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Estimation of the Number of Orthogonal Signals with Unknown Non-Energy Parameters[1]

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Abstract—The synthesis and analysis of an algorithm for estimation of the number of orthogonal signals with unknown non-energy parameters have been performed by using the maximum likelihood method.

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The need of estimating the number of received signals arises in case of the unknown number of signal sources and in case of the unknown channel structure through which the signal is transmitted. For example, in the case of using a multipath radio communication channel in MIMO systems [1, 2], the number of beams is often a priori unknown and it should be determined. In case of radar and acousto-detection-and-ranging (active or passive) observation, it is quite a common situation when the number of signals arriving at the antenna array is unknown [3–9].

However, today the problem of estimating the number of signals has been solved only partially. The difficulties arise in determining the structure of estimation algorithm. In practice the results of theoretical analysis of the performance quality of algorithms for estimating the number of signals are not available. Moreover, the generally accepted and well-defined quantitative characteristic of such algorithms is also lacking. Without the introduction of quantitative characteristics of algorithms for estimating the number of signals, difficulaties arise in comparing the specified algorithms and choosing the most effective one.

The problem of estimating the number of signals with unknown amplitudes was investigated in [10] where it was assumed that the received signals could be nonorthoonal. The signal amplitude is an energy parameter [11] because the signal energy depends on it. At the same time, there is often a need of estimating the number of signals, which contain unknown non-energy parameters. They include the time of signal arrival, signal frequency, initial phase, etc.

Below we consider the problem of synthesis and analysis of the algorithm for estimation of the number of orthogonal signals with unknown non-energy parameters that are received against the background of the Gaussian white noise. The synthesis of estimation algorithm is performed on the basis of the maximum likelihood method [11]. The algorithm efficiency is characterized by the error probability in estimating the number of signals. The analysis of the obtained algorithm involved the use of the signal detection theory [12] and the notion of the truncated error probability [10].

Following [10] we shall consider the maximum likelihood estimate of the number of orthogonal deterministic signals. Let us assume that a sum of ν signals $s_i(t)$ is observed, so that the signal received will have the form:

$$s(t,\nu)=\sum_{i=1}^{\nu} s_i(t), \tag{1}$$

where $\nu=\overline{1,\nu_{\max}}$, $\forall i \;\; s_i(t)\in L_2(0,T)$.

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Let us assume that signal (1) is observed during the time interval $[0, T]$ against the background of additive Gaussian white noise $n(t)$ with one-sided spectral density N_0. Therefore, the following realization is available for processing:

$$x(t) = n(t) + \sum_{i=1}^{\nu_0} s_i(t), \tag{2}$$

where ν_0 is the true number of signals.

Let us impose the orthogonality condition on functions of set $\{s_i(t)\}_{i=1}^{\nu_{\max}}$:

$$\int_0^T s_i(t) s_j(t) \mathrm{d}t = \begin{cases} E_i, & i = j, \\ 0, & i \neq j. \end{cases}$$

For estimation of the number of signals ν_0 we shall use the maximum likelihood method. Paper [11] presented a formula for the logarithm of likelihood ratio functional (LRF) when the interference is additive white Gaussian noise:

$$L(l) = \frac{2}{N_0} \int_0^T x(t) s(t, l) \mathrm{d}t - \frac{1}{N_0} \int_0^T s^2(t, l) \mathrm{d}t, \tag{3}$$

where l is the collection of unknown parameters of signal $s(t, l)$.

Substituting expression (1) into (3) we can write the LRF logarithm in the form:

$$L(\nu) = \frac{2}{N_0} \sum_{m=1}^{\nu} \int_0^T x(t) s_m(t) \mathrm{d}t - \frac{1}{N_0} \sum_{m=1}^{\nu} E_m, \tag{4}$$

where $\nu \in \overline{1, \nu_{\max}}$, $E_m = \int_0^T s_m^2(t) \mathrm{d}t$ is the energy of the mth signal in expression (1).

Using expression (4) we can find the maximum likelihood algorithm for the estimation of the number of signals:

$$\hat{\nu} = \arg\sup_{\nu} L(\nu), \quad \nu = \overline{1, \nu_{\max}}. \tag{5}$$

Let us consider now properties of the LRF logarithm (4). To this end, substituting the realization of observed data (2) into expression (4), we obtain:

$$L(\nu, \mathbf{z}) = \sum_{i=1}^{\min(\nu_0, \nu)} z_i^2 - \frac{1}{2} \sum_{i=1}^{\nu} z_i^2 + \sum_{i=1}^{\nu} z_i \eta_i, \tag{6}$$

where $z_i^2 = 2E_i / N_0$ is the signal-to-noise ratio (SNR) for the ith signal, η_i are the independent Gaussian random quantities with parameters (0, 1), $\mathbf{z} = \| z_i \|$ is the SNR vector.

As is shown in [10], from expression (6) it follows that estimate (5) is consistent [11].

The efficiency of the algorithm of estimating the number of signals can be characterized by the quantity of total error probability $p_e = p(\hat{\nu} \neq \nu_0)$. However the calculation of this probability involves the need of considerable computational resources.

In order to obtain a simplified approximate formula for error probability, it should be noted that any algorithm $\Re$ of estimating the number of signals can be presented in the form:

$$\hat{\nu} = \arg\sup_{\nu} R(\nu, x(t)),$$

where $R(\nu, x(t))$ is the functional determined by the structure of algorithm $\Re$ and dependent on the number signals and the realization of observed data. Using this representation the total error probability for algorithm $\Re$ can be written in the form:

$$p_e = 1 - p(R(\nu_0, x(t)) > R(i, x(t)),\ i \neq \nu_0,\ i = \overline{1, \nu_{\max}}). \quad (7)$$

Next, for the total error probability approximation on condition $1 < \nu_0 < \nu_{\max}$ we shall introduce into consideration the truncated error probability of algorithm $\Re$ that can be determined in the form:

$$p_t = 1 - p(R(\nu_0, x(t)) > R(\nu_0 + 1, x(t)),$$

$$R(\nu_0, x(t)) > R(\nu_0 - 1, x(t))). \quad (8)$$

From (8) it follows that the truncated error probability is the low limit for the total error probability (7) when $1 < \nu_0 < \nu_{\max}$. In case $\nu_0 = 1$ or $\nu_0 = \nu_{\max}$, instead of (8) it is necessary to use formulas

$$p_t = 1 - p(R(\nu_0, x(t)) > R(\nu_0 + 1, x(t))$$

or

$$p_t = 1 - p(R(\nu_0, x(t)) > R(\nu_0 - 1, x(t))),$$

respectively. It should be noted that truncated error probability (8) coincides with total error probability (7) at $\nu_{\max} = 3$ and $\nu_0 = 2$.

Let us find the truncated error probability (8) for algorithm (5):

$$p_{t0} = 1 - p(L(\nu_0) > L(\nu_0 + 1), L(\nu_0) > L(\nu_0 - 1)). \quad (9)$$

Substituting (6) into (9) we obtain the formula for the truncated error probability of algorithm (5):

$$p_{t0} = 1 - p\left(\eta_{\nu_0} > -\frac{z_{\nu_0}}{2}, \eta_{\nu_0+1} < \frac{z_{\nu_0+1}}{2}\right) = 1 - \Phi\left(\frac{z_{\nu_0}}{2}\right)\Phi\left(\frac{z_{\nu_0+1}}{2}\right), \quad (10)$$

where $\Phi(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} \exp(-t^2/2) dt$ is the probability integral.

Expression (10) is a particular case of the truncated error probability for the algorithm of estimating the number of signals with known amplitudes [10].

Statistical simulation of the algorithm for estimating the number of signals (5) was carried out for the functional check of algorithm (5) and establishment of the applicability limits of formula (10). For the sake of illustration of simulation results thereafter it is assumed that all energies of signals in expression (1) are equal and for all signals $z^2 = z_i^2$, $i \in \overline{1, \nu_{\max}}$. The simulation results are shown in Fig. 1.

Solid line in Fig. 1 shows the relationship of the error probability in determining the number of signals ν by using algorithm (5) as a function of SNR at $\nu_{\max} = 3$ and $\nu_0 = 2$. This probability coincides with truncated error probability (10) at $\nu_{\max} \geq 3$. In addition, Fig. 1 presents the values of error probability in determining the number of signals ν by using algorithm (5) that were obtained by statistical simulation for the following cases: $\nu_{\max} = 3$ and $\nu_0 = 2$ (circles); $\nu_{\max} = 9$ and $\nu_0 = 5$ (squares); and $\nu_{\max} = 101$ and $\nu_0 = 51$ (triangles).

From Fig. 1 it follows that at SNR of more than 2.5–3, formula (10) can be used for the calculation of total error probability (7) of algorithm (6) at any $\nu_{\max} \leq 101$. In addition, Fig. 1 confirms that the truncated error probability is the low limit of total error probability (7) and tends to this limit with the rise of SNR.

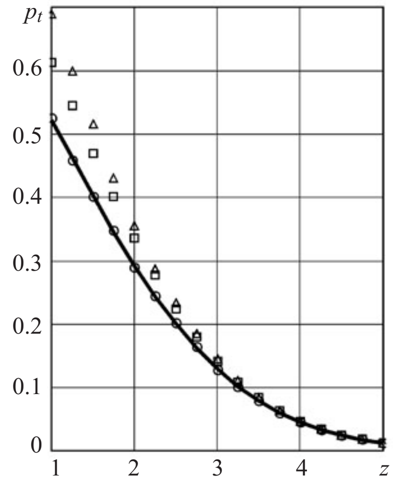

Fig. 1.

Let us consider the extension of the above statements to the case where signals have unknown non-energy parameters. We shall assume that the sum of ν signals $s_i(t, \mathbf{l}_i)$ can be observed, and each of these signals depends on the vector of unknown parameters $\mathbf{l}_i$:

$$s(t, \nu, \mathbf{L}) = \sum_{i=1}^{\nu} s_i(t, \mathbf{l}_i), \tag{11}$$

where $\nu = \overline{1, \nu_{\max}}$; $\forall\, i, \mathbf{l}_i : s_i(t, \mathbf{l}_i) \in L_2(T_1, T_2)$; $\mathbf{L} = ||\mathbf{l}_1, \ldots, \mathbf{l}_{\nu_{\max}}||$ is the block vector combining all unknown parameters, $\mathbf{l}_i = ||l_{i1}, \ldots, l_{i\mu_i}||$; $\mathbf{l}_i \in \Lambda_i$; Λ_i is the a priori domain of possible values of the ith vector of unknown parameters, μ_i is the number of unknown parameters of the ith signal.

Let us assume that signal (11) is observed as before during time interval $[0, T]$ against the background of additive Gaussian white noise $n(t)$ with one-sided spectral density N_0. Hence, the following realization is available for processing:

$$x(t) = n(t) + \sum_{i=1}^{\nu_0} s_i(t, \mathbf{l}_{0i}), \tag{12}$$

where ν_0 and $\mathbf{L}_0 = ||\mathbf{l}_{01}, \ldots, \mathbf{l}_{0\nu_0}||$ are the true values of corresponding parameters.

Let us impose the orthogonality condition on functions of set $\{s_i(t, \mathbf{l}_i)\}_{i=1}^{\nu_{\max}}$:

$$\int_0^T s_i(t, \mathbf{l}_i) s_j(t, \mathbf{l}_j) \mathrm{d}t = \begin{cases} E_i, & i = j, \\ 0, & i \neq j. \end{cases} \tag{13}$$

Equality (13) should be fulfilled at any values of unknown parameters $\mathbf{l}_i \in \Lambda_i$. We shall assume that unknown parameters $\mathbf{L} = ||\mathbf{l}_1, \ldots, \mathbf{l}_{\nu_{\max}}||$ are non-energy ones [11], i.e., $\forall\, i$:

$$\int_0^T s_i^2(t, \mathbf{l}_i) \mathrm{d}t = E_i = \mathrm{const}(\mathbf{l}_i).$$

Let us introduce into consideration the following functionals:

$$L_{zi}(\mathbf{l}_i) = \frac{2}{N_0}\int_0^T x(t)s_i(t,\mathbf{l}_i)\mathrm{d}t. \tag{14}$$

Substituting expression (11) into (3) and using expression (14), we shall write the LRF logarithm in the form:

$$L(\nu, \mathbf{L}) = \sum_{i=1}^{\nu}[L_{zi}(\mathbf{l}_i) - z_i^2/2]. \tag{15}$$

In accordance with the maximum likelihood method the values of unknown parameters of signals in expression (15) should be replaced with their maximum likelihood estimates. This procedure is reduced to maximization of LRF logarithm (15) in terms of the values of vector of unknown parameters **L**:

$$L_m(\nu) = \sup_{\mathbf{L}} L(\nu,\mathbf{L}) = \sum_{i=1}^{\nu}[L_{mzi} - z_i^2/2], \tag{16}$$

where $L_{mzi} = \sup_{\mathbf{l}_i} L_{zi}(\mathbf{l}_i)$, $\mathbf{l}_i \in \Lambda_i$ is the absolute (the largest) maximum of LRF logarithm (3).

Using (16) we shall write the maximum likelihood algorithm for the estimate of the number of signals:

$$\hat{\nu} = \arg\sup_{\nu} L_m(\nu), \quad \nu = \overline{1, \nu_{\max}}. \tag{17}$$

Truncated error probability (8) for algorithm (17) has the form:

$$p_{t0} = 1 - p(L_m(\nu_0) > L_m(\nu_0+1), L_m(\nu_0) > L_m(\nu_0-1))$$

$$= 1 - p(L_m(\nu_0) - L_m(\nu_0+1) > 0, L_m(\nu_0) - L_m(\nu_0-1) > 0). \tag{18}$$

Substituting expression (16) into (18), we obtain

$$p_t = 1 - p\left(L_{mz\nu_0} > \frac{z_{\nu_0}^2}{2}, L_{mz\nu_0+1} < \frac{z_{\nu_0+1}^2}{2}\right). \tag{19}$$

It can be shown that processes $L_{z\nu_0}(\mathbf{l}_{\nu_0})$ and $L_{z(\nu_0+1)}(\mathbf{l}_{\nu_0+1})$ are uncorrelated and, therefore, independent, hence the absolute maxima $L_{mz\nu_0}$, $L_{mz\nu_0+1}$ of these processes are independent random quantities. Therefore, expression (19) can be written in the form:

$$p_t = 1 - p\left(L_{mz\nu_0} > \frac{z_{\nu_0}^2}{2}\right) p\left(L_{mz\nu_0+1} < \frac{z_{\nu_0+1}^2}{2}\right). \tag{20}$$

Probability p_t will be determined by using expression (20). Then first probability p has the form:

$$p\left(L_{mz\nu_0} > \frac{z_{\nu_0}^2}{2}\right) = 1 - p\left(L_{mz\nu_0} < \frac{z_{\nu_0}^2}{2}\right)$$

$$=1-p\left\{\sup_{\mathbf{l}_{\nu_0}}\left[z_{\nu_0}^2 S_{\nu_0}(\mathbf{l}_{0\nu_0},\mathbf{l}_{\nu_0})+z_{\nu_0}N_{\nu_0}(\mathbf{l}_{\nu_0})\right]<\frac{z_{\nu_0}^2}{2}\right\}, \tag{21}$$

where $S_k(\mathbf{l}_{1k},\mathbf{l}_{2k})$ is the normalized signal function determined by the following expression [11]:

$$S_k(\mathbf{l}_{1k},\mathbf{l}_{2k})=\frac{2}{N_0}\int_0^T s_k(t,\mathbf{l}_{1k})s_k(t,\mathbf{l}_{2k})\frac{\mathrm{d}t}{z_k^2}, \tag{22}$$

$N_k(\mathbf{l}_k)$ is the normalized noise function that represents a realization of the centered Gaussian uniform field with correlation function (22) [12].

The problem of detection of signal with unknown non-energy parameters by the maximum likelihood method is considered in [12]. In this case a decision about the signal presence or absence is made on the basis of comparison of the absolute maximum of LRF logarithm $L_{mz\nu}$ (16) with the threshold. Comparing expression (21) with formulas from paper [12], we can come to a conclusion that probability $p(L_{mz\nu_0}<z_{\nu_0}^2/2)$ in (21) coincides with the probability of error of second kind (probability of signal skip) in case of detecting signal $s_{\nu_0}(t,\mathbf{l}_{\nu_0})$ by the maximum likelihood method if the detection threshold is equal to $z_{\nu_0}^2/2$ [12].

Let us designate the probability of error of second kind in detecting signal $s_k(t,\mathbf{l}_{0k})$ as follows:

$$\beta_k=p\left\{\sup_{\mathbf{l}_k}\left[z_k^2 S_k(\mathbf{l}_{0k},\mathbf{l}_k)+z_k N_k(\mathbf{l}_k)\right]<\frac{z_k^2}{2}\right\}=p(L_{mzk}<z_k^2/2). \tag{23}$$

According to [12] the approximate expression for probability (23) assumes the form

$$\beta_k=\frac{z_k^{\mu_k/2}}{\sqrt{2\pi}}\exp\left[-\frac{\xi_k}{(2\pi)^{(\mu_k+1)/2}}\left(\frac{z_k}{2}\right)^{\mu_k-1}\exp\left(-\frac{z_k^2}{8}\right)\right]$$

$$\times\int_{-\infty}^{z_k/2}\exp\left(\frac{2z_k^2-x^2}{4}\right)D_{-\mu_k/2}(2z_k-x)\mathrm{d}x \tag{24}$$

at $z_k>2\sqrt{\mu_k-1}$ and $\beta_k\approx 0$ at $z_k<2\sqrt{\mu_k-1}$. The accuracy of formula (24) improves with the rise of ξ_k and z_k.

The following designations are used in (24): $D_\mu(x)$ is the function of parabolic cylinder [13],

$$\xi_k=V_k\sqrt{\det\left\|\left.\frac{\partial^2 S_k(\mathbf{l}_{1k},\mathbf{l}_{2k})}{\partial l_{1ki}\partial l_{2kj}}\right|_{\mathbf{l}_{1k}=\mathbf{l}_{2k}}\right\|},$$

$$i,j=\overline{1,\mu_k} \tag{25}$$

ξ_k is the reduced volume of a priori domain Λ_k of possible values of parameters $\mathbf{l}_k$, V_k is the Euclidean volume of this domain.

The reduced volume (25) characterizes the number of different values of unknown parameters in a priori domain of their possible values. If the interference represents a white Gaussian noise, the reduced volume (25) can be expressed in terms of derivatives of the useful signal

$$\xi_k = V_k \sqrt{\det\left\| \frac{1}{E_k} \int_0^T \frac{\mathrm{d}s_k(t,\mathbf{l}_k)}{\mathrm{d}l_{ki}} \frac{\mathrm{d}s_k(t,\mathbf{l}_k)}{\mathrm{d}l_{kj}} \mathrm{d}t \right\|},$$

$$i, j = \overline{1,\mu_k}. \tag{26}$$

Next we can determine second probability p in (20) as follows:

$$p\left(L_{mz\nu_0+1} < \frac{z^2_{\nu_0+1}}{2}\right) = 1 - p\left(L_{mz\nu_0+1} > \frac{z^2_{\nu_0+1}}{2}\right)$$

$$= 1 - p\left\{\sup_{\mathbf{l}_{\nu_0+1}} \left[N_{\nu_0+1}(\mathbf{l}_{\nu_0+1})\right] > \frac{z_{\nu_0+1}}{2}\right\} = 1 - \alpha_{\nu_0+1}. \tag{27}$$

Quantity α_{ν_0+1} can be interpreted as the probability of error of first kind (false alarm) in detecting signal $s_{\nu_0+1}(t,\mathbf{l}_{0\nu_0+1})$ by the maximum likelihood method if the detection threshold is equal to $z^2_{\nu_0+1}/2$. Indeed, as is shown in [12], the probability of false alarm in detecting signal $s_{\nu_0+1}(t,\mathbf{l}_{0\nu_0+1})$ by the maximum likelihood method is equal to the exceedence of the specified detection threshold by random field $N_{\nu_0+1}(\mathbf{l}_{\nu_0+1})$.

Using the formula obtained in [12], we can write an expression for the false alarm probability in (27) in the form:

$$\alpha_k = \begin{cases} 1 - \exp\left[-\dfrac{\xi_k}{(2\pi)^{(\mu_k+1)/2}} \left(\dfrac{z_k}{2}\right)^{\mu_k - 1} \exp\left(-\dfrac{z_k^2}{8}\right)\right], & z_k > 2\sqrt{\mu_k - 1}, \\ 1, & z_k < 2\sqrt{\mu_k - 1}. \end{cases} \tag{28}$$

Using (21)–(27), we can find the truncated error probability (19):

$$p_t = 1 - (1 - \beta_{\nu_0})(1 - \alpha_{\nu_0+1}) = \beta_{\nu_0} + \alpha_{\nu_0+1} - \beta_{\nu_0}\alpha_{\nu_0+1}, \tag{29}$$

where β_{ν_0} and α_{ν_0+1} are determined from expressions (24) and (28), respectively.

At sufficiently large values of SNR z_k formulas (24) and (28) can be simplified by replacing their asymptotically exact expressions at $z_k \to \infty$ [12]

$$\beta_k = 1 - \Phi(z_k/2),$$

$$\alpha_k = \frac{\xi_k}{(2\pi)^{(\mu_k+1)/2}} \left(\frac{z_k}{2}\right)^{\mu_k - 1} \exp\left(-\frac{z_k^2}{8}\right). \tag{30}$$

At $\xi_{\nu_0} = \xi_{\nu_0+1} = \xi$, $\mu_{\nu_0} = \mu_{\nu_0+1} = \mu$ and sufficiently large SNR $z^2 = z^2_{\nu_0} = z^2_{\nu_0+1}$ expression (29) can be written in the following form by using (30):

$$p_t = \frac{1}{z}\sqrt{\frac{2}{\pi}} \exp\left(-\frac{z^2}{8}\right)\left(1 + \frac{\xi z^{\mu}}{2^{3\mu/2}\pi^{\mu/2}}\right). \tag{31}$$

If the signal does not contain unknown parameters, then at large values of SNR z^2 expression (10) assumes the form

$$p_{t0} = \frac{2}{z}\sqrt{\frac{2}{\pi}}\exp\left(-\frac{z^2}{8}\right).$$

The degradation of performance quality of algorithm (17) for the estimation of the number of signals due to the presence of unknown non-energy parameters can be characterized by the following quantity:

$$\chi = \frac{p_t}{p_{t0}} = \frac{1}{2}\left(1 + \frac{\xi z^{\mu}}{2^{3\mu/2}\pi^{\mu/2}}\right).$$

Therefore, the relative degradation of the estimation quality of the number of signals due to the presence of unknown non-energy parameters takes place with the increase of the following parameters: SNR, the number of unknown parameters and the reduced volume of a priori domain of possible values of unknown parameters.

As an example, let us consider a case where signal $s_{\nu_0}(t, \mathbf{l}_{\nu_0})$ represents a radio pulse with bell-shaped envelope:

$$s(t, \varphi, \omega, \lambda) = a\exp\left(-\frac{(t-\lambda)^2}{\tau^2}\right)\cos(\omega t - \varphi), \quad (32)$$

where phase $\varphi \in [-\pi, \pi]$, frequency $\omega \in [\Omega_1, \Omega_2]$ and delay $\lambda \in [T_1, T_2]$ are the unknown parameters of signal. It is assumed that the time of signal observation [0, *T*] is much more than signal duration τ, $T_1 > 0$ and $T_2 < T$.

Using (26), we shall find the reduced volumes of a priori domains of possible values of unknown parameters for the following situations:

1) when only phases are unknown: $\xi_1 = 2\pi$, $\mu_1 = 1$;

2) when phases and frequencies are unknown: $\xi_2 = \pi\tau(\Omega_2 - \Omega_1)$, $\mu_2 = 2$; and

3) when all three parameters of signal (phase, frequency and delay) are unknown: $\xi_3 = \pi(T_2 - T_1)(\Omega_2 - \Omega_1)$, $\mu_3 = 3$.

Substituting the values of determined volumes of a priori domains and the numbers of unknown parameters into expressions (24) and (27), we shall calculate the truncated error probability (28) for the above situations.

Figure 2 displays the relationships of the truncated error probability of estimate of the number of signals as a function of SNR when all parameters of signal (32) are a priori known (dash-dotted line) and in the presence of unknown non-energy parameters in signal (32) (solid lines). Figure 2 shows the following cases: only phases of received signals are unknown: $\xi_1 = 2\pi$ (curve *1*); phases and frequencies of received signals are unknown: $\xi_2 = 10\pi$ (curve *2*); and the phases, frequencies and delays of received signals are unknown: $\xi_3 = 50\pi$ (curve *3*).

As can be seen from Fig. 2, an increase of the number of unknown parameters and an increase of the reduced volume of a priori domain of possible values of unknown parameters significantly affect the performance quality of algorithm for estimation of the number of signals that coincides with the results of calculations. For example, at SNR $z = 7$ the error probability for algorithm (17) due to the lack of knowledge of the initial phase increases by a factor of 5 in relation to situation where all parameters are known; the lack of knowledge of the phase and frequency results in the rise of this probability by a factor of 50; and the lack of knowledge of all three parameters of the signal leads to a 100-fold rise of the error probability for algorithm (17).

The proposed truncated error probability of estimate of the number of signals makes it possible to quantitatively characterize the efficiency of algorithms for estimation of the number of signals. The truncated error probability is the lower limit of the total error probability and tends to it with the rise of SNR. The obtained results of analysis of the maximum likelihood algorithm for estimation of the number of signals with unknown non-energy parameters enable us to assess losses in the performance quality of the

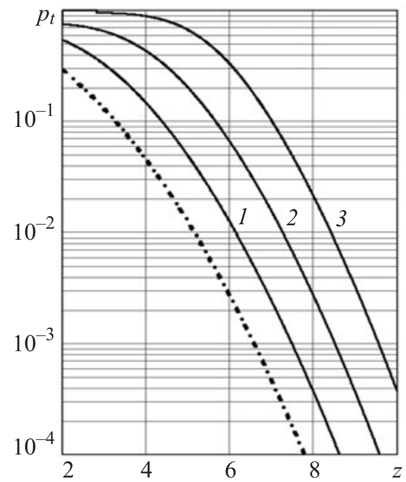

Fig. 2.

algorithm depending on the number and character of unknown parameters. It has been shown that the rise of the reduced volume of a priori domain of possible values of unknown parameters results in the degraded performance quality of the discussed algorithm for estimation of the number of signals.

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