u)$, we employ the method of [7, 12, 13] and introduce auxiliary random process $y(\lambda_2) = u - \mu_{21}(\lambda_2)$, which is the Gaussian Markovian process with the drift and diffusion coefficients

$$k_{12}(\lambda_2) = \frac{1}{2}\begin{cases}-1, & \lambda_2 < \lambda_{02},\\ 1, & \lambda_2 \ge \lambda_{02},\end{cases} \quad k_{22}(\lambda_2) = 1. \quad (41)$$

Then, distribution function (40) at $i = 2$

$$F_{21}(u) = P\{y(\lambda_2) > 0, \lambda_2 \in [\Lambda_{2\min}, \Lambda_{2\max}]\}$$

is the probability with which Markovian random process $y(\lambda_2)$ does not reach boundaries $y = 0$ and $y = +\infty$ at interval $\lambda_2 \in [\Lambda_{2\min}, \Lambda_{2\max}]$. The desired probability is represented as [10]

$$F_{21}(u) = \int_0^\infty W(y, \Lambda_{2\max})\, dy. \tag{42}$$

Here, $W(y, \lambda_2)$ is the solution to the Fokker–Planck–Kolmogorov equation [7, 10]

$$\frac{\partial W(y,\lambda_2)}{\partial \lambda_2} + \frac{\partial}{\partial y}[k_{12} W(y,\lambda_2)] - \frac{1}{2}\frac{\partial^2}{\partial y^2}[k_{22} W(y,\lambda_2)] = 0 \tag{43}$$

under the boundary conditions $W(y=0,\lambda_2) = W(y=\infty,\lambda_2) = 0$ and the initial condition

$$W(y,\lambda_2)\big|_{\lambda_2=\Lambda_{2\min}} = \frac{1}{\sqrt{2\pi\Lambda_{2\min}}} \exp\left[-\frac{(y-u+\Lambda_{2\min}/2)^2}{2\Lambda_{2\min}}\right].$$

Using the method of reflection with sign changing [10], we use the approach of [4] to find the solutions to Eq. (43) with coefficients (41):

$$W(y,\lambda_2) = \frac{\exp[-y/2 - (\lambda_2 - \Lambda_{2\min})/8]}{2\pi\sqrt{\Lambda_{2\min}(\lambda_2 - \Lambda_{2\min})}} \times \int_0^\infty \exp\left[-\frac{(\xi - u + \Lambda_{2\min}/2)^2}{2\Lambda_{2\min}} + \frac{\xi}{2}\right] \times \left\{\exp\left[-\frac{(y-\xi)^2}{2(\lambda_2 - \Lambda_{2\min})}\right] - \exp\left[-\frac{(y+\xi)^2}{2(\lambda_2 - \Lambda_{2\min})}\right]\right\} d\xi$$

at $\lambda_2 \le \lambda_{02}$

and

$$W(y,\lambda_2) = \frac{\exp[y/2 - (\lambda_2 - \Lambda_{2\min})/8]}{2\pi\sqrt{(\lambda_2 - \lambda_{02})(\lambda_{02} - \Lambda_{2\min})}} \times \int_0^\infty\int_0^\infty \frac{\exp\left[-(\xi - u + \Lambda_{2\min}/2)^2/2\Lambda_{2\min} + \xi/2 - \xi_1\right]}{\sqrt{2\pi\Lambda_{2\min}}} \times \left\{\exp\left[-\frac{(\xi_1-\xi)^2}{2(\lambda_{02} - \Lambda_{2\min})}\right] - \exp\left[-\frac{(\xi_1+\xi)^2}{2(\lambda_{02} - \Lambda_{2\min})}\right]\right\} \times \left\{\exp\left[-\frac{(y-\xi_1)^2}{2(\lambda_2 - \lambda_{02})}\right] - \exp\left[-\frac{(y+\xi_1)^2}{2(\lambda_2 - \lambda_{02})}\right]\right\} d\xi d\xi_1 \tag{44}$$

at $\lambda_2 > \lambda_{02}$.

Substituting solution (44) at $\lambda_2 = \Lambda_{2\max}$ in formula (42), we represent the expression for function $F_{21}(u)$ as

$$F_{21}(u) = \frac{\exp[-(\lambda_{02} - \Lambda_{2\min})/8]}{2\pi\sqrt{\Lambda_{2\min}(\lambda_{02} - \Lambda_{2\min})}} \times \int_0^\infty\int_0^\infty \exp\left[-\frac{(\xi - u + \Lambda_{2\min}/2)^2}{2\Lambda_{2\min}} + \frac{\xi - \xi_1}{2}\right] \times \left\{\exp\left[-\frac{(\xi_1-\xi)^2}{2(\lambda_{02} - \Lambda_{2\min})}\right] - \exp\left[-\frac{(\xi_1+\xi)^2}{2(\lambda_{02} - \Lambda_{2\min})}\right]\right\} \times \varphi(1, \Lambda_{2\max} - \lambda_{02}, \xi_1)\, d\xi d\xi_1, \tag{45}$$

where

$$\varphi(y_1,y_2,y_3) = \Phi\left(y_1\sqrt{y_2}/2 + y_3/y_1\sqrt{y_2}\right) - \exp(-y_3)\Phi\left(y_1\sqrt{y_2}/2 - y_3/y_1\sqrt{y_2}\right),$$

$$\Phi(x) = \int_{-\infty}^{x} \exp\left(-t^2/2\right) dt \Big/ \sqrt{2\pi}$$

is the probability integral.

A similar procedure for random process $\mu_{11}(\lambda_1)$ yields an approximate expression for the distribution function of its absolute maximum:

$$F_{11}(u) = \frac{\exp[-(\lambda_{01} - \Lambda_{1\min})/8]}{2\pi\sqrt{\Lambda_{1\min}(\lambda_{01} - \Lambda_{1\min})}} \times \int_0^\infty\int_0^\infty \exp\left[-\frac{(\xi - u + \Lambda_{1\min}/2)^2}{2\Lambda_{1\min}} + \frac{\xi - \xi_1}{2}\right] \times \left\{\exp\left[-\frac{(\xi_1-\xi)^2}{2(\lambda_{01} - \Lambda_{1\min})}\right] - \exp\left[-\frac{(\xi_1+\xi)^2}{2(\lambda_{01} - \Lambda_{1\min})}\right]\right\} \times \varphi(1, \Lambda_{1\max} - \lambda_{01}, \xi_1)\, d\xi d\xi_1. \tag{46}$$

Substituting functions (45) and (46) in formula (39), we derive an asymptotically exact expression for the omission probability:

$$\beta(\theta_{01},\theta_{02}) = \exp\left(-\frac{\mu_2 + \mu_1}{8}\right) \times \int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty \exp\left[-\frac{\left(\xi_1 + \xi_3 - h + z_{\min}^2/2\right)^2}{2z_{\min}^2} + \frac{\xi_1 - \xi_2 + \xi_3 - \xi_4}{2}\right] \chi(\xi_1,\xi_2,0,\mu_1)\,\chi(\xi_3,\xi_4,0,\mu_2) \times \varphi(1,\nu_1,\xi_2)\,\varphi(1,\nu_2,\xi_4) \times \frac{\xi_1 + \xi_3 - h + z_{\min}^2/2}{2\pi z_{\min}^3\sqrt{2\pi\mu_1\mu_2}}\, d\xi_1 d\xi_2 d\xi_3 d\xi_4. \tag{47}$$

Here, we use the notation

$$\nu_1 = q(\theta_{1\min}, \theta_{01}), \quad \nu_2 = q(\theta_{02}, \theta_{2\max}),$$
$$\mu_1 = q(\theta_{01}, \theta_{1\max}), \quad \mu_2 = q(\theta_{2\min}, \theta_{02}),$$

$$z_{\min}^2 = q(\theta_{1\max}, \theta_{2\min}),$$

$$\chi(y_1, y_2, y_3, y_4) = \exp\left(-\frac{(y_1 - y_2 + y_3)^2}{2y_4}\right) - \exp\left(-\frac{(y_1 + y_2 + y_3)^2}{2y_4}\right).$$

The above asymptotically exact (with an increase in the SNR) expression (47) for the omission probability of the radio signal with unknown moments of appearance and disappearance, amplitude, and phase coincides with the exact formula for the omission probability of a quasi-deterministic signal with unknown moments of appearance and disappearance and the a priori known amplitude [4] and the asymptotic expression for the omission probability of a quasi-deterministic signal with the unknown amplitude [5].

By way of example, we consider the detection of a rectangular pulse with the skewed upper part of the envelope [14]. We choose a priori intervals (2) of the possible moments of appearance and disappearance in such a way that parameters $\theta_{1\min}$ and $\theta_{2\max}$ are fixed and the maximum duration of the signal $T_{\max} = \theta_{2\max} - \theta_{1\min}$ remains unchanged. We assume that points $\theta_{1\max}$ and $\theta_{2\min}$ are symmetric relative to center θ of the interval $[\theta_{1\min}, \theta_{2\max}]$: $\theta = (\theta_{1\min} + \theta_{2\max})/2$. Parameters $\theta_{1\max}$ and $\theta_{2\min}$ may exhibit simultaneous variations with a variation in ratio $k = T_{\max}/T_{\min}$, where $T_{\min} = \theta_{2\min} - \theta_{1\max}$ is the minimum duration of the signal. The dynamic range of variation in the signal duration that is characterized by parameter k is $[1, \infty)$. When $k = 1$, the a priori intervals shrink to a point, so that the signal is detected at the known moments of appearance and disappearance. The duration of such a signal is $T_{\max}$. The normalized lengths of the a priori intervals are

$$\eta_i = \frac{\theta_{i\max} - \theta_{i\min}}{T_{\max}} = \frac{k-1}{2k}, \quad i = 1; 2.$$

The shape of the skewed upper part of the pulse is described using the following function:

$$f(t) = \left[1 + 2\frac{1-d}{1+d}\frac{t-\theta}{T_{\max}}\right]\frac{1+d}{2}\sqrt{\frac{3}{d^2+d+1}}, \qquad (48)$$

where parameter $d = f(\theta_{1\min})/f(\theta_{2\max})$ characterizes the slope of the skewed peak. Factor $(1+d) \times \left[3/(d^2+d+1)\right]^{1/2}/2$ is introduced in such a way that the energy of the normalized signal with the maximum duration

$$E = a_0^2 \int_{\theta_{1\min}}^{\theta_{2\max}} f^2(t)\,dt = a_0^2 T_{\max}$$

is independent of the slope of the pulse peak. Thus, we can compare the efficiencies of the signal detection for different slopes and identical energies. We calculate function (21) for signal shape (48)

$$q(\theta_1, \theta_2) = z_r^2 \frac{\xi_1 + \xi_2 + b(\xi_2^2 - \xi_1^2) + b^2(\xi_2^3 + \xi_1^3)/3}{1 + b^2/12},$$

where $b = 2(1-d)/(1+d)$,

$$z_r^2 = a_0^2 T_{\max}/N_0 \qquad (49)$$

is the SNR of the output signal of the MP receiver for the rectangular pulse with the duration $T_{\max}$, and $\xi_1 = (\theta - \theta_1)/T_{\max}$ and $\xi_2 = (\theta_2 - \theta)/T_{\max}$ are the normalized moments of the appearance and disappearance.

We assume that the true values of the moments of appearance θ_{01} and disappearance θ_{02} are located at the centers of the a priori intervals, so that $\theta_{0i} = (\theta_{i\max} + \theta_{i\min})/2$, $i = 1; 2$.

Figures 3 and 4 show the dependences of false-alarm probability (31) on threshold h for QO algorithm (17) of the detection of the rectangular radio signal with the skewed upper part of the envelope. The curves in Fig. 3 are plotted for the radio pulse with the rectangular envelope (d = 1) and several dynamic ranges of the signal duration (parameter k). The solid,

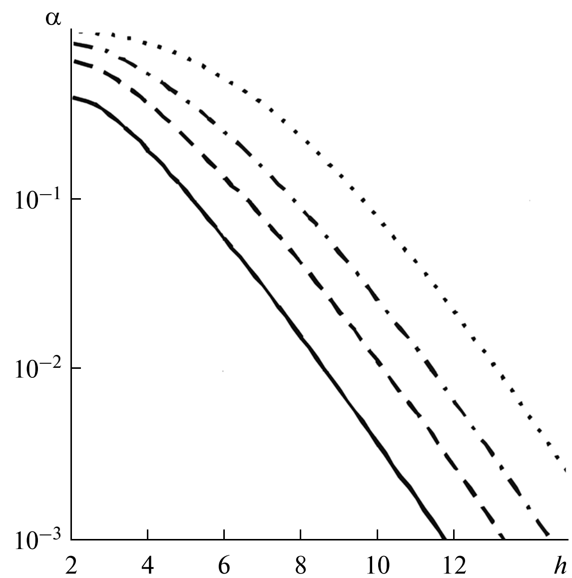

Fig. 3. Plots of the false-alarm probability of the QO detection algorithm vs. threshold for several a priori intervals of the moments of appearance and disappearance.

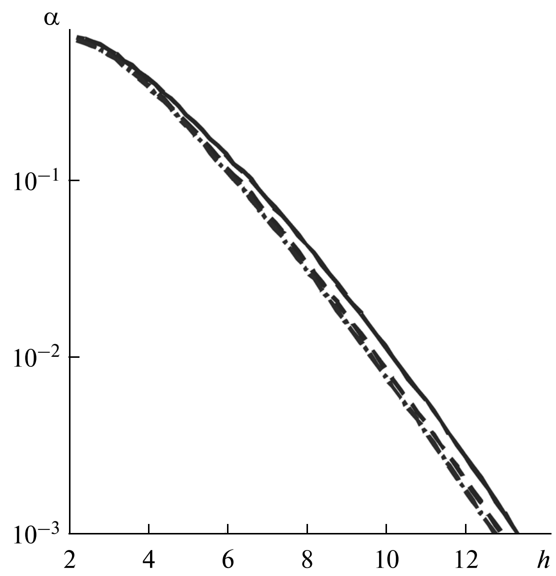

Fig. 4. Plots of the false-alarm probability of the QO detection algorithm vs. threshold for several slopes of the pulse envelope.

dashed, dashed-and dotted, and dotted lines correspond to $k = 2$, 4, 10, and 100, respectively. The curves in Fig. 4 are plotted for $k = 4$ and several slopes of the pulse peak. The solid, dashed, and dashed-and dotted lines correspond to $d = 1, 10^{\pm 1}$, and $100^{\pm 1}$, respectively. The analysis of Figs. 3 and 4 shows that an increase in the lengths of the a priori intervals of the possible moments of appearance and disappearance leads to an increase in the false-alarm probability. The deviations of the shape of the radio-pulse envelope from the rectangular shape cause a decrease in the false-alarm probability.

Figures 5 and 6 present the dependences of omission probability (47) on SNR z_r (49) for the QO algorithm (17) of the detection of the radio pulse with the skewed upper part of the envelope for the false-alarm probability $\alpha = 10^{-2}$. The curves in Fig. 5 are plotted for the radio pulse with the rectangular envelope ($d = 1$) and several dynamic ranges of the signal duration (k). The solid, dashed, and dashed-and dotted lines correspond to $k = 2$, 4, and 10, respectively. The curves in Fig. 6 are plotted for $k = 4$ and several slopes of the upper part of the radio-pulse envelope. The solid, dashed, and dashed-and-dotted lines correspond to $d = 1$, $10^{\pm 1}$, and $100^{\pm 1}$, respectively. The analysis of Figs. 5 and 6 shows that an increase in the dynamic range of variations in the signal duration (parameter k) and the deviations of the envelope shape from the rectangular shape lead to a decrease in the detection efficiency.

Omission probability (47) is invariant with respect to the absence of the a priori data on the amplitude and phase of the signal. Thus, the effect of the a priori absence of information on the initial phase of the radio signal on the detection efficiency can be quantitatively characterized using the quantity

$$\zeta = \alpha(h_a)/p, \tag{50}$$

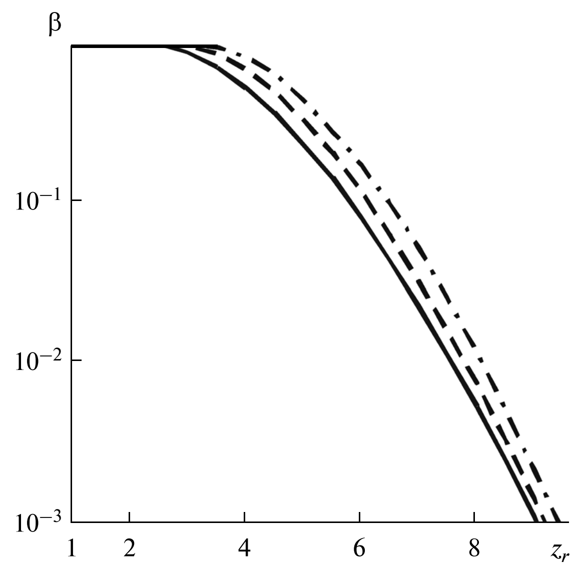

Fig. 5. Plots of the omission probability vs. SNR for a fixed false-alarm probability of 10^{-2} and several dynamic ranges of the signal duration.

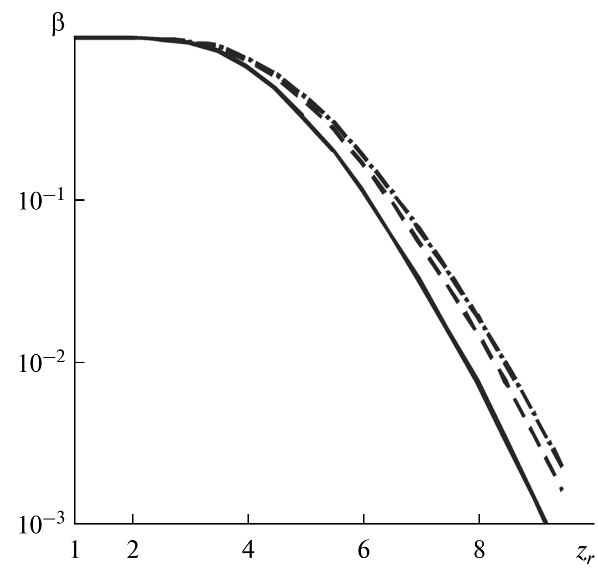

Fig. 6. Plots of the omission probability vs. SNR for a fixed false-alarm probability of 10^{-2} and several slopes of the pulse envelope.

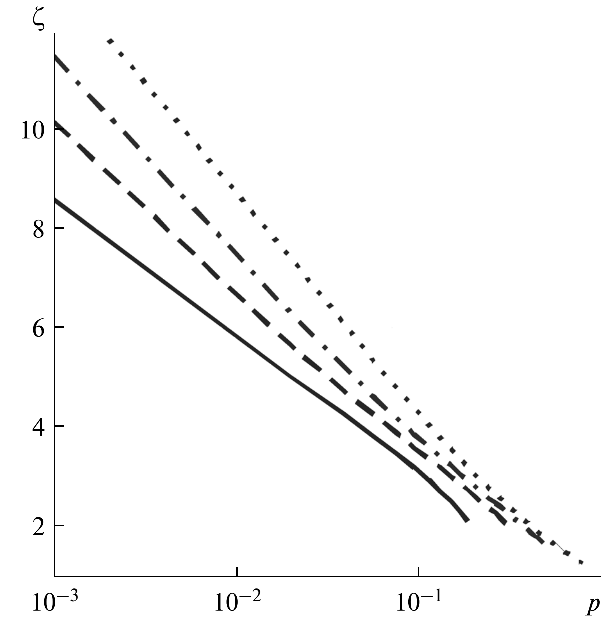

Fig. 7. Plots of the loss in the probability of the false alarm for the QO detection algorithm owing to the absence of information on the initial phase of the signal.

where h_a is the threshold that is found from the solution to the equation $\alpha_a(h_a) = p$ and

$$\alpha_a(h) = 1 - H(h-1)$$
$$\times\left[\exp\left(-\tilde{m}_1\sqrt{\frac{1}{2\pi}}\exp\left(-\frac{1}{2}\right) - \tilde{m}_2\sqrt{\frac{h-1/2}{\pi}}\exp\left(\frac{1}{2}-h\right)\right)\right.$$
$$+ \frac{\tilde{m}_1}{\sqrt{\pi}}\int_{1/2}^{h-1/2}\frac{2y-1}{2\sqrt{y}}\exp\left[-\tilde{m}_2\sqrt{\frac{h-x}{\pi}}\exp(y-h)\right.$$
$$\left.\left. - \tilde{m}_1\sqrt{\frac{y}{\pi}}\exp(-y) - y\right]dy\right]$$

is the formula of [5] for the false-alarm probability of the QO algorithm for the detection of the signal with the unknown amplitude and the moments of appearance and disappearance and the a priori known initial phase. Quantity ζ (50) characterizes an increase in the false-alarm probability of the QO detection algorithm related to the a priori absence of information on the initial phase of the signal at a constant omission probability. Figure 7 presents the dependence of parameter (50) on p for the radio pulse with the rectangular envelope ($d = 1$) and several dynamic ranges of the signal duration. The solid, dashed, dashed-and dotted, and dotted lines correspond to k = 2, 4, 10, and 100, respectively. It is seen that a relative increase in the false-alarm probability owing to the absence of the a priori information on initial phase of the radio signal increases with a decrease in the needed level of the false alarm and an increase in the a priori intervals of the moments of appearance and disappearance.

CONCLUSIONS

The application of the QO detection algorithm in accordance with which the decision statistics is represented as a sum of two independent terms and each term is maximized with respect to the amplitude and phase makes it possible to propose a two-channel scheme of the detector and analyze the detection algorithm. The analysis of the characteristics of the proposed QO detection algorithm shows that an increase in the relative lengths of the a priori intervals of the possible moments of appearance and disappearance substantially contributes to a decrease in the efficiency of the QO detection algorithm. A decrease in the allowed level of the false alarm leads to an increase in the relative loss of the efficiency of the QO detection algorithm owing to the a priori absence of information on the initial phase of the signal.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research (project no. 14-49-00079).

REFERENCES

1. A. G. Tartakovskii, Probl. Peredachi Inf. **24** (2), 39 (1988).
2. V. G. Repin, Probl. Peredachi Inf. **27** (1), 61 (1991).
3. A. P. Trifonov and Yu. E. Korchagin, Probl. Peredachi Inf. **37** (1), 52 (2001).
4. A. P. Trifonov and Yu. E. Korchagin, Probl. Peredachi Inf. **45** (2), 91 (2009).
5. A. P. Trifonov, Yu. E. Korchagin, and P. A. Kondratovich, Nelin. Mir **10**, 287 (2012).
6. V. I. Tikhonov, *Optimal Detection of Signals* (Radio i Svyaz', Moscow, 1983) [in Russian].
7. A. P. Trifonov and Yu. S. Shinakov, *Simultaneous Separation of Signals and Estimation of Their Parameters in the Presence of Noise* (Radio i Svyaz', Moscow, 1986) [in Russian].
8. P. S. Akimov, P. A. Bakut, V. A. Bogdanovich, et al., *Signal Detection Theory*, Ed. by P. A. Bakut (Radio i Svyaz', Moscow, 1984) [in Russian].
9. B. I. Shakhtarin, *Signal Detection* (Goryachaya Liniya-Telekom, Moscow, 2014) [in Russian].
10. V. I. Tikhonov and M. A. Mironov, *Markovian Processes* (Radio i Svyaz', Moscow, 1977) [in Russian].
11. Yu. G. Sosulin, *Detection and Estimation Theory for Stochastic Signals* (Sovetskoe Radio, Moscow, 1978) [in Russian].
12. A. P. Trifonov and Yu. E. Korchagin, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **45**, 625 (2002).
13. A. P. Trifonov and Yu. E. Korchagin, J. Commun. Technol. Electron. **50**, 406 (2005).
14. M. I. Gryaznov, M. L. Gurevich, and Yu. A. Ryabinin, *Measurement of Pulse Parameters* (Radio i Svyaz', Moscow, 1991) [in Russian].

Translated by A. Chikishev