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The Estimate of the Dispersion of the Random Radio Pulse with Unknown Time of Arrival in the Presence of the Interference with Unknown Intensity

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Abstract

We are considering a problem of the measurement of the dispersion of the random pulse signal with unknown time of arrival against the white noise and the correlated Gaussian interference. By applying a maximum likelihood method, we synthesize quasi-optimal, quasi-likelihood and adaptive estimation algorithms. We also find out the theoretical and experimental dependences for the characteristics of the obtained dispersion estimates that are then used in the study of the efficiency of the introduced algorithms and in the further investigation revea-

ling the loss in estimation accuracy, due to the absence of the prior information on intensity of operational interferences. We are to show that, with the adaptation in terms of intensity of the correlated interference, it is possible to obtain the dispersion estimate independent from the intensity of white noise, and its characteristics coincide asymptotically with the corresponding characteristics of the dispersion estimate, obtained under the a priori known intensities of interference and white noise.

Keywords: random Gaussian pulse, band interference, maximum likelihood method, parametrical prior uncertainty, adaptive estimate, dispersion measurer, local Markov approximation method

1 Introduction

The problem of estimation of the dispersion of the stationary random processes is considered in a number of researches [1-3, etc.]. At the same time, in many applications of statistical radio engineering and radio physics it is necessary to estimate the dispersion of the essentially non-stationary random pulses against hindrances.

In the study [4] the way of hardware implementation is suggested and the efficiency is tested of the maximum likelihood measurer of the dispersion of the random pulse signal with unknown time of arrival against Gaussian white noise. However, besides the effect of the receiver noise (approximated by Gaussian white noise), the useful signal is often rather distorted by the additive external disturbance with a generally unknown intensity. As examples of such disturbances, there can be considered an external unintentional (interburst) interference passed through the input receiver filter (preselector) [3], or a barrage jamming [5, 6].

2 The Problem Definition

As opposed to [4], we assume that the realization of the random process

$$x(t) = s(t, \lambda_0, D_0) + n(t) + \nu(t). \tag{1}$$

is observed over the interval $[T_1, T_2]$. Here $s(t, \lambda_0, D_0)$ is the random Gaussian pulse

$$s(t, \lambda_0, D_0) = \xi(t)\, I\left(\frac{t-\lambda_0}{\tau}\right), \qquad I(x) = \begin{cases} 1, & |x| < 1/2, \\ 0, & |x| \ge 1/2, \end{cases} \tag{2}$$

representing the segment of the realization of stationary centered Gaussian random process $\xi(t)$ with dispersion D_0, time of arrival λ_0 and duration τ. We now write down the process $\xi(t)$ spectral density (SD) in the form of [4, 7]

$$G_\xi(\omega) = (\pi D_0/\Omega_0)\left\{ I[(\vartheta-\omega)/\Omega_0] + I[(\vartheta+\omega)/\Omega_0] \right\},$$

where ϑ is the band center, and Ω_0 is the bandwidth of the process $\xi(t)$. As in

[4], we approximate an additive interference $n(t)$ by Gaussian white noise with one-sided SD N_0. As model of the external interference $\nu(t)$, we choose stationary centered Gaussian random process possessing the SD [5-7]

$$G_\nu(\omega) = (\gamma_0/2)\{ I[(\vartheta - \omega)/\Omega_1] + I[(\vartheta + \omega)/\Omega_1] \}. \tag{3}$$

In Eq. (3) Ω_1 and γ_0 are bandwidth and SD magnitude (intensity) of the process $\nu(t)$, correspondently. We assume that $\Omega_1 \geq \Omega_0$, and the processes $s(t)$, $n(t)$ and $\nu(t)$ are statistically independent. Besides, we suppose that the radio pulse (2) duration τ is much greater than the correlation time of the process $\xi(t)$, i.e. the following condition is satisfied:

$$\mu = \tau\Omega_0/2\pi >> 1. \tag{4}$$

With observable realization (1), it is necessary to estimate the dispersion D_0 of the random process $\xi(t)$. Time of arrival λ_0 of the radio pulse (2) and intensity γ_0 of the interference $\nu(t)$ can be generally unknown and possess the values within prior intervals $\lambda_0 \in [\Lambda_1, \Lambda_2]$, $\gamma_0 \in [0, \infty)$. And the values T_1 and T_2 are such that $T_1 \leq \Lambda_1 - \tau/2 < \Lambda_2 + \tau/2 \leq T_2$, i.e. the signal (2) with any λ_0 is always within the observation interval.

3 Quasi-Optimal Estimate of the Dispersion

To estimate the dispersion D_0, there can be used the measurer considered in [4]. In this case we get the following estimate

$$\tilde{D}_m = \max[0, M(\lambda_m)/\tau - E_N]. \tag{5}$$

Here $E_N = N_0\Omega_0/2\pi$ is the average power of the noise $n(t)$ within bandwidth of the process $\xi(t)$, λ_m is the estimate of the time of arrival λ_0 of the radio pulse (2):

$$\lambda_m = \arg\sup M(\lambda), \quad \lambda \in [\Lambda_1, \Lambda_2], \tag{6}$$

and the function $M(\lambda)$ is defined by the expression

$$M(\lambda) = \int_{\lambda-\tau/2}^{\lambda+\tau/2} y_0^2(t)\, dt, \tag{7}$$

where $y_0(t) = \int_{-\infty}^{\infty} x(t') h_0(t - t')\, dt'$ is the output signal of the filter, its transfer function $H_0(\omega)$ satisfying the condition

$$|H_0(\omega)|^2 = I[(\vartheta - \omega)/\Omega_0] + I[(\vartheta + \omega)/\Omega_0].$$

Let us determine the characteristics of the estimate (5). Introducing the dimensionless parameter $l = \lambda/\tau$, we present the functional $M(\lambda)$ in the form of the sum [8] of the signal $S(l)$ and noise $N(l)$ functions

$$M(l) = S(l) + N(l). \tag{8}$$

Here $S(l)=\langle M(l)\rangle$, $N(l)=M(l)-\langle M(l)\rangle$, and averaging is carried out over all the realizations of the observable data $x(t)$ (1), the unknown parameters λ_0, γ_0 and D_0 having the fixed values. According to Eq. (7), the signal function has a form:

$$S(l)=A\max\left(0,1-|l-l_0|\right)+C, \tag{9}$$

where $A=\tau D_0$, $l_0=\lambda_0/\tau$, $C=\tau\left(E_N+E_\gamma\right)$, and $E_\gamma=\gamma_0\Omega_0/2\pi$ is the average power of the external interference $\nu(t)$ within bandwidth of the process $\xi(t)$.

As follows from Eqs. (7) and (8), $\langle N(l)\rangle=0$ and under $|l_i-l_0|\le 1$, $i=1,2$ we get

$$\langle N(l_1)N(l_2)\rangle=\sigma_S^2\begin{cases}1-|l_1-l_2|-g\min\left(|l_1-l_0|,|l_2-l_0|\right), & (l_1-l_0)(l_2-l_0)\ge 0,\\ 1-|l_1-l_2|, & (l_1-l_0)(l_2-l_0)<0,\end{cases} \tag{10}$$

$$\sigma_S^2=\frac{\left[\tau E_N\left(1+q_\nu+q_0\right)\right]^2}{\mu},\quad g=\frac{q_0\left(2+q_\nu+q_0\right)}{\left(1+q_\nu+q_0\right)^2},\quad q_\nu=\frac{E_\nu}{E_N},\quad q_0=\frac{D_0}{E_N}.$$

If $\max\left(|l_1-l_0|,|l_2-l_0|\right)>1$, then

$$\langle N(l_1)N(l_2)\rangle=\sigma_N^2\max\left(0,1-|l_1-l_2|\right),\quad \sigma_N^2=\left[\tau E_N\left(1+q_\nu\right)\right]^2/\mu. \tag{11}$$

Using Eqs. (9)-(11) and referring to the results in the researches [8, 9], it is possible to obtain the approximate expression for the distribution function $F_U(x)$ of the random variable $U=\left[M(\lambda_m)-C\right]/\sigma_S$

$$F_U(x)=F_S(x)F_N(x\sigma_S/\sigma_N)=F_S(x)F_N\left[x(1+q_\nu+q_0)/(1+q_\nu)\right], \tag{12}$$

$$F_S(x)=\Phi(x-z)-2\exp\left[\psi^2z^2/2+\psi z(z-x)\right]\Phi\left[x-z(\psi+1)\right]+$$
$$+\ \exp\left[2\psi^2z^2+2\psi z(z-x)\right]\Phi\left[x-z(2\psi+1)\right],$$

$$F_N(x)=\begin{cases}\exp\left[-\left(mx/\sqrt{2\pi}\right)\exp\left(-x^2/2\right)\right], & x\ge 1,\\ 0, & x<1,\end{cases}$$

where

$$z^2=\frac{\mu q_0^2}{\left(1+q_\nu+q_0\right)^2},\quad \psi=\frac{2}{2-g}=\frac{2\left(1+q_\nu+q_0\right)^2}{\left(1+q_\nu\right)^2+\left(1+q_\nu+q_0\right)^2},\quad m=\frac{\Lambda_2-\Lambda_1}{\tau},$$

and $\Phi(x)=\int_{-\infty}^{x}\exp\left(-t^2/2\right)dt/\sqrt{2\pi}$ is the probability integral.

According to [8, 9], the formulas (12) are valid while the conditions $\mu>>1$ (4), $z>>1$, $m>>1$ are satisfied, and their accuracy increases with μ, z, m. If $m\lesssim 1$ (order of unit or less) and $\mu>>1$ ($z>>1$), then

$$F_U(x)\approx F_S(x). \tag{13}$$

The distribution function $\tilde{F}_m(x|D_0)=P\left[\tilde{D}_m<x\right]$ of the estimate $\tilde{D}_m$ is linked to the function $F_U(x)$ by following relations

$$\tilde{F}_m(x) = F_U\left[z\left(x/D_0 - q_\nu/q_0\right)\right], \tag{14}$$

if $x \geq 0$, and $\tilde{F}_m(x) = 0$, if $x < 0$.

Taking into account Eq. (14), we get the expressions for bias (systematic error) $b\left(\tilde{D}_m \middle| D_0\right) = \left\langle \tilde{D}_m - D_0 \right\rangle$ and variance (error mean square) $V\left(\tilde{D}_m \middle| D_0\right) = \left\langle \left(\tilde{D}_m - D_0\right)^2 \right\rangle$ of the estimate $\tilde{D}_m$ (5)

$$b\left(\tilde{D}_m \middle| D_0\right) = \int_0^\infty \left[1 - \tilde{F}_m\left(x \middle| D_0\right)\right] dx - D_0, \quad V\left(\tilde{D}_m \middle| D_0\right) = 2\int_0^\infty (x - D_0)\left[1 - \tilde{F}_m\left(x \middle| D_0\right)\right] dx + D_0^2 . \tag{15}$$

We can analytically integrate Eq. (15) in case when $m \tilde{<} 1$ only. Using approximation (13) for the function $F_U(x)$, for the characteristics of the estimate $\tilde{D}_m$ (5) we obtain

$$\begin{aligned} b\left(\tilde{D}_m \middle| D_0\right) = D_0 \Bigg\{ & \left(1 + \frac{q_\nu}{q_0} + \frac{3}{2\psi z^2}\right) \Phi\left[z\left(1 + \frac{q_\nu}{q_0}\right)\right] + \frac{1}{z\sqrt{2\pi}} \exp\left[-\frac{z^2}{2}\left(1 + \frac{q_\nu}{q_0}\right)^2\right] + \\ & + \frac{2}{\psi z^2} \exp\left[\psi z^2 \left(\frac{\psi}{2} + \frac{q_\nu}{q_0} + 1\right)\right]\left[1 - \Phi\left(z\left(\psi + 1 + \frac{q_\nu}{q_0}\right)\right)\right] - \\ & - \frac{1}{2\psi z^2} \exp\left[2\psi z^2 \left(\psi + 1 + \frac{q_\nu}{q_0}\right)\right]\left[1 - \Phi\left(z\left(2\psi + 1 + \frac{q_\nu}{q_0}\right)\right)\right] - 1 \Bigg\}, \end{aligned} \tag{16}$$

$$\begin{aligned} V\left(\tilde{D}_m \middle| D_0\right) = D_0^2 \Bigg\{ & 1 - \left(1 - \frac{q_\nu^2}{q_0^2} - \frac{1}{z^2} - \frac{3q_\nu}{q_0 \psi z^2} - \frac{7}{2\psi^2 z^4}\right) \Phi\left[z\left(1 + \frac{q_\nu}{q_0}\right)\right] - \\ & - \frac{1}{z\sqrt{2\pi}}\left(1 - \frac{q_\nu}{q_0} - \frac{3}{\psi z^2}\right) \exp\left[-\frac{z^2}{2}\left(1 + \frac{q_\nu}{q_0}\right)^2\right] - \\ & - \left(1 - \frac{1}{\psi z^2}\right)\frac{4}{\psi z^2} \exp\left[\psi z^2\left(\frac{\psi}{2} + 1 + \frac{q_\nu}{q_0}\right)\right]\left[1 - \Phi\left(z\left(\psi + 1 + \frac{q_\nu}{q_0}\right)\right)\right] + \\ & + \left(1 - \frac{1}{2\psi z^2}\right)\frac{1}{\psi z^2} \exp\left[2\psi z^2\left(\psi + 1 + \frac{q_\nu}{q_0}\right)\right]\left[1 - \Phi\left(z\left(2\psi + 1 + \frac{q_\nu}{q_0}\right)\right)\right] \Bigg\} . \end{aligned}$$

The accuracy of the formulas (12), (14), (15) increases with μ, *z*, *m*, as well as the accuracy of the formulas (16) increases with μ and *z*. Formulas (15), (16) become considerably simpler in case of large μ (*z*) values when the probability of the anomal error

$$P_a = P\left[\left|\lambda_m - \lambda_0\right| > \tau\right] \tag{17}$$

can be neglected while estimating the time of arrival. Then we get

$$b\left(\tilde{D}_m \middle| D_0\right) \approx E_\gamma + 3D_0 / 2\psi z^2 \approx E_\gamma,\ V\left(\tilde{D}_m \middle| D_0\right) \approx E_\gamma^2 + D_0^2 / z^2 . \qquad (18)$$

Assuming that $q_\nu = 0$ in Eqs. (12)-(16), (18), we procced to the earlier found (in [4]) expressions for the distribution function and the characteristics of the estimate of the dispersion of the random radio pulse (2) in absence of the external interference $\nu(t)$.

The loss in the measurement accuracy produced by the presence of the external interference can be quantitatively characterized by the relation $\tilde{\rho} = V\left(\tilde{D}_m \middle| D_0\right) / V\left(D_{m0} \middle| D_0\right)$. Here $V\left(D_{m0} \middle| D_0\right) = V\left(\tilde{D}_m \middle| D_0\right)\Big|_{q_\nu = 0}$ is the variance of the estimate (5) in absence of the external interference [4]. In Fig. 1 for $m = 20$ the dependences $\tilde{\rho}(q_\nu)$ calculated by means of the formulas (12) and (15) are drawn by dashed lines under $\mu = 100$, and by solid lines – under $\mu = 200$. Curves 1 correspond to $q_0 = 0.25$, curves 2 – to $q_0 = 0.5$ and curves 3 – to $q_0 = 1$. As follows from Fig. 1, the loss in the estimate (5) accuracy increases with hindrance-to-noise ratio q_ν, and then it can reach considerable values. At the same time, the influence of the external interference upon the accuracy of the estimate $\tilde{D}_m$ (5) increases with the increase of μ and the decrease of q_0.

4 Maximum Likelihood and Quasi-Likelihood Estimates of the Dispersion

Accuracy of the estimate of the dispersion D_0 of the random radio pulse (2) can be improved, if, while synthesizing the estimation algorithm in accordance with the maximum likelihood method, we take into account the presence of the external interference $\nu(t)$. For this purpose we designate the logarithm of the functional of likelihood ratio for the hypothesis $x(t) = s(t, \lambda_0, D_0) + \nu(t) + n(t)$ against alternative $x(t) = n(t)$ as $L(\lambda, D, \gamma)$. If condition (4) holds, then, according to [10, 11], we get

$$L(\lambda, D, \gamma) = \tau \left\{ \frac{1}{\mu(N_0 + \gamma)} \left[\frac{d}{N_0 + \gamma + d} \int_{\lambda - \tau/2}^{\lambda + \tau/2} y_0^2(t)\, dt + \right.\right.$$
$$\left.\left. + \frac{\gamma}{N_0} \int_{T_1}^{T_2} y_1^2(t)\, dt \right] - \ln\left(1 + \frac{\gamma + d}{N_0}\right) - (K - 1)\ln\left(1 + \frac{\gamma}{N_0}\right) \right\}, \qquad (19)$$

where $y_0(t)$ is defined from Eq. (7), $y_1(t) = \int_{-\infty}^{\infty} x(t') h_1(t - t')\, dt'$ is the output signal of the filter with the transfer function $H_1(\omega)$, satisfying the condition

$\left|H_1(\omega)\right|^2 = I[(\vartheta-\omega)/\Omega_1] + I[(\vartheta+\omega)/\Omega_1]$, $d = 2\pi D/\Omega_0$ and $K = (T_2 - T_1)\Omega_1/\tau\Omega_0$. It should be noted that the relation

$$K > m \tag{20}$$

is always satisfied, as $T_1 < \Lambda_1$, $T_2 > \Lambda_2$ and $\Omega_1 \geq \Omega_0$.

If the magnitude γ_0 of the interference $\nu(t)$ SD (3) is a priori known, then the maximum likelihood estimate (MLE) D_m of the dispersion D_0 presents itself as

$$D_m = \arg\sup_{D \geq 0} L(\lambda_m, D, \gamma_0). \tag{21}$$

Here λ_m is MLE of the time of arrival λ_0 of the radio pulse (2):

$$\lambda_m = \arg\sup_{\lambda \in [\Lambda_1, \Lambda_2]} L(\lambda, D_m, \gamma_0). \tag{22}$$

As follows from Eq. (19), MLE (22) agrees with the estimate (6).

Substituting Eq. (19) in Eq. (21) we find

$$D_m = \max\left[0, M(\lambda_m)/\tau - E_N - E_\gamma\right], \tag{23}$$

where the function $M(\lambda)$ is defined from Eq. (7).

For the start, we assume that the magnitude γ_0 of SD (3) is a priori unknown, but it is possible to specify its some approximate expected (predicted) value γ^*. Besides, we presuppose that, in a general case, SD N_0 of the white noise $n(t)$ is also known inexactly, i.e. while measurer is being synthesized, a particular expected value N^* is used, instead of the true value N_0, and $N^* \neq N_0$. Then, in Eq. (23) we substitute the expected values γ^* and N^* for the unknown parameters γ_0 and N_0 and obtain the estimate

$$D_m^* = \max\left[0, M(\lambda_m)/\tau - E_N^* - E_\gamma^*\right], \tag{24}$$

where $E_N^* = N^*\Omega_0/2\pi$, $E_\gamma^* = \gamma^*\Omega_0/2\pi$. We name Eq. (24) as a quasi-likelihood estimate (QLE), unlike MLE (23). Indeed, if $\gamma^* = \gamma_0$ and $N^* = N_0$, then QLE (24) transforms into MLE (23).

Algorithm of QLE D_m^* (24) of the dispersion D_0 of the random radio pulse (2) can be implemented by means of the measurer presented in [4, Fig. 3]. For this purpose it is necessary to submit the expected average power $E_N^* + E_\gamma^*$ of the total interference $n(t) + \nu(t)$ to the measurer‘s subtractor, instead of the average power E_N of the noise $n(t)$.

Let us consider how a deviation of expected values γ^* and N^* from true values γ_0 and N_0 of the interference and noise SDs influences on characteristics of QLE (24). Similarly to Eqs. (12)-(14), it can be shown that the distribution function $F_m^*(x|D_0)$ of the estimate D_m^* (24) is linked to the function $F_U(x)$ by the expression

$$F_m^*(x|D_0) = F_U\{z[x/D_0 + (1+q_\nu)\delta_E/q_0]\}, \quad x \ge 0. \tag{25}$$

Here $\delta_E = (E_N^* + E_\gamma^* - E_N - E_\gamma)/(E_N + E_\gamma)$, and $F_U(x)$ is defined from Eq. (12), if $m >> 1$, or from Eq. (13) if $m \tilde{<} 1$. Accordingly, the expressions for bias $b(D_m^*|D_0)$ and variance $V(D_m^*|D_0)$ of the estimate D_m^* (24) can be found from Eq. (15) changing $\tilde{F}_m(x|D_0)$ for $F_m^*(x|D_0)$. While carrying out integration in Eq. (15) in case of $m \tilde{<} 1$, we obtain

$$\begin{aligned}
b(D_m^*|D_0) = D_0 \Bigg\{ & \left(1 - \frac{1+q_\nu}{q_0}\delta_E + \frac{3}{2\psi z^2}\right)\Phi\left[z\left(1 - \frac{1+q_\nu}{q_0}\delta_E\right)\right] - 1 + \\
& + \frac{1}{z\sqrt{2\pi}}\exp\left[-\frac{z^2}{2}\left(1 - \frac{1+q_\nu}{q_0}\delta_E\right)^2\right] + \frac{2}{\psi z^2}\exp\left[\psi z^2\left(\frac{\psi}{2} - \frac{1+q_\nu}{q_0}\delta_E + 1\right)\right] \times \\
& \times \left[1 - \Phi\left(z\left(\psi + 1 - \frac{1+q_\nu}{q_0}\delta_E\right)\right)\right] - \frac{1}{2\psi z^2}\exp\left[2\psi z^2\left(\psi + 1 - \frac{1+q_\nu}{q_0}\delta_E\right)\right] \times \\
& \times \left[1 - \Phi\left(z\left(\psi + 1 - \frac{1+q_\nu}{q_0}\delta_E\right)\right)\right]\Bigg\},
\end{aligned} \tag{26}$$

$$\begin{aligned}
V(D_m^*|D_0) = D_0^2 \Bigg\{ & 1 - \left(1 - \frac{(1+q_\nu)^2}{q_0^2}\delta_E^2 - \frac{1}{z^2} + \frac{3(1+q_\nu)}{q_0 \psi z^2}\delta_E - \frac{7}{2\psi^2 z^4}\right) \times \\
& \times \Phi\left[z\left(1 - \frac{1+q_\nu}{q_0}\delta_E\right)\right] - \frac{1}{z\sqrt{2\pi}}\left(1 + \frac{1+q_\nu}{q_0}\delta_E - \frac{3}{\psi z^2}\right)\exp\left[-\frac{z^2}{2} \times \right. \\
& \left. \times \left(1 - \frac{1+q_\nu}{q_0}\delta_E\right)^2\right] - \left(1 - \frac{1}{\psi z^2}\right)\frac{4}{\psi z^4}\exp\left[\psi z^2\left(\frac{\psi}{2} - \frac{1+q_\nu}{q_0}\delta_E + 1\right)\right] \times \\
& \times \left[1 - \Phi\left(z\left(\psi + 1 - \frac{1+q_\nu}{q_0}\delta_E\right)\right)\right] + \left(1 - \frac{1}{2\psi z^2}\right)\frac{1}{\psi z^2} \times \\
& \times \exp\left[2\psi z^2\left(\psi + 1 - \frac{1+q_\nu}{q_0}\delta_E\right)\right]\left[1 - \Phi\left(z\left(2\psi + 1 - \frac{1+q_\nu}{q_0}\delta_E\right)\right)\right]\Bigg\}.
\end{aligned}$$

If the probability of the anomal error (17) during the estimation of time of arrival of the random pulse (2) can be neglected, i.e. when conditions $\mu >> 1$ (4), $z >> 1$ hold, then from Eq. (26) we find

$$b\left(D_m^*\middle|D_0\right)\approx -E_N\delta_E\left(1+q_\nu\right),\quad V\left(D_m^*\middle|D_0\right)\approx E_N^2\left[\left(1+q_\nu+q_0\right)^2/\mu+\delta_E^2\left(1+q_\nu\right)^2\right]. \quad (27)$$

In case of $\gamma^*=\gamma_0$, $N^*=N_0$ ($\delta_E=0$) the QLE (24) characteristics go over corresponding MLE (23) characteristics.

Formulas (14), (25) allow us to find a gain in accuracy of the estimate of the dispersion of the random radio pulse (2), when the $\nu(t)$ interference effect with a priori known γ_0 and N_0 is taken into consideration. For this purpose we introduce the relation $\rho=V\left(\tilde{D}_m\middle|D_0\right)/V\left(D_m\middle|D_0\right)$ of the estimate (5) variance $V\left(\tilde{D}_m\middle|D_0\right)$ to the estimate (23) variance $V\left(D_m\middle|D_0\right)$. In Fig. 2, in case of $m=20$, the dependences $\rho(q_\nu)$ calculated by means of the formulas (12), (15), (25) are drawn by dashed lines, if $\mu=100$, and by solid lines, if $\mu=200$. Curves 1 correspond to $q_0=0.25$, curves 2 – to $q_0=0.5$, curves 3 – to $q_0=1$. According to Fig. 2, the measurer (23) provides a considerable gain in accuracy of the estimate of the dispersion of process $\xi(t)$ in comparison with the measurer (5), especially under large values μ and q_ν.

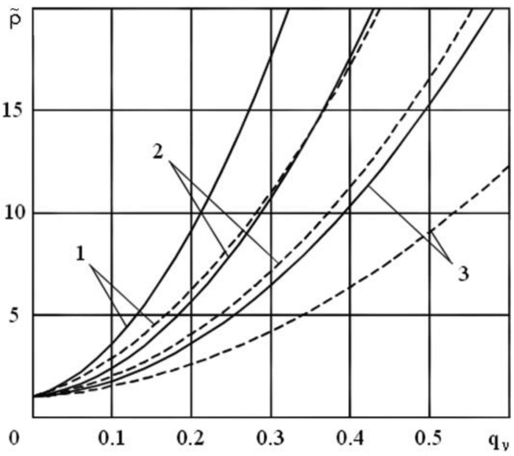

Fig. 1. Loss in the accuracy of the estimate of the random radio pulse dispersion in case of the external interference availability

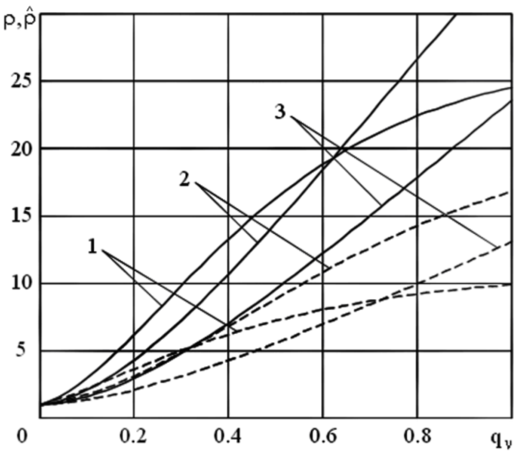

Fig. 2. Gain in accuracy of the estimate of the dispersion of the random radio pulse, with the external interference effect taken into consideration

However, the realization of this gain is not always possible, as the intensities γ_0 and N_0 of the interference $\nu(t)$ and noise $n(t)$ can be a priori unknown, or known inexactly. We characterize the effect of deviations of the expected values γ^* and N^* from their true values γ_0 and N_0 upon QLE (24) accuracy by the relation $\rho^*=V\left(D_m^*\middle|D_0\right)/V\left(D_m\middle|D_0\right)$. For $m=20$ and $\mu=200$, the dependences $\rho^*(\delta_E)$ are plotted in Fig. 3. The curve 1 is calculated for values $q_0 = 0.25$, $q_\nu = 1$, the curve 2 – for $q_0 = 0.25$, $q_\nu = 0$, the curve 3 – for $q_0 = 1$, $q_\nu = 1$ and the curve 4 – for $q_0 = 1$, $q_\nu = 0$. The analysis of the curves in Fig. 3 shows that the ignorance of the intensities of the external interference and the white noise can lead to the considerable loss in the accuracy of the dispersion estimate (24). Fulfillment of the condition $q_\nu = 0$ means that the external interference is absent. Therefore, in this case QLE (24) coincides with the similar estimate considered in [4] and synthesized in the assumption that the SD N_0 of the white noise $n(t)$ is known inexactly. Accordingly, under $q_\nu = 0$, the loss in accuracy of the estimate of the dispersion D_0 of the random radio pulse (2) is caused only by the deviation of the expected value N^* from N_0. We can see from curves 2 and 4 that QLE (24) accuracy decreases considerably, even if the relative deviation of N^* from the true SD value N_0 of the noise $n(t)$ is not too large.

5 Adaptive Estimate of the Dispersion

We can reduce the loss in accuracy of the estimate of the dispersion D_0 of the random radio pulse (2) due to the interference $\nu(t)$ ignorance by realizing the unknown parameter γ_0 adaptation. In this case MLE $\hat{D}_m$ of the dispersion D_0 is written down in the form of

$$\hat{D}_m = \arg\sup_{D \geq 0} L(\lambda_m, D), \tag{28}$$

where

$$L(\lambda, D) = \sup_{\gamma \geq 0} L(\lambda, D, \gamma), \tag{29}$$

and $\lambda_m = \sup_{\lambda \in [\Lambda_1, \Lambda_2]} L(\lambda, \hat{D}_m)$, and it coincides with the estimate (6).

Substituting Eq. (19) into Eqs. (28), (29) we find

$$\hat{D}_m = \max\left[0, \hat{M}(\lambda_m)/\tau\right], \quad \hat{M}(\lambda) = \frac{1}{K-1}\left[KM(\lambda) - \int_{T_1}^{T_2} y_1^2(t)\, dt\right]. \tag{30}$$

From Eq. (30) follows that the structure of the synthesized estimate algorithm of the dispersion is independent from the external interference γ_0 and the white noise N_0 SDs.

The estimate (30) can be obtained by the measurer, its block diagram is shown in Fig. 4. Here the designations are: 1 is the filter with transfer function $H_0(\omega)\sqrt{K/\tau(K-1)}$, 2 is the squarer, 3 is the integrator, 4 is the delay line for the time τ, 5 is the switch that is open for time $[\Lambda_1 + \tau/2, \Lambda_2 + \tau/2]$, 6 is the peak detector, 7 is the switch that is open for time $[T_1, T_2]$, 8 is the filter with transfer function $H_1(\omega)\sqrt{\tau(K-1)}$, 9 is the nonlinear element with the characteristic $f(x) = \max(0, x)$, 10 is the gate circuit forming the signal sample at the moment of time T_2. The sample magnitude at the gate circuit 10 output is the estimate $\hat{D}_m$ (30).

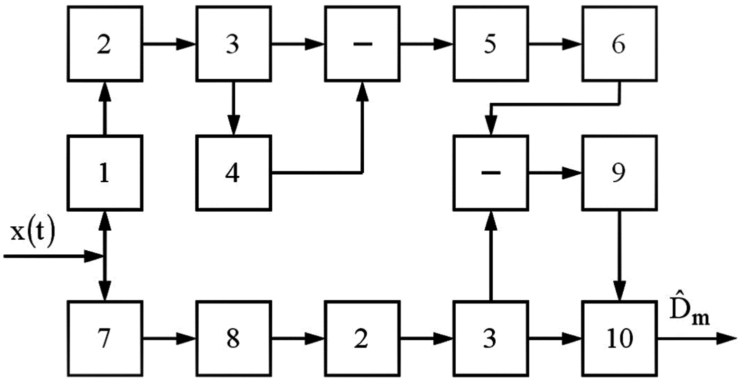

Fig. 4. Block diagram of the adaptive measurer of the random radio pulse dispersion against interferences with unknown intensities

Let us determine the MLE $\hat{D}_m$ (30) characteristics. For this purpose, similarly to Eq. (8) we present the functional $\hat{M}(\lambda)$ as the sum of the signal $\hat{S}(l)$ and noise $\hat{N}(l)$ functions

$$\hat{M}(l) = \hat{S}(l) + \hat{N}(l). \tag{31}$$

Here $\hat{S}(l) = \langle \hat{M}(l) \rangle$, $\hat{N}(l) = \hat{M}(l) - \langle \hat{M}(l) \rangle$, and averaging is carried out over all realizations of the observable data $x(t)$ (1) under the fixed values of unknown parameters λ_0, γ_0 and D_0, as before. We now write down the signal function in the form of

$$\hat{S}(l) = A \begin{cases} 1 - K|l - l_0|/(K-1), & |l - l_0| \le 1, \\ -1/(K-1), & |l - l_0| > 1, \end{cases} \tag{32}$$

where A is defined from Eq. (9).

According to Eq. (31), $\langle \hat{N}(l) \rangle = 0$, and under $|l_i - l_0| \le 1$, $i = 1,2$

$$\langle \hat{N}(l_1)\hat{N}(l_2) \rangle = \hat{\sigma}_S^2 \begin{cases} 1 - |l_1 - l_2| - g\min(|l_1 - l_0|, |l_2 - l_0|) - \\ \quad - 2g[1 - |l_0 - (l_1 + l_2)/2|]/K - \hat{A}, & (l_1 - l_0)(l_2 - l_0) \ge 0, \\ 1 - (1 - g/K)|l_1 - l_2| - 2g/K - \hat{A}, & (l_1 - l_0)(l_2 - l_0) < 0, \end{cases} \tag{33}$$

where $\hat{\sigma}_S^2 = K^2\sigma_S^2/(K-1)^2$, $\hat{A} = (1 + q_\nu)^2 / K(1 + q_\nu + q_0)^2 - g/K$, and σ_S^2, g are defined in Eq. (10).

If $\max(|l_1 - l_0|, |l_2 - l_0|) > 1$, then

$$\left\langle \hat{N}(l_1)\hat{N}(l_2) \right\rangle = \hat{\sigma}_N^2 \begin{cases} 1 - |l_1 - l_2| - \hat{B}, & |l_1 - l_2| \le 1, \\ -\hat{B}, & |l_1 - l_2| > 1, \end{cases} \quad (34)$$

$$\hat{\sigma}_N^2 = K^2 \sigma_N^2 / (K-1)^2, \qquad \hat{B} = 1/K - q_0 (2 + 2q_\nu + q_0) / K^2 (1 + q_\nu)^2 .$$

The specified properties of the functional $\hat{M}(l)$ (31) allow us to write down the distribution function $\hat{F}_m(x|D_0)$ of the estimate $\hat{D}_m$ (30) under $m \tilde{<} 1$, similarly to Eqs. (13), (14), as

$$\hat{F}_m(x|D_0) = \hat{F}_S(zx/D_0 H), \qquad x \ge 0, \quad (35)$$

where

$$\hat{F}_S(x) = \Phi(x - \hat{z}) - 2\exp\left[\hat{\psi}^2 z^2 / 2 + \hat{\psi} z (\hat{z} - x)\right] \Phi(x - \hat{\psi} z - \hat{z}) + \\ + \exp\left[2\hat{\psi}^2 z^2 + 2\hat{\psi} z (\hat{z} - x)\right] \Phi(x - 2\hat{\psi} z - \hat{z}),$$

$$\hat{\psi} = (K-1)^2 H \psi / K^2, \qquad \hat{z} = z/\sqrt{H}, \qquad H = 1 + (1 + q_\nu)^2 / (K-1)(1 + q_\nu + q_0)^2 .$$

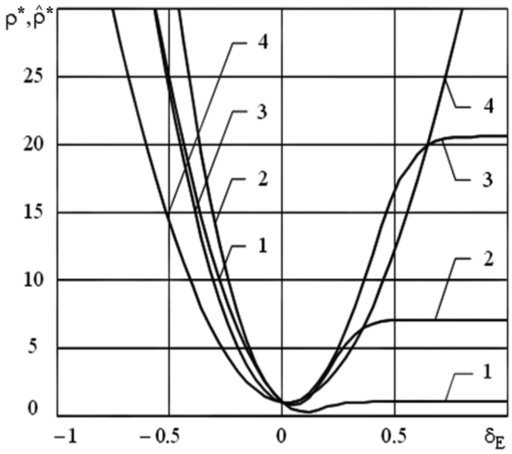

Fig. 3. Loss in accuracy of the estimate of the casual radio pulse dispersion, if the external interference and the white noise intensities are known inexactly

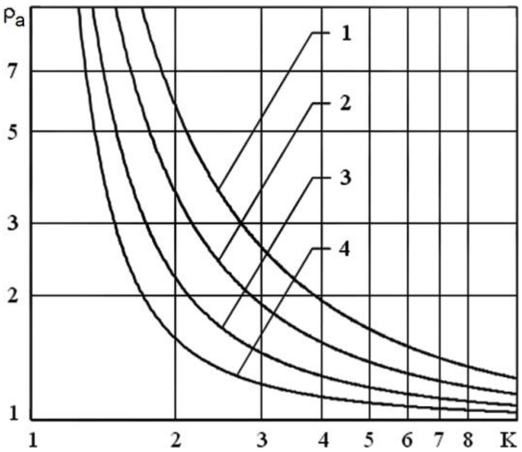

Fig. 5. Loss in the accuracy of the adaptive estimate of the casual radio pulse dispersion due to the ignorance of the external interference and the white noise intensities

In Eq. (15) we substitute the approximation $\hat{F}_m(x|D_0)$ (35) for $\tilde{F}_m(x|D_0)$, and, upon completing integrating, we now obtain the bias $b(\hat{D}_m|D_0)$ and variance $V(\hat{D}_m|D_0)$ of MLE $\hat{D}_m$ (30) for the case when $m \tilde{<} 1$:

$$b\left(\hat{D}_m\middle|D_0\right)=D_0\left\{\left(1+\frac{3}{2\hat{\psi}\hat{z}z}\right)\Phi(\hat{z})+\frac{1}{\hat{z}\sqrt{2\pi}}\exp\left(-\frac{\hat{z}^2}{2}\right)+\frac{2}{\hat{\psi}\hat{z}z}\exp\left(\frac{\hat{\psi}^2z^2}{2}+\hat{\psi}\hat{z}z\right)\times\right.$$
$$\left.\times\left[1-\Phi(\hat{\psi}z+\hat{z})\right]-\frac{1}{2\hat{\psi}\hat{z}z}\exp\left[2\hat{\psi}z(\hat{\psi}z+\hat{z})\right]\left[1-\Phi(2\hat{\psi}z+\hat{z})\right]-1\right\},$$

(36)

$$V\left(\hat{D}_m\middle|D_0\right)=D_0^2\left\{1-\left(1-\frac{1}{\hat{z}^2}-\frac{7}{2\hat{\psi}^2\hat{z}^2z^2}\right)\Phi(\hat{z})-\left(1-\frac{3}{\hat{\psi}\hat{z}z}\right)\frac{1}{\hat{z}\sqrt{2\pi}}\times\right.$$
$$\times\exp\left(-\frac{\hat{z}^2}{2}\right)-\left(1-\frac{1}{\hat{\psi}\hat{z}z}\right)\frac{4}{\hat{\psi}\hat{z}z}\exp\left(\frac{\hat{\psi}^2z^2}{2}+\hat{\psi}\hat{z}z\right)\left[1-\Phi(\hat{\psi}z+\hat{z})\right]+$$
$$\left.+\left(1-\frac{1}{2\hat{\psi}\hat{z}z}\right)\frac{1}{\hat{\psi}\hat{z}z}\exp\left[2\hat{\psi}z(\hat{\psi}z+\hat{z})\right]\left[1-\Phi(2\hat{\psi}z+\hat{z})\right]\right\}.$$

For sufficiently large values μ and z more simple approximations of bias and variance of the adaptive MLE (30) instead of (36) can be used:

$$b\left(\hat{D}_m\middle|D_0\right)\approx 0,\quad V\left(\hat{D}_m\middle|D_0\right)\approx E_N^2\left[(1+q_\nu+q_0)^2+(1+q_\nu)^2/(K-1)\right]/\mu. \quad (38)$$

If $m>>1$, then, according to Eq. (20), it is necessary to set $K>>1$ in Eqs. (32)-(34). Thus, the characteristics of the functional $\hat{M}(\lambda)$ (30) coincide with the characteristics of the centered functional $[M(\lambda)-C]$ (7). As a result, under $m>>1$, the characteristic of MLE $\hat{D}_m$ (30) with unknown magnitude γ_0 of the external interference SD (3) coincide with the corresponding characteristics of MLE (23) under a priori known magnitude γ_0 of SD. Therefore, curves in Fig. 2 also show the gain $\hat{\rho}=V\left(\tilde{D}_m\middle|D_0\right)/V\left(\hat{D}_m\middle|D_0\right)$ in the accuracy of MLE (30) in comparison with the accuracy of the estimate (5) synthesized in study [4]. Accordingly, curves in Fig. 3 show the gain $\hat{\rho}^*=V\left(D_m^*\middle|D_0\right)/V\left(\hat{D}_m\middle|D_0\right)$ in the accuracy of MLE (30) in comparison with the accuracy of QLE (24). According to Figs. 2, 3, the gain in accuracy of the estimate of the dispersion, provided by the adaptive measurer (30) shown in Fig. 4, may be considerable.

If $m \lesssim 1$, then the value K may be small. Therefore, the accuracy of the adaptive estimate $\hat{D}_m$ (30) will be lower than the accuracy of MLE D_m (23), in general. We characterize the loss in the accuracy of MLE (30) in comparison with MLE (23) by the relation $\rho_a=V\left(\hat{D}_m\middle|D_0\right)/V\left(D_m\middle|D_0\right)$, where $V\left(D_m\middle|D_0\right)$ and $V\left(\hat{D}_m\middle|D_0\right)$ are defined from Eqs. (26) (under $\delta_E=0$) and (36), accordingly. Dependences $\rho_a=\rho_a(K)$ are presented in Fig. 5. It is assumed that $\mu=200$. The curve 1 is calculated for values $q_0=0.25$, $q_\nu=1$, the curve 2 – for $q_0=0.25$, $q_\nu=0$, the curve 3 – for $q_0=1$, $q_\nu=1$, the curve 4 – for $q_0=1$,

$q_\nu = 0$. As it can be seen from Fig. 5, the loss in the accuracy of the adaptive estimate (30) in comparison with the estimate obtained in (23) increases monotonously with the increasing q_ν and the decreasing *K* and q_0, and it reaches substantial values under small *K*. Curves 2 and 4 calculated for $q_\nu = 0$ describe only the influence of the white noise adaptation carried out in the absence of the external interference upon the accuracy of the estimate of the dispersion. The analysis of these curves shows that even while receiving the random radio pulse against intrinsic noise with unknown intensity, the loss in the accuracy of the adaptive estimate $\hat{D}_m$ (30) in relation to MLE D_m (23) may be considerable in case of small *K*. However, under $K > 8 \div 10$ the characteristics of MLE $\hat{D}_m$ (30) and MLE D_m (23) practically coincide, so that there is no loss in the accuracy of the estimation of the dispersion due to the ignorance of the intensities of the interference and the white noise. From here specifically follows that the adaptive dispersion measurer (30) should be applied even in the absence of the external interference even, if the white noise SD is a priori unknown and there can be made sufficiently large *K*.

6 Results of Statistical Simulation

In order to test the serviceability of the considered measurers and to establish the borders of applicability for the found asymptotically exact formulas for the estimation characteristics, we are to demonstrate the statistical computer simulation of the algorithms (5), (23), (24), (30), using a procedure presented in [12]. During simulation within the interval $\left[\tilde{\Lambda}_1, \tilde{\Lambda}_2\right]$, $\tilde{\Lambda}_{1,2} = \Lambda_{1,2}/\tau$, with step $\Delta l = 10^{-2}$, the samples of realizations of the normalized functional $\tilde{M}(l) = M(\lambda)/N_0$ (7) are generated and the value of the normalized random variable $\tilde{M}_T = \int_{T_1}^{T_2} y_1^2(t)\,dt / N_0(K-1)$ (30) is also formed. For each realization of $x(t)$ (1), the position $l_m = \arg\sup_{l \in [\tilde{\Lambda}_1, \tilde{\Lambda}_2]} \tilde{M}(l)$ of the functional $\tilde{M}(l)$ absolute maximum is determined and, according to Eqs. (5), (23), (24), (30), the estimates $\tilde{D}_m$, D_m, D_m^*, $\hat{D}_m$ are defined, and their variances are calculated.

In Figs. 6, 7 the theoretical dependences (12), (14), (15) and (12), (15), (25) (under $\delta_E = 0$) are drawn for the normalized variances $\tilde{V}_q = V\left(\tilde{D}_m \middle| D_0\right)/E_N^2$, $V_q = V\left(D_m \middle| D_0\right)/E_N^2$ of the estimates $\tilde{D}_m$ (5), D_m (23). The solid lines are calculated for $q_\nu = 0.5$, and the dashed lines – for $q_\nu = 0.25$. Curves 1 correspond to $\mu = 50$, curves 2 – to $\mu = 100$, curves 3 – to $\mu = 200$. Experimental values of the variances $\tilde{V}_q$, V_q for $\mu = 50$, 100 and 200 are designated by squares, crosses, rhombuses under $q_\nu = 0.5$, and by pluses, circlets, triangles under $q_\nu = 0.25$.

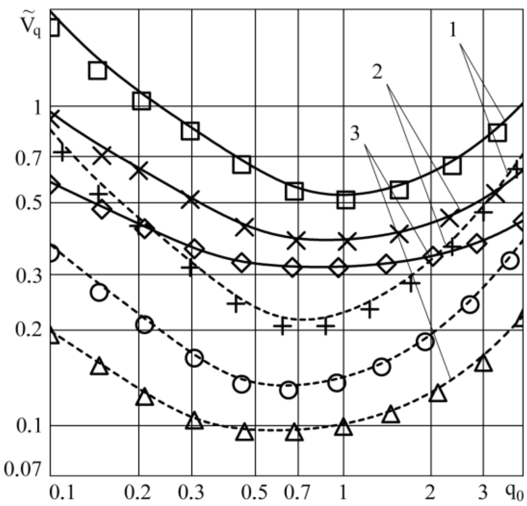

Fig. 6. Theoretical and experimental dependences of the variance of the estimate of the random radio pulse dispersion in case of the ignorance of the external interference effect

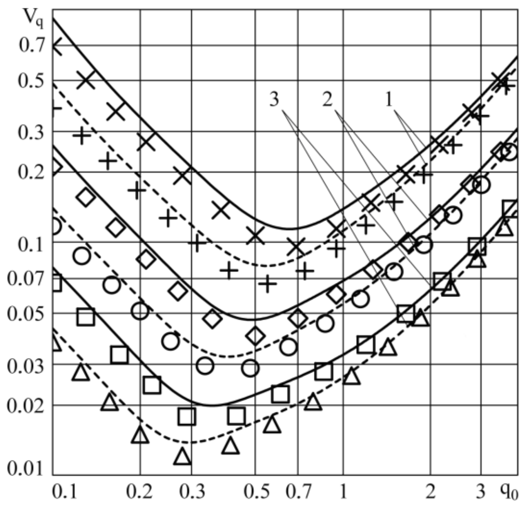

Fig. 7. Theoretical and experimental dependences of the variance of the maximum likelihood estimate of the random radio pulse dispersion

In Fig. 8 the theoretical dependences (12), (15), (25) are plotted for the normalized variance $V_q^* = V\left(D_m^* \middle| D_0\right) \big/ E_N^2$ of the estimate D_m^* (24). The solid lines are calculated for $\delta_E = 0.2$, and the dashed lines – for $\delta_E = -0.2$. Curves 1 correspond to $\mu = 100$, $q_\nu = 0.5$, curves 2 – to $\mu = 200$, $q_\nu = 0.5$, curves 3 – to $\mu = 200$, $q_\nu = 0$. Experimental values of the variance V_q^* are designated by squares, crosses, rhombuses for $\delta_E = -0.2$, and by pluses, circlets, triangles for $\delta_E = 0.2$.

Finally, in Figs. 9a and 9b the theoretical dependences are shown for the normalized variance $\hat{V}_q = V\left(\hat{D}_m \middle| D_0\right) \big/ E_N^2$ of the adaptive MLE $\hat{D}_m$ (30), if $K = 2$ and $K = 21$, accordingly. All curves are calculated by formulas (36) and (12), (15), (25) (under $\delta_E = 0$): curves 1 – for $\mu = 100$, $q_\nu = 1$, curves 2 – for $\mu = 200$, $q_\nu = 1$, curves 3 – for $\mu = 100$, $q_\nu = 0$, curves 4 – for $\mu = 200$, $q_\nu = 0$. Appropriate experimental values of the variance $\hat{V}_q$ are designated by squares, crosses, rhombuses and circlets.

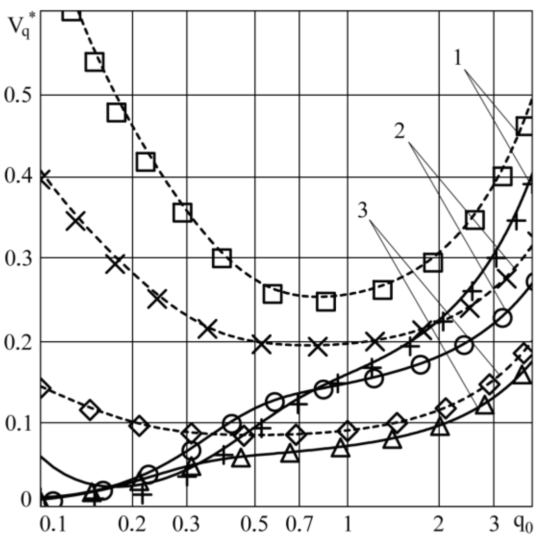

Fig. 8. Theoretical and experimental dependences of the variance of the estimate of the random radio pulse dispersion when the external interference and the white noise intensities are known inexactly

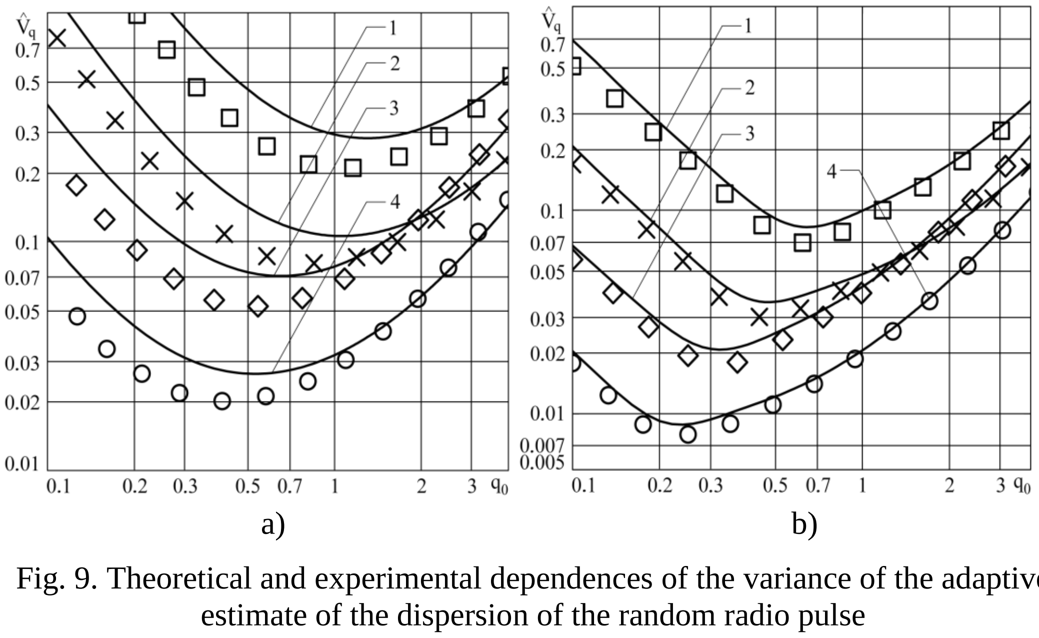

Fig. 9. Theoretical and experimental dependences of the variance of the adaptive estimate of the dispersion of the random radio pulse

The obtained results lead us to the following conclusions. Formulas (12), (14), (15) and (12), (14), (25) for the characteristics of the estimates $\tilde{D}_m$, D_m, D_m^*, $\hat{D}_m$ well approximate the experimental data under $m \geq 20$ and $\mu \geq 50$, $z \geq 0.5...1$. If $m \leq 1$, and the value of parameter K is small, then the asymptotic expressions (36) for the characteristics of the adaptive algorithm (30) are in satisfactory agreements with corresponding experimental dependences under

$\mu \geq 50$, $z \geq 3\ldots4$. If $z > 4...5$, so the probability of the anomal error (17) can be neglected, then the variances $V\left(\tilde{D}_m \middle| D_0\right)$, $V\left(D_m \middle| D_0\right)$, $V\left(D_m^* \middle| D_0\right)$, $V\left(\hat{D}_m \middle| D_0\right)$ of the estimates $\tilde{D}_m$ (5), D_m (23), D_m^* (24), $\hat{D}_m$ (30) calculated with help of Eqs. (12), (14), (15), (25) and Eqs. (16), (26), (36) practically coincide. In case of $z > 6...7$, for the calculation of the characteristics of the dispersion estimate algorithms (5), (23), (24), (30) it is possible to use the formulas (18), (27), (38), instead of Eqs. (16), (26), (36), accordingly, without appreciable loss in accuracy.

7 Conclusion

The quality of the band random pulse dispersion estimation algorithms constructed for the expected (predicted) values of the interference and the white noise SDs particularly depends upon the presence of the prior data concerning the unknown spurious parameters. If the prior information about the intensities of the band interference and the white noise is absent, or is inexact, then the quality of the estimate of the dispersion of the pulse signal may deteriorate considerably. In this case a negative error in determining the average capacity of the total interference is generally less desirable than a corresponding positive error.

Application of the adaptive approach for the removal of the prior uncertainty concerning unknown band interference intensity allows us to obtain the algorithm for the estimate of the dispersion of the band random pulse, which is also independent from the intensity of the white noise. Besides, if the length of the observation interval is much greater than the useful signal duration, or if the interference bandwidth surpasses the interference bandwidth of the pulse random substructure considerably, then the loss in the accuracy of the maximum likelihood estimate of the dispersion due to the ignorance of the interference and the white noise spectral densities is absent asymptotically. Conclusions and recommendations are valid, if the output signal-to-noise ratio is greater than 0.5…1. Thus, with the found expressions we can now make a reasonable choice between estimates (5), (23), (24) and (30), having accounted for the available prior information concerning analyzed process and for the requirements to the estimate accuracy and to the simplicity of the measurer hardware implementation.

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