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STATISTICAL RADIOPHYSICS

Quasi-Optimal Estimation of Distance and Velocity Using Laser Ranging

A. P. Trifonov[a], A. V. Kurbatov[a], O. V. Chernoyarov[b], and B. I. Shakhtarin[c]

[a]*Voronezh State University, Universitetskaya pl. 1, Voronezh, 394007 Russia*
[b]*Moscow Power Engineering Institute (National Research University), Krasnokazarmennaya ul. 14, Moscow, 111250 Russia*
[c]*Bauman State Technical University, Vtoraya Baumanskaya ul. 5, str. 1, Moscow, 105005 Russia*
e-mail: trifonov@phys.vsu.ru

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Abstract—An algorithm for estimation of distance and velocity using laser ranging in each repetition period of the sounding train of optical pulses is synthesized. It is demonstrated that the algorithm allows a simpler technical implementation in comparison with the maximum-likelihood algorithm. The characteristics of estimations are found with allowance for anomalous errors. Accuracy loss of quasi-optimal estimations is determined in comparison with the maximum-likelihood estimations. Methods of statistical simulation are used to demonstrate the efficiency of the synthesized algorithm and the applicability limits are determined for the asymptotic formulas that make it possible to determine the characteristics.

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In several LIDAR problems [1–4], the estimation of the distance to the target, which is easily provided by laser rangers, is supplemented with the estimation of the radial velocity. Optical pulse trains are widely used for determination of the parameters of moving targets. In this regard, it is of interest to study the accuracy of the corresponding methods for the estimation of distance and velocity.

The maximum-likelihood estimations of distance and velocity upon sounding of object with the aid of an optical pulse train are considered in [5, 6] and the potential characteristics of the distance and velocity estimations are determined. However, significant problems are encountered in the technical implementation of the maximum-likelihood algorithms for simultaneous estimation of two parameters of motion. The estimations of distance and velocity can substantially be simplified with the aid of the below quasi-optimal estimation based on the laser ranging for each pulse in the train.

We assume that the intensity of the emitted optical pulse train is given by

$$s_N(t) = \sum_{k=0}^{N-1} \hat{s}(t-(k-\mu)\vartheta-\lambda), \tag{1}$$

where $\hat{s}(t)$ is the function that describes the intensity of a single optical pulse, λ is the time position of the train, and ϑ is the pulse repetition period. Parameter μ determines the point of the series that is related to time position λ. In particular, λ determines the position of the first pulse, middle of the train, and the last pulse when $\mu = 0$, $(N-1)/2$, and $N-1$, respectively.

We assume that the detected (processed) signal results from the scattering of optical pulse train (1) by an object that is located at distance R_0 and moves at radial velocity V_0. In this case, the intensity of the detected signal is represented as [1, 5, 6]

$$s(t, R_0, V_0) = \sum_{k=0}^{N-1} s(t-2R_0/c-(k-\mu)(1+2V_0/c)\vartheta), \tag{2}$$

where c is the velocity of light. In general, intensity profile $s(t)$ of the detected pulses may differ from intensity profile $\hat{s}(t)$ of the sounding pulses (1).

We assume that

$$|V_0| \ll c.$$

Note that interval of observation $[0, T]$ is greater than the duration of the pulse train ($T > N\vartheta$) and the pause-to-pulse ratio is no less than 2, so that the pulses are not overlapped.

Signal with intensity (2) is observed over time interval $[0, T]$ in the presence of optical noise with constant intensity ν. Therefore, we process a sample of Poisson process $\pi(t)$ with the intensity

$$\beta(t, R_0, V_0) = s(t, R_0, V_0) + \nu$$

and possible distances R_0 and velocities V_0 belong to a priori intervals

$$[R_{\min}, R_{\max}], \quad [V_{\min}, V_{\max}].$$

We introduce the following notation:

$$R_{apr} = (R_{min} + R_{max})/2, \quad \Delta R_{apr} = R_{max} - R_{min}, \quad \Delta R_0 = R_{apr} - R_0,$$
$$V_{apr} = (V_{min} + V_{max})/2, \quad \Delta V_{apr} = V_{max} - V_{min}, \quad \Delta V_0 = V_{apr} - V_0.$$

A functional of likelihood ratio (FLR) needs to be formed for the maximum-likelihood estimation [7–10]. Accurate to insignificant terms, such a functional is represented as [11]

$$L(R, V) = \int_0^T \ln(1 + s(t, R, V)/\nu)\, d\pi(t). \tag{3}$$

For the estimation of parameters (R_0, V_0), we may employ quantities

$$(\hat{R}, \hat{V}) = \underset{(R, V) \in \mathbf{W}}{\arg\sup} L(R, V), \tag{4}$$

where $\mathbf{W} = [R_{min}, R_{max}][V_{min}, V_{max}]$ is the a priori set of possible values of parameters.

The following expressions are obtained in [6] for the conditional bias and spread of the maximum-likelihood estimations of distance and velocity with allowance for anomalous errors:

$$\begin{gathered} b(\hat{R}|R_0, V_0) = \langle \hat{R} - R_0 \rangle = (1 - P_0)\Delta R_0, \\ b(\hat{V}|R_0, V_0) = \langle \hat{V} - V_0 \rangle = (1 - P_0)\Delta V_0, \\ B(\hat{R}|R_0, V_0) = \langle (\hat{R} - R_0)^2 \rangle = P_0 B_0(\hat{R}|R_0, V_0) \\ + (1 - P_0)\Delta R_{apr}^2/12 + (1 - P_0)\Delta R_0^2, \\ B(\hat{V}|R_0, V_0) = \langle (\hat{V} - V_0)^2 \rangle = P_0 B_0(\hat{V}|R_0, V_0) \\ + (1 - P_0)\Delta V_{apr}^2/12 + (1 - P_0)\Delta V_0^2. \end{gathered} \tag{5}$$

Here, the angular brackets denote the statistical averaging over samples of process $\pi(t)$ at fixed values of all unknown parameters. In these formulas, probability P_0 of reliable estimation is given by [6]

$$P_0 = \frac{1}{\sqrt{2\pi}} \times \int_\kappa^{+\infty} \exp\left[-\frac{(x - z)^2}{2} - \frac{\xi x}{\kappa(2\pi)^{3/2}} \exp\left(-\frac{x^2}{2\kappa^2}\right)\right] dx, \tag{6}$$

where

$$\kappa^2 = \frac{\int_{-\infty}^{+\infty} \ln^2(1 + s(t)/\nu)\, dt}{\int_{-\infty}^{+\infty} (1 + s(t)/\nu)\ln^2(1 + s(t)/\nu)\, dt}, \tag{7}$$

$$z^2 = N z_1^2 \tag{8}$$

is the signal-to-noise ration (SNR) for the pulse train with intensity (2),

$$z_1^2 = \frac{\left[\int_{-\infty}^{+\infty} s(t)\ln(1 + s(t)/\nu)\, dt\right]^2}{\nu \int_{-\infty}^{+\infty} (1 + s(t)/\nu)\ln^2(1 + s(t)/\nu)\, dt} \tag{9}$$

is the SNR for a single optical pulse with intensity $s(t)$,

$$\xi = 2\Delta R_{apr}\Delta V_{apr}\vartheta(\beta/c)^2\sqrt{(N^2 - 1)/3} \tag{10}$$

is the reduced area of the a priori region of the possible values of unknown distance and velocity [12], and

$$\beta^2 = \int_{-\infty}^{+\infty} \left[\frac{ds(t)/dt}{1 + s(t)/\nu}\right]^2 dt \Bigg/ \nu^2 \int_{-\infty}^{+\infty} \ln^2(1 + s(t)/\nu)\, dt. \tag{11}$$

The reduced area determines the number of distinguishable values of distance and velocity in domain **W**.

At relatively high SNR (8), probability of reliable estimation (6) is $P_0 \approx 1$ for entire train (2). Hence, the spread of the maximum-likelihood estimations (5) is represented as

$$\begin{gathered} B(\hat{R}|R_0, V_0) \simeq B_0(\hat{R}|R_0, V_0) \\ = \frac{c^2}{4\alpha^2} \frac{12(\mu^2 + \mu) + 4N^2 - 6(2\mu + 1)N + 2}{N(N^2 - 1)}, \\ B(\hat{V}|R_0, V_0) \simeq B_0(\hat{V}|R_0, V_0) \\ = \frac{c^2}{4\alpha^2\vartheta^2} \frac{12}{N(N^2 - 1)}, \end{gathered} \tag{12}$$

where

$$\alpha^2 = \frac{1}{\nu} \int_{-\infty}^{+\infty} \frac{[ds(t)/dt]^2}{1 + s(t)/\nu}\, dt.$$

Expressions (12) determine the spread of reliable maximum-likelihood estimations of distance and velocity. Such spreads coincide with variances of efficient estimations [1, 2, 4]. Therefore, the maximum-likelihood estimations of distance and velocity are asymptotically efficient (with an increase in the SNR for the entire train) [1, 2, 4].

In accordance with formula (4), logarithm of FLR (3) must be formed to obtain the maximum-likelihood estimation as a function of two variables (R and V) for all possible values $(R, V) \in \mathbf{W}$. Such a procedure is difficult to implement, since a multichannel (with respect to velocity) scheme must be employed [7]. In this case, the ranger contains several parallel channels each of which produces logarithm of FLR $L(R, V_j)$ at points $V_j \in [V_{\min}, V_{\max}]$ that are chosen with a step that provides the desired measurement accuracy. Each channel of the measurement system must contain a matched filter for a single pulse and a perfect comb filter [13]. However, the technical implementation of the comb filter at a relatively large number of pulses and a large a priori interval of possible values of unknown velocity meets difficulties related to strict requirements on the stability of the parameters of the delay line and high accuracy of the positioning of leads that provides the synchronous accumulation of pulses.

To simplify the implementation of the device for measurement of distance and velocity, we consider the possibility of determination of distance and velocity using the estimated time positions of single pulses in train (2):

$$\lambda_k = 2R/c + (k-\mu)(1+2V/c)\vartheta. \tag{13}$$

Intensity of the detected signal (2) can be represented as

$$s(t, R_0, V_0) = \sum_{k=0}^{N-1} s(t-\lambda_{0k}), \tag{14}$$

where

$$\lambda_{0k} = 2R_0/c + (k-\mu)(1+2V_0/c)\vartheta.$$

We assume that we know time intervals $[t_{k-1}, t_k]$ (the kth repetition period) in which signals $s(t - \lambda_{0k})$ are localized. With allowance for expression (13), the estimations of distance and velocity can be based on the estimations of time positions λ_{0k} that correspond to the laser pulsed ranging $R_k = c\lambda_{0k}/2$.

In accordance with the most-likelihood method, estimations of time positions λ_{0k} are based on the FLR logarithm that is represented for each pulse (accurate to insignificant terms) as [8]

$$L_k(\lambda_k) = \int_{t_{k-1}}^{t_k} \ln(1+s(t-\lambda_k)/\nu)\,d\pi(t). \tag{15}$$

For the estimation of time positions λ_{0k} of single pulses, we employ the positions of the greatest maxima of decision statistics (15):

$$\hat{\lambda}_k = \underset{\lambda_k \in [\Lambda_{k\,\min}, \Lambda_{k\,\max}]}{\arg\sup} L_k(\lambda_k), \tag{16}$$

where

$$\Lambda_{k\,\min} = \begin{cases} 2R_{\min}/c + (k-\mu)(1+2V_{\min}/c)\vartheta & \text{at } k > \mu, \\ 2R_{\min}/c + (k-\mu)(1+2V_{\max}/c)\vartheta & \text{at } k \le \mu, \end{cases}$$
$$\Lambda_{k\,\max} = \begin{cases} 2R_{\max}/c + (k-\mu)(1+2V_{\max}/c)\vartheta & \text{at } k > \mu, \\ 2R_{\max}/c + (k-\mu)(1+2V_{\min}/c)\vartheta & \text{at } k \le \mu. \end{cases} \tag{17}$$

Here, $[\Lambda_{k\,\min}, \Lambda_{k\,\max}]$ is the a priori interval of possible values of time position λ_k of the kth pulse. We assume that $[\Lambda_{k\,\min}, \Lambda_{k\,\max}] \subseteq [t_{k-1}, t_k]$. We introduce additional notation

$$\Delta\Lambda_{k\,\text{apr}} = \Lambda_{k\,\max} - \Lambda_{k\,\min}$$
$$= 2\Delta R_{\text{apr}}/c + 2|k-\mu|\Delta V_{\text{apr}}\vartheta/c,$$
$$\Lambda_{k\,\text{apr}} = (\Lambda_{k\,\max} + \Lambda_{k\,\min})/2$$
$$= 2R_{\text{apr}}/c + (k-\mu)(1+2V_{\text{apr}}/c)\vartheta,$$
$$\Delta\Lambda_{0k} = \Lambda_{k\,\text{apr}} - \lambda_{0k}.$$

Thus, a relatively simple procedure can be used to obtain the maximum-likelihood estimations of time positions λ_{0k}: it is suffice to employ a filter that is matched with a single pulse of train (14) and, then, find the positions of the absolute maxima of the output signal of the matched filter at a priori intervals $[\Lambda_{k\,\min}, \Lambda_{k\,\max}]$.

In the presence of possible anomalous errors, estimations (16) are characterized using probability densities [11, 12, 14]:

$$W_k(\hat{\lambda}_k | R_0, V_0) = P_{0k} W_{0k}(\hat{\lambda}_k | R_0, V_0) + (1-P_{0k}) W_{ak}(\hat{\lambda}_k), \tag{18}$$

where the Gaussian probability density of a reliable estimation is written as [8, 11, 13]

$$W_{0k}(\hat{\lambda}_k | R, V) = \frac{1}{\sqrt{2\pi D_0(\hat{\lambda}_k)}} \times \exp\left[-\frac{(\hat{\lambda}_k - m_{0k}(R_0, V_0))^2}{2D_0(\hat{\lambda}_k)}\right]. \tag{19}$$

Here,

$$m_{0k}(R_0, V_0) = \langle \hat{\lambda}_k \rangle = \lambda_{0k}$$
$$= 2R_0/c + (k-\mu)\vartheta(1+2V_0/c),$$
$$D_0(\hat{\lambda}_k) = \alpha^{-2},$$

$W_{ak}(\hat{\lambda}_k)$ is the probability density of anomalous estimation that is constant in a priori interval $[\Lambda_{k\,\min}, \Lambda_{k\,\max}]$, and P_{0k} is the probability of reliable estimation that is given by [11, 13]

$$P_{0k} = \frac{1}{\sqrt{2\pi}} \times \int_0^{\infty} \exp\left[-\frac{(x-z_1)^2}{2} - \frac{\xi_k}{2\pi}\exp\left(-\frac{x^2}{2\kappa^2}\right)\right]dx, \tag{20}$$

where

$$\xi_k = \Delta\Lambda_{k\mathrm{apr}}\beta$$

is the reduced length of a priori interval $[\Lambda_{k\,\min}, \Lambda_{k\,\max}]$ of the possible values of the time positions for the kth pulse [11, 14]. It characterizes the number of optical pulses that can be located in a priori interval $[\Lambda_{k\,\min}, \Lambda_{k\,\max}]$. Quantities κ^2, z_1^2, and β^2 are given by expressions (7), (9), and (11), respectively.

The distribution of random quantity $\hat{\lambda}_k$ can be approximated using Gaussian distribution (19) provided that the following condition is satisfied [15]:

$$\int_{-\infty}^{+\infty} s(t)dt + \nu\tau \gg 1.$$

Here, τ is the equivalent duration of pulses of received train (2).

For the conditional bias and spread of the estimation of time positions (16), we have [14]

$$b(\hat{\lambda}_k|\lambda_{0k}) = \langle\hat{\lambda}_k - \lambda_{0k}\rangle = (1-P_{0k})\Delta\Lambda_{0k},$$

$$B(\hat{\lambda}_k|\lambda_{0k}) = \langle(\hat{\lambda}_k - \lambda_{0k})^2\rangle = P_{0k}/\alpha^2 + (1-P_{0k})(\Delta\Lambda_{k\mathrm{apr}}^2/12 + \Delta\Lambda_{0k}^2).$$

Quantities R_0 and V_0 can be estimated using estimations of time positions $\hat{\lambda}_k$ of single pulses. Formula (18) shows that the probability densities of the estimations of time positions $\hat{\lambda}_k$ are not Gaussian. This circumstance impedes the synthesis of the quasi-optimal estimation using expression (18). Therefore, we search for the quasi-optimal estimation on the assumption of Gaussian approximation (19) of distribution (18) that is valid if P_{0k} is close to unity. Thus, we use approximate formula (19) for the approximation of the conditional probability density of random quantity $\hat{\lambda}_k$.

A set of N independent random quantities $\hat{\lambda}_k$ (16) is used as the initial statistics for the quasi-optimal estimations of quantities $\tilde{R}$ and $\tilde{V}$. The corresponding likelihood function is written as

$$W(\hat{\lambda}_0, \ldots, \hat{\lambda}_{N-1}|R, V) = \prod_{k=0}^{N-1} W_{0k}(\hat{\lambda}_k|R, V) = \prod_{k=0}^{N-1} \frac{1}{\sqrt{2\pi D_0(\hat{\lambda}_k)}} \exp\left[-\frac{(\hat{\lambda}_k - 2R/c - (k-\mu)(1+2V/c)\vartheta)^2}{2D_0(\hat{\lambda}_k)}\right].$$

Accurate to insignificant constant terms, the logarithm of the likelihood function is represented as

$$L_N(R, V) = \ln W(\hat{\lambda}_0, \ldots, \hat{\lambda}_{N-1}|R, V) = -\frac{1}{2}\sum_{k=0}^{N-1} \frac{(\hat{\lambda}_k - 2Rc - (k-\mu)(1+2V/c)\vartheta)^2}{D_0(\hat{\lambda}_k)}. \tag{21}$$

In accordance with the maximum-likelihood method, estimations $\hat{R}$ and $\hat{V}$ of the parameters of motion are the values at which function $L(R, V)$ (21) reaches maximum. Using the solution to the system of likelihood equations

$$\left.\frac{\partial L_N(R, V)}{\partial R}\right|_{(\tilde{R}, \tilde{V})} = 0, \quad \left.\frac{\partial L_N(R, V)}{\partial V}\right|_{(\tilde{R}, \tilde{V})} = 0,$$

we find the quasi-optimal estimations of the distance and velocity:

$$\tilde{R} = \sum_{k=0}^{N-1} \delta_{Rk}\hat{\lambda}_k, \quad \tilde{V} = \sum_{k=0}^{N-1} \delta_{Vk}\hat{\lambda}_k - \frac{c}{2}, \tag{22}$$

where

$$\delta_{Rk} = \frac{c[k(6\mu - 3N + 3) + (N-1)(2N - 1 - 3\mu)]}{N(N^2-1)},$$

$$\delta_{Vk} = -\frac{3c(N - 2k - 1)}{\vartheta N(N^2-1)}.$$

Such a quasi-optimal estimation algorithm can be implemented using the block diagram of Fig. 1. The input signal of the detector is a series of short pulses that represent derivative $\pi'(t)$ of a sample of Poisson random process $\pi(t)$. A series of pulses passes through filter Φ with response function $h(t) = h_0\ln(1 + s(t^* - t)/\nu)$, where h_0 is the filter gain and t^* is the delay, such that $t^* > \tau$, where τ is the duration of a single optical pulse with intensity $s(t)$. The output signal of the filter is multiplied by function

$$\chi_k(t) = \begin{cases} 1, & t \in [\Lambda_{k\,\min}, \Lambda_{k\,\max}], \\ 0, & t \notin [\Lambda_{k\,\min}, \Lambda_{k\,\max}] \end{cases}$$

sequentially for all $k = 0, \ldots, N - 1$. An extremator determines the time position of the greatest maximum of the signal at interval $[\Lambda_{k\,\min}, \Lambda_{k\,\max}]$ (17) for the kth input pulse and produces a series of estimations $\hat{\lambda}_k$. Computing device (CD) calculates the estimations of distance and velocity using formulas (22). Figure 1 shows the single-channel implementation of the receiver of the quasi-optimal algorithm for the estimation of distance and velocity that differs from the implementation of the maximum-likelihood receiver.

To calculate the accuracy characteristics of the quasi-optimal estimations of distance and velocity, we change random quantities $\hat{\lambda}_k$ in formulas (22) by conditional mathematical mean values and obtain conditional mathematical means of estimations $\tilde{R}$ and $\tilde{V}$. When true values λ_{0k} are substituted for $\hat{\lambda}_k$ in formulas (22), we obtain true distances R_0 and velocities V_0, since the reliable estimations are unbiased. Subtracting the first result from the second one, we derive formulas for conditional biases of estimations $\tilde{R}$ and $\tilde{V}$:

$$b(\tilde{R}|R_0, V_0) = \langle \tilde{R} - R_0 \rangle = \sum_{k=0}^{N-1} \delta_{Rk}(1 - P_{0k})\Delta\Lambda_{0k},$$

$$b(\tilde{V}|R_0, V_0) = \langle \tilde{V} - V_0 \rangle = \sum_{k=0}^{N-1} \delta_{Vk}(1 - P_{0k})\Delta\Lambda_{0k}.$$

For the conditional spreads, we have

$$\begin{aligned} B(\tilde{l}|R_0, V_0) &= \langle (\tilde{l} - l_0)^2 \rangle = \sum_{k=0}^{N-1} \delta_{lk}^2 P_{0k}/\alpha^2 \\ &+ [\chi_{l00}\Delta R_{\mathrm{apr}}^2 + 2\vartheta\chi_{l01}\Delta R_{\mathrm{apr}}\Delta V_{\mathrm{apr}} \\ &+ \vartheta^2\chi_{l11}\Delta V_{\mathrm{apr}}^2]/3c^2 + 4[\psi_{l00}\Delta R_0^2 \\ &+ 2\vartheta\psi_{l01}\Delta R_0\Delta V_0 + \vartheta^2\psi_{l11}\Delta V_0^2]/c^2, \end{aligned} \tag{23}$$

where l is the estimated parameter of motion (R or V) and

$$\chi_{lmn} = \sum_{k=0}^{N-1} \delta_{lk}^2 (1 - P_{0k})|k - \mu|^{m+n}, \quad m, n = 0, 1,$$

$$\begin{aligned} \psi_{lmn} &= \sum_{k=0}^{N-1} \delta_{lk}^2 P_{0k}(1 - P_{0k})(k - \mu)^{m+n} \\ &+ \left(\sum_{k=0}^{N-1} \delta_{lk}(1 - P_{0k})(k - \mu)^m\right) \\ &\times \left(\sum_{k=0}^{N-1} \delta_{lk}(1 - P_{0k})(k - \mu)^n\right). \end{aligned}$$

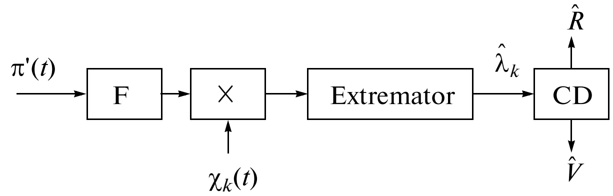

Fig. 1. Block diagram of the quasi-optimal device for measurement of distance and velocity.

In a particular case, when true values R_0 and V_0 coincide with the centers of a priori intervals R_{apr} and V_{apr} (i.e., $R_0 = R_{\mathrm{apr}}$ and $V_0 = V_{\mathrm{apr}}$), formulas (32) are simplified:

$$\begin{aligned} B(\tilde{l}|R_0, V_0) &= \sum_{k=0}^{N-1} \delta_{lk}^2 \frac{P_{0k}}{\alpha^2} + \frac{1}{3c^2} \\ &\times \sum_{k=0}^{N-1} \delta_{lk}^2 (1 - P_{0k})[\Delta R_{\mathrm{apr}} + \vartheta|k - \mu|\Delta V_{\mathrm{apr}}]^2. \end{aligned} \tag{24}$$

At a relatively high SNR (9), probability of reliable estimations (20) is $P_{0k} \approx 1$ for each pulse of train (2). Thus, expressions (23) for the spreads of quasi-optimal estimations of distance and velocity are given by expression (12) (i.e., coincide with the spreads of reliable maximum-likelihood estimations and, hence, with the spreads of efficient estimations [1, 2, 4]). Therefore, the asymptotic quasi-optimal estimations are efficient when the SNR increases for each pulse.

In accordance with the results of [3, 4], a relatively high accuracy of the maximum-likelihood estimations of distance and velocity is reached at high SNRs $z^2 = Nz_1^2$ (8) for entire train (2) even if SNR z_1^2 (9) for a single pulse is relatively low. In this regard, accuracy of quasi-optimal estimations (22) may be significantly less than the accuracy of maximum-likelihood estimations (4). Indeed, anomalous errors of estimations (4) of distance and velocity are almost absent if z^2 (8) is relatively high. At small values of z_1^2 (9), we may obtain anomalous errors of estimations $\hat{\lambda}_k$ (16), that may lead to a significant decrease in accuracy of estimations (22).

For the comparison of the characteristics of the simultaneously efficient estimations, maximum-likelihood estimations, and quasi-optimal estimations, we specify the intensity profiles of optical pulses and present formulas for conditional normalized spreads at $R_0 = R_{\mathrm{apr}}$ and $V_0 = V_{\mathrm{apr}}$.

We assume that the intensity profile of a single pulse of train (2) is represented using the Gaussian curve:

$$s(t) = a\exp(-\pi t^2/2\tau^2), \tag{25}$$

where

$$a = \max s(t)$$

is the maximum pulse intensity and

$$\tau = \int_{-\infty}^{+\infty} s^2(t)ds/\max s^2(t)$$

is the equivalent pulse duration.

For the quasi-optimal estimations, the formulas for conditional normalized spreads at $R_0 = R_{\text{apr}}$ and $V_0 = V_{\text{apr}}$ are written in accordance with expression (24) as

$$\begin{gathered} b_{\tilde{R}}(R_0, V_0) = \frac{B(\tilde{R}|R_0, V_0)}{\Delta R_{\text{apr}}^2} \\ = \sum_{k=0}^{N-1} \tilde{\delta}_{Rk}^2 \left[\frac{P_{0k}}{4m_R^2 q\mu_S\alpha_0^2} + \frac{1-P_{0k}}{12}\left(1 + \frac{m_V}{m_R}|k-\mu|\right)^2\right], \\ b_{\tilde{V}}(R_0, V_0) = \frac{B(\tilde{V}|R_0, V_0)}{\Delta V_{\text{apr}}^2} \\ = \sum_{k=0}^{N-1} \tilde{\delta}_{Vk}^2 \left[\frac{P_{0k}}{4m_V^2 q\mu_S\alpha_0^2} + \frac{1-P_{0k}}{12}\left(\frac{m_R}{m_V} + |k-\mu|\right)^2\right], \\ \tilde{\delta}_{Rk} = \frac{2[k(6\mu - 3N + 3) + (N-1)(-3\mu + 2N - 1)]}{N(N^2-1)}, \\ \tilde{\delta}_{Vk} = \frac{6(2k - N + 1)}{N(N^2-1)}. \end{gathered} \tag{26}$$

In accordance with expression (5), the formulas for conditional normalized spreads for the maximum-likelihood estimations at $R_0 = R_{\text{apr}}$ and $V_0 = V_{\text{apr}}$ are represented as

$$\begin{gathered} b_{\hat{R}}(R_0, V_0) = \frac{B(\hat{R}|R_0, V_0)}{\Delta R_{\text{apr}}^2} = \frac{P_0}{4m_R^2 q\mu_S\alpha_0^2} \\ \times \frac{2(6\mu^2 + 6\mu + 2N^2 - 3(2\mu+1)N + 1)}{N(N^2-1)} + \frac{1-P_0}{12}, \\ b_{\hat{V}}(R_0, V_0) = \frac{B(\hat{V}|R_0, V_0)}{\Delta V_{\text{apr}}^2} = \frac{P_0}{4m_V^2 q\mu_S\alpha_0^2} \\ \times \frac{12}{N(N^2-1)} + \frac{1-P_0}{12}. \end{gathered} \tag{27}$$

Assuming that $P_0 = 1$ in expressions (27), we obtain the normalized characteristics of reliable maximum-likelihood estimations (12) that coincide with the characteristics of the simultaneous efficient estimations [11, 14]:

$$\begin{gathered} b_{RE}(R_0, V_0) = \frac{B_0(\hat{R}|R_0, V_0)}{\Delta R_{pr}^2} = \frac{1}{4m_R^2 q\mu_S\alpha_0^2} \\ \times \frac{2(6\mu^2 + 6\mu + 2N^2 - 3(2\mu+1)N + 1)}{N(N^2-1)}, \\ b_{VE}(R_0, V_0) = \frac{B_0(\hat{V}|R_0, V_0)}{\Delta V_{pr}^2} \\ = \frac{1}{4m_V^2 q\mu_S\alpha_0^2}\frac{12}{N(N^2-1)}. \end{gathered} \tag{28}$$

In the expressions for normalized spreads of estimations (26)–(28), we introduce notation

$$\begin{gathered} \alpha_0^2 = \pi^2 \int_{-\infty}^{\infty} \frac{x^2\exp(-\pi x^2)}{1 + q\exp(-\pi x^2/2)}dx, \\ m_R = \Delta R_{\text{apr}}/c\tau \end{gathered} \tag{29}$$

is the number of pulses with intensity (29) that are located in the a priori interval of possible distances, $c\tau$ is the spatial equivalent duration of a single pulse,

$$m_V = \vartheta\Delta V_{\text{apr}}/c\tau$$

is the fraction of the spatial length of pulse that may be equal to the displacement of target over the repletion period of train (2),

$$q = a/\nu$$

is the signal-to-background ratio, and

$$\mu_S = a\tau$$

is the mean number of signal points (photoelectrons) that correspond to a single pulse.

To calculate probability of reliable estimation P_0 in expression (27), we must change quantity κ^2 (7) in formula (6) by the quantity

$$\kappa^2 = \frac{\int_{-\infty}^{+\infty} \ln^2[1 + q\exp(-\pi x^2/2)]dx}{\int_{-\infty}^{+\infty} [1 + q\exp(-\pi x^2/2)]\ln^2[1 + q\exp(-\pi x^2/2)]dx} \tag{30}$$

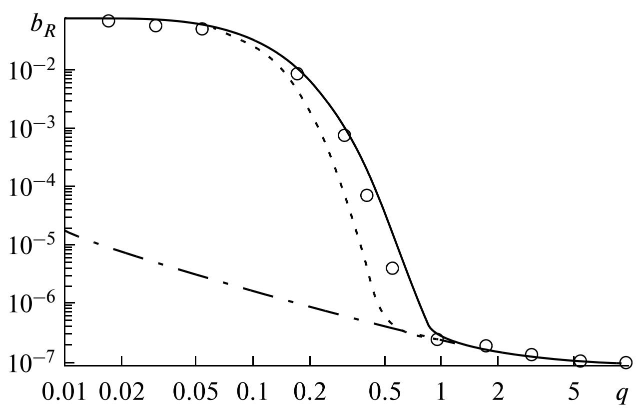

Fig. 2. Normalized spreads of the estimations of distance at $N = 2$, $\mu_S = 100$, $m_R = 100$, and $m_V = 1$.

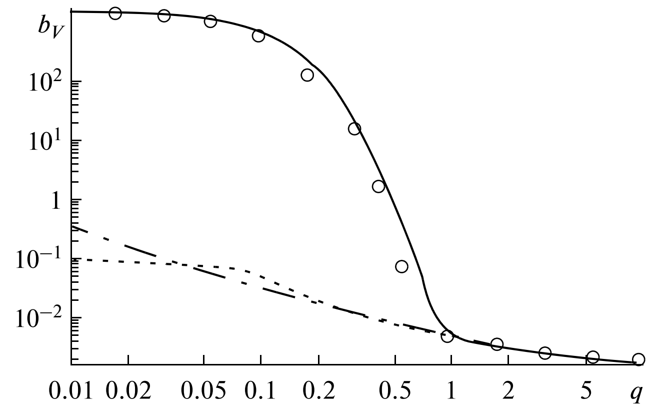

Fig. 3. Normalized spreads of the estimations of velocity at $N = 2$, $\mu_S = 100$, $m_R = 100$, and $m_V = 1$.

and employ quantity z^2 (8) under the condition that SNR z_1^2 (9) is represented as

$$z_1^2 = q\mu_S \frac{\left[\int_{-\infty}^{+\infty} \exp(-\pi x^2/2)\ln[1 + q\exp(-\pi x^2/2)]dx\right]^2}{\int_{-\infty}^{+\infty} [1 + q\exp(-\pi x^2/2)]\ln^2[1 + q\exp(-\pi x^2/2)]dx} \tag{31}$$

and quantity

$$\xi = 2q^2 m_R m_V \beta_0^2 \sqrt{(N^2 - 1)/3}.$$

Here, we have

$$\beta_0^2 = \pi^2 \int_{-\infty}^{+\infty} \left[\frac{x\exp\left(-\frac{\pi x^2}{2}\right)}{1 + q\exp\left(-\frac{\pi x^2}{2}\right)}\right]^2 dx \Bigg/ \int_{-\infty}^{+\infty} \ln^2[1 + q\exp(-\pi x^2/2)]dx.$$

For the calculation of probability of reliable estimation P_{0k} in expression (26), we must substitute quantities (30) and (31) in formula (20) and assume that

$$\xi_k = 2q\beta_0(m_R + |k - \mu| m_V).$$

Figures 2–7 present the dependences of conditional normalized biases b_R and b_V (expressions (26)–(28)) of the estimations of distance and velocity on signal-to-background ratio q at $\mu = 0$. The solid lines show the theoretical dependences for the quasi-optimal estimation with allowance for anomalous errors (26), the dashed lines correspond to maximum-likelihood estimations with allowance for anomalous errors (27), and the dashed-and-dotted lines correspond to simultaneous efficient estimation (28). The dots show the results of the statistical simulation.

It is seen that the characteristics of quasi-optimal estimations and maximum-likelihood estimations coincide with the characteristics of the simultaneous efficient estimations in the domain of reliable estimation. This circumstance illustrates the asymptotic efficiency of the quasi-optimal and the maximum-likelihood methods. In the domain of anomalous errors, the accuracy of the quasi-optimal estimation is significantly less than the maximum-likelihood estimation. Note that the asymptotic efficiency of the maximum-likelihood estimations is reached at lower signal-to-background ratios in comparison with the quasi-optimal estimations.

The comparison of the results of Figs. 2 and 4 and Figs. 3 and 5 shows that an increase in the number of pulses in the sounding pulse train leads to an increase in the estimation accuracy. Note a developed increase in the velocity estimations. In addition, an increase in the number of pulses does not lead to changes of the domain of anomalous errors for the quasi-likelihood method and a decrease in such a domain for the maximum-likelihood estimations. An increase in the reduced length of the a priori interval of possible values of distance m_R does not cause variations in reliable characteristics. However, an increase in the a priori

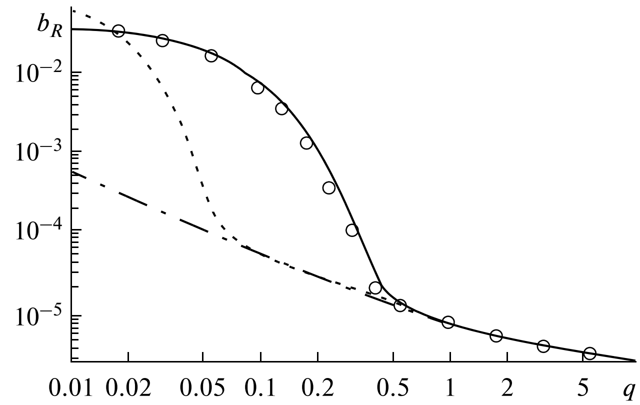

Fig. 4. Normalized spreads of the estimations of distance at $N = 10$, $\mu_S = 100$, $m_R = 10$, and $m_V = 1$.

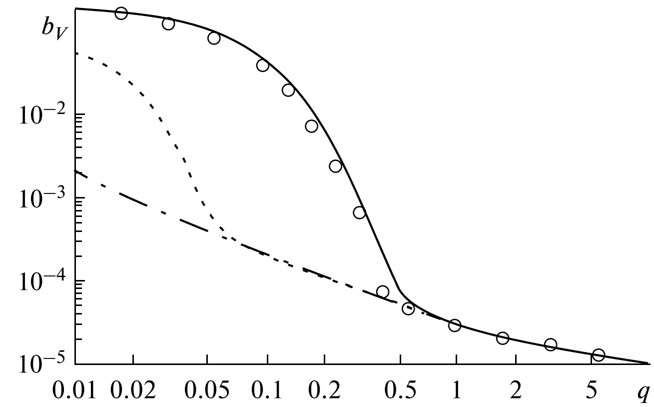

Fig. 5. Normalized spreads of the estimations of velocity at $N = 10$, $\mu_S = 100$, $m_R = 10$, and $m_V = 1$.

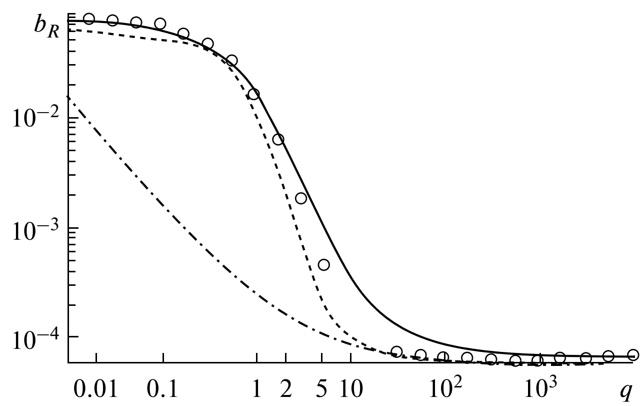

Fig. 6. Normalized spreads of the estimations of distance at $N = 2$, $\mu_S = 10$, $m_R = 10$, and $m_V = 0.5$.

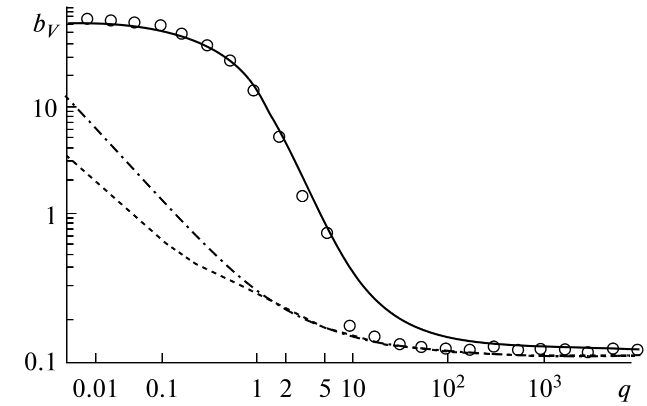

Fig. 7. Normalized spreads of the estimations of velocity at $N = 2$, $\mu_S = 10$, $m_R = 10$, and $m_V = 0.5$.

interval of possible distances results in an increase in the domain of threshold effects for both quasi-optimal and maximum-likelihood methods.

The above expressions for the characteristics of quasi-optimal estimations are only asymptotically accurate at relatively large parameters μ_S and m_R. The accuracies of the derived formulas can hardly be determined using analytical methods at finite values of the parameters. In this regard, we employ the computer statistical simulation to study the efficiency of the quasi-optimal algorithm and determine the applicability limits of the asymptotic expressions for the characteristics of estimations of distance and velocity.

In the statistical simulation, we form discrete samples of FLR logarithm (15) for each pulse with a step of $\Delta\lambda = \tau/25$. In this case, the integral in expression (15) is approximated using a finite sum of samples of the integrand with a step of $\Delta t = \tau/25$. Poisson process $\pi(t)$ is formed using a conventional device that generates independent random values uniformly distributed over interval [0, 1] for the intensity of process $s(t - \lambda_{0k}) + \nu$, where quantity $s(t)$ is given by formula (25). For sampling interval $\Delta\lambda = \tau/2$, the relative mean-square error of approximation of FLR logarithm (15) using step functions based on the corresponding samples is no greater than 2.5%. Using the position of the greatest maximum of the approximation of FLR logarithm (15), we determine estimation $\hat{\lambda}_k$ (16) of the time position of the kth pulse. A set of estimations $\hat{\lambda}_k$ is used to obtain quasi-optimal estimations of distance and velocity (22). Then, we calculate selective normalized spreads of estimations.

Size N_e of the experimental sample ranges from 10^3 to 5×10^3 depending on parameters q, μ_S, and m_R. Such sizes provide a mean-square error of experimental data of 10–20%. Figures 2–7 show that theoretical dependences (26) of the spread of quasi-optimal estimations of distance and velocity with allowance for anomalous errors reasonably approximate the experimental data at $\mu_S \geq 10$ and $m_R \geq 10$. Such lower limits of the domain of applicability of the derived theoretical formulas for the characteristics of quasi-optimal estimations can hardly be significantly improved. Indeed, the theoretical and experimental data are already in agreement at $\mu_S = 5$.

Quasi-optimal estimations (22) and maximum-likelihood estimations (4) are asymptotically efficient when the SNR increases. Maximum-likelihood estimations (4) are close to efficient estimations provided that SNR (8) is relatively high for the entire train of optical pulses with intensity (2). The quasi-optimal estimation is close to the efficient estimation if SNR (9) is relatively high for each pulse of the observed train of optical pulses. Therefore, a significantly higher energy of signal is needed for a high a posteriori accuracy of the quasi-optimal estimations.

When the conditions for a relatively high a posteriori accuracy of the estimations of time position of each pulse are satisfied, a relatively simple quasi-optimal algorithm (22) can be used instead of a difficult-to-implement maximum-likelihood algorithm (4) almost without loss of accuracy. In addition, algorithm (22) can be used for the processing of the results in the existing high-accuracy LIDARs to obtain additional information on the velocity of target.

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