Quasi-likelihood detection of rectangle ultra-wideband quasi-radiosignal

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Abstract**—*The quasi-likelihood algorithm detection of rectangle ultra-wideband quasi-radiosignal with unknown amplitude, initial phase and duration has been synthesized. The statistical characteristics of the efficiency synthesized detection algorithm – false alarm probability and probability of missing a signal, have been found.*

Keywords—signal detection; ultra-wideband quasi-radiosignal, false-alarm probability, missing probability a signal.

I. Introduction

There is an acute task of radiosignal detection in practical applications of radio- and hydrolocation, navigation, seismology, radiocommunication and others, which has been considered in literature [1-3]. It was supposed that the radiosignal is narrowband [1,3]. Ultra-wideband signals find a wider application in the modern world [4], but algorithms of optimal processing of such signals have not been researched enough. The work [5] considers the task of detection of a radiosignal, which does not satisfy the condition of relative narrowbandness and is called ultra-wideband quasi-radiosignal. Amplitude and initial phase of the ultra-wideband quasi-radiosignal were considered in [5] unknown. However, often apart from amplitude and initial phase the duration of the received signal appears to be unknown. Quasi-likelihood algorithm of ultra-wideband quasi-radiosignal detection with unknown amplitude, initial phase, and duration is researched in [6], where instead of duration, unknown in advance, its expected, predicted value is used. This work considers quasi-likelihood algorithm of ultra-wideband quasi-radiosignal detection with adaptation in duration.

II. Problem Statement

Let the signal be a subject to duration

$$s\left(t,a,\varphi,\tau\right)=\begin{cases}a\cos\left(\omega\, t-\varphi\right), & 0\le t\le\tau,\\ 0, & t<0, \quad t>\tau,\end{cases} \tag{1}$$

where a, φ, ω, τ – amplitude, initial phase, frequency, and duration [5]. If frequency band $\Delta\omega$ and frequency ω satisfy condition

$$\Delta\omega << \omega. \tag{2}$$

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The signal (1) is a narrowband quasi-radiosignal [2,7]. If condition (2) is not performed, formula (1) describes ultra-wideband quasi-radiosignal [5,6]. Values a, φ, ω are parametres of harmonic oscillations, used for its forming. Nevertheless, similarly to [5,6] we will then name a, φ, ω amplitude, initial phase, and frequency of ultra-wideband quasi-radiosignal (1). Let us consider that signal (1) is received at the background of white Gaussian noise $n(t)$ with unilateral spectral density N_0, and true values of amplitude a_0, initial phase φ_0 and duration τ_0 are prior unknown. We will submit additive mixture of signal (1) and noise $n(t)$, observed during time period $t\in[0,T]$ in a form

$$\xi(t)=\gamma_0 s\left(t,a_0,\varphi_0,\tau_0\right)+n(t), \tag{3}$$

where γ is a discrete parameter, taking the value $\gamma=0$ in the absence of signal and $\gamma=1$ in the presence, and γ_0 as its unknown true value. We will consider that the signal duration can take values from the prior interval $\tau\in\left[T_1,T_2\right]$. Having the accepted realization (3), the receiving device should make a solution about signal presence or absence. Then the task of detection comes down to the evaluation of the parameter γ on the basis of the observed data.

III. Synthesis of the Detection Algorithm

For the synthesis of the algorithm of ultra-wideband quasi-radiosignal detection (estimation of the parameter γ) we will use the maximum likelihood (ML) method [1,2,7]. Under unknown signal parameters there is prior parameter uncertainty in relation to amplitude, initial phase, and duration. In any case, the logarithm of the likelihood ratio functional (LRF) depends on four unknown parameters

$$L(\gamma,a,\varphi,\tau)=\frac{2\gamma}{N_0}\int_0^T\xi(t)s(t,a,\varphi,\tau)dt-\frac{\gamma}{N_0}\int_0^T s^2(t,a,\varphi,\tau)dt. \tag{4}$$

A range of detection algorithms can be received after setting in the formula (4) the unknown values instead of a, φ

and τ. These values can be fixed as in [6], and can be determined by the realization of the observed data. Instead of unknown amplitude and initial phase in formula (4) we will by analogy with [6] use some of their expected values a^* and φ^* respectively, and instead of unknown duration its quasi-likelihood estimation (which is equal to adaptation of the algorithm of detection by duration). Then the estimation $\hat{\gamma}$ of the γ parameter, determined as γ value, under which the logarithm LRF reaches its absolute maximum, is quasi-likelihood [7]. Quasi-likelihood algorithm of the signal detection (estimation of the γ parameter) can by analogy with [6] be represented as

$$\hat{\gamma} = \begin{cases} 1, & L \geq h, \\ 0, & L < h, \end{cases} \tag{5}$$

there

$$L = \sup_{\tau} L(\tau), \quad L(\tau) = L(\gamma = 1, \ a = a^*, \ \varphi = \varphi^*, \ \tau). \tag{6}$$

The threshold h in formula (5) is chosen in coordination with a specified criteria of optimality [1,2]. Formulas (4) – (6) determine the structure of the receiver. A detector should form a random process (6) for all possible values of duration and find its maximum. The solution about absence or presence of a signal is made on the basis of the comparison of the maximum value (6) with threshold h. Let us put into the formula (4) the explicit form of the ultra-wideband quasi-radiosignal (1) and transform the logarithm LRF to the form

$$\begin{aligned} L(\tau) = a^* \left(X(\tau)\cos\varphi^* + Y(\tau)\sin\varphi^* \right) - \\ - a^{*2} \frac{Q(\tau) + P_c(\tau)\cos 2\varphi^* + P_s(\tau)\sin 2\varphi^*}{2}, \end{aligned} \tag{7}$$

where

$$\begin{aligned} X(\tau) = \frac{2}{N_0}\int_0^{\tau} \xi(t)\cos\omega t dt, \quad Y(\tau) = \frac{2}{N_0}\int_0^{\tau} \xi(t)\sin\omega t dt, \\ Q(\tau) = \frac{\tau}{N_0}, \quad P_c(\tau) = \frac{\sin 2\omega\tau}{2\omega N_0}, \quad P_s(\tau) = \frac{1 - \cos 2\omega\tau}{2\omega N_0}. \end{aligned} \tag{8}$$

Quasi-likelihood detector of the ultra-wideband quasi-radiosignal (5) can be realized on the basis of the flow chart at the Fig.1, where integrators (I) work at the time interval $[0, t]$, $t \in [0, T_2]$, F – frequency doubler, RG – ramp generator, PD - peak detector, TD – thresholder, that makes the comparison of the maximum value L with threshold h and makes the solution about absence or presence of a signal.

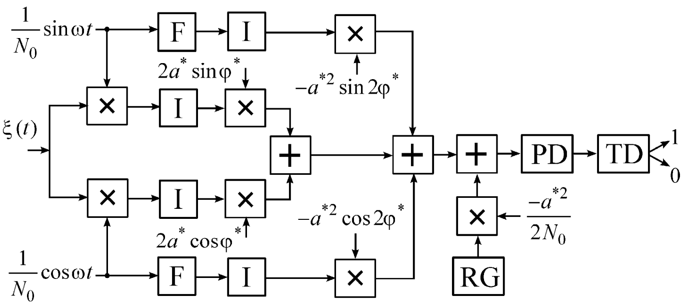

Fig.1. Block diagram of the ultra-wideband quasi-radiosignal detector

IV. Characteristics of the detection algorithm

Let us make the analysis of the quasi-likelihood algorithm of detection (5), that find the probabilities of the false-alarm and the signal omission [1,2,8]. It is obvious that ignorance of amplitude and initial phase impacts the efficiency of detection. Thus, let us introduce values, characterizing detuning of quasi-likelihood detector by amplitude $\Delta_a = a^*/a_0$ and initial phase $\Delta_\varphi = \varphi^* - \varphi_0$. Then expected amplitude and initial phase can be expressed by true values and detuning of corresponding parameters $a^* = a_0\Delta_a$ and $\varphi^* = \varphi_0 + \Delta_\varphi$. Putting expected values a^* and φ^* in formula (7), we will write the logarithm LRF as

$$\begin{aligned} L(\tau) = a_0\Delta_a \left(X(\tau)\cos(\varphi_0 + \Delta_\varphi) + Y(\tau)\sin(\varphi_0 + \Delta_\varphi) \right) - \\ - \frac{(a_0\Delta_a)^2}{2}\left(Q(\tau) + P_c(\tau)\cos(2\varphi_0 + 2\Delta_\varphi) + P_s(\tau)\sin(2\varphi_0 + 2\Delta_\varphi) \right). \end{aligned} \tag{9}$$

According to (9) a random process $L(\tau)$ is Gaussian. Thus, for its full statistical description it is enough to find expected value and correlation function. Let us consider $L_1(\tau) = \{L(\tau) | \gamma_0 = 1\}$ to be the logarithm LRF in the presence of signal in accepted realization, and $L_0(\tau) = \{L(\tau) | \gamma_0 = 0\}$ – in its absence. Making averaging out, we get mathematical expectations in the presence of signal

$$\begin{aligned} S_1(\tau) = \langle L_1(\tau) \rangle = a_0^2\Delta_a \Big[Q(\min(\tau, \tau_0))\cos(\Delta_\varphi) + \\ + P_c(\min(\tau, \tau_0))\cos(2\varphi_0 + \Delta_\varphi) + P_s(\min(\tau, \tau_0))\sin(2\varphi_0 + \Delta_\varphi) \Big] - \\ - \frac{a_0^2\Delta_a^2}{2}\Big[Q(\tau) + P_c(\tau)\cos(2\varphi_0 + 2\Delta_\varphi) + P_s(\tau)\sin(2\varphi_0 + 2\Delta_\varphi) \Big] \end{aligned} \tag{10}$$

and in its absence

$$\begin{aligned} S_0(\tau) = \langle L_0(\tau) \rangle = - \frac{a_0^2\Delta_a^2}{2}\Big[Q(\tau) + \\ + P_c(\tau)\cos(2\varphi_0 + 2\Delta_\varphi) + P_s(\tau)\sin(2\varphi_0 + 2\Delta_\varphi) \Big], \end{aligned} \tag{11}$$

and correlation function

$$\begin{aligned} K(\tau_1,\tau_2) &= \left\langle \left[L_1(\tau_1) - \left\langle L_1(\tau_1) \right\rangle \right] \left[L_1(\tau_2) - \left\langle L_1(\tau_2) \right\rangle \right] \right\rangle = \\ &= \left\langle \left[L_0(\tau_1) - \left\langle L_0(\tau_1) \right\rangle \right] \left[L_0(\tau_2) - \left\langle L_0(\tau_2) \right\rangle \right] \right\rangle = \\ &= a_0^2 \Delta_a^2 \left[Q\left(\min(\tau_1,\tau_2)\right) + P_c\left(\min(\tau_1,\tau_2)\right) \cos\left(2\varphi_0 + 2\Delta_\varphi\right) + \right. \\ &\qquad \left. + P_s\left(\min(\tau_1,\tau_2)\right) \sin\left(2\varphi_0 + 2\Delta_\varphi\right) \right]. \end{aligned} \tag{12}$$

Let us then consider that the out signal to noise ratio (SNR) is big enough for the received signal. To find the false-alarm probability let us research determinant statistics near its maximum position. With the growth of the SNR the maximum position of determinant statistics converges in mean square to the maximum position of its mathematical expectation [8]. Let us differentiate mathematical expectation in the absence of signal (11)

$$\frac{\partial S_0(\tau)}{\partial \tau} = -a_0^2 \Delta_a^2 \cos^2\left(\omega\tau - \varphi_0 - \Delta_\varphi\right) / N_0 .$$

It is evident that the derivative of the mathematical expectation in the absence of signal is negative for all possible meanings of duration. Thus, the maximum position of the mathematical expectation $S_0(\tau)$ of the determinant statistics coincides with the left border of the prior interval of possible meanings of duration T_1. Putting formulas (11) and (12) in Taylor's series by τ near T_1, we will get asymptotic expressions for mathematical expectation in the absence of signal

$$S_0(\tau) \approx -\lambda_0/2 - (\tau - T_1)\psi_0/2T_2 \tag{13}$$

and correlation function

$$K_{q0}(\tau_1,\tau_2) \approx \lambda_0 + \psi_0 \min(\tau_1 - T_1, \tau_2 - T_1)/T_2 , \tag{14}$$

where

$$\lambda_0 = (a_0\Delta_a)^2 \left(Q(T_1) + P_c(T_1)\cos\left(2\varphi_0 + 2\Delta_\varphi\right) + P_s(T_1)\sin\left(2\varphi_0 + 2\Delta_\varphi\right)\right), \quad \psi_0 = \Delta_a^2 z^2 \cos^2\left(\omega T_1 - \varphi_0 - \Delta_\varphi\right),$$

$$z^2 = 2a_0^2 T_2 / N_0 \tag{15}$$

– SNR on the output of the ML receiver for square pulse of amplitude a_0 and duration T_2. Let us approximate under big SNR the logarithm LRF $L_0(\tau)$ by the Gaussian random process $\mu_0(\tau)$ with mathematical expectation (13) and correlation function (14) along the whole prior interval of duration values. Using formulas (13), (14) and Doob's theorem [9,10], we can show that the determinant statistics $\mu_0(\tau)$ is a Gaussian Markov process with drift coefficient k_{10} and diffusion coefficient k_{20} [9,10]

$$k_{10} = -\psi_0/2T_2 , \quad k_{20} = \psi_0/T_2 . \tag{16}$$

The false-alarm probability is by definition $\alpha = 1 - F_0(h)$, where

$$F_0(h) = P\left\{\mu_0(\tau) < h,\ \tau \in [T_1, T_2]\right\} \tag{17}$$

- probability of failure to achieve borders $y = -\infty$ and $y = h$ by Markov random process $\mu_0(\tau)$ at the interval $\tau \in [T_1, T_2]$. The required probability (17) can be expressed through the probability density $W(y,\tau)$ of realizations of a random process $\mu_0(\tau)$, which have never reached borders $y = -\infty$, $y = h$ [9]

$$F_0(h) = \int_{-\infty}^{h} W(y, T_2)\, dy . \tag{18}$$

The probability density $W(y,\tau)$ is the solution to the Fokker-Planck-Kolmogorov (FPK) equation [8,9]

$$\frac{\partial W(y,\tau)}{\partial \tau} + \frac{\partial}{\partial y}\left[k_1 W(y,\tau)\right] - \frac{1}{2}\frac{\partial^2}{\partial y^2}\left[k_2 W(y,\tau)\right] = 0 , \tag{19}$$

with coefficients (16) $k_1 = k_{10}$, $k_2 = k_{20}$ under initial condition

$$W(y, T_1) = \frac{1}{\sqrt{2\pi\lambda_0}} \exp\left(-\frac{(y + \lambda_0/2)^2}{2\lambda_0} \right)$$

and boundary conditions

$$W(-\infty, \tau) = W(h, \tau) = 0 .$$

Solving the FPK equation by the reflection technique with the sign variation [8-10], putting found solution in (18), and then (18) in (17), we get a formula for false-alarm probability

$$\begin{aligned} \alpha = 1 - \frac{1}{\sqrt{2\pi\lambda_0}} \int_0^\infty \exp\left(-\frac{(h - \xi + \lambda_0/2)^2}{2\lambda_0} \right) \times \\ \times \left[\Phi\left(\frac{1}{2}\sqrt{\frac{\psi_0 (T_2 - T_1)}{T_2}} + \frac{\xi}{\sqrt{\psi_0 (T_2 - T_1)/T_2}} \right) - \right. \\ \left. - e^{-\xi}\Phi\left(\frac{1}{2}\sqrt{\frac{\psi_0 (T_2 - T_1)}{T_2}} - \frac{\xi}{\sqrt{\psi_0 (T_2 - T_1)/T_2}} \right) \right] d\xi, \end{aligned} \tag{20}$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-t^2/2\right) dt$ – error function integral.

Let us then find a formula for conditional the missing probability a signal, for which we will research determinant statistics $L_1(\tau)$ near the maximum position of its mathematical expectation(10) $\tau_s = \arg\sup S_1(\tau)$. The derivative of mathematical expectation (10) of determinant statistics (9) looks like

$$\frac{\partial S_1(\tau)}{\partial \tau} = \frac{a_0^2 \Delta_a}{N_0} \left[\left(\cos\left(\varphi^* - \varphi_0\right) - \frac{\Delta_a}{2} \right) + \right.$$
$$\left(\cos\left(\varphi^* + \varphi_0\right) - \Delta_a \cos\left(2\varphi^*\right)/2 \right) \cos(2\omega\tau) +$$
$$\left. + \left(\sin\left(\varphi^* + \varphi_0\right) - \Delta_a \sin\left(2\varphi^*\right)/2 \right) \sin(2\omega\tau) \right], \quad \tau < \tau_0,$$

$$\frac{\partial S_1(\tau)}{\partial \tau} = -a_0^2 \Delta_a^2 \cos^2\left(\omega\tau - \varphi_0 - \Delta_\varphi\right)/N_0, \quad \tau \geq \tau_0.$$

We will consider then combinations of expected and true values of amplitude and initial phase, in which the maximum position of mathematical expectation (10) coincides with the true value of unknown duration, so that $\tau_s = \tau_0$. Let us decompose functions (10) and (12) in Taylor's series by τ in the neighborhood of τ_0, we will get asymptotic expressions for mathematical expectation in the presence of signal

$$S_1(\tau) \approx \frac{\lambda_1}{2} + \frac{\tau - \tau_0}{2T_2} \begin{cases} \psi_1, & \tau \leq \tau_0, \\ -\psi_2, & \tau > \tau_0, \end{cases} \tag{21}$$

and correlation function

$$K_{q1}(\tau_1, \tau_2) \approx \lambda_1 + \psi_2 \min(\tau_1 - \tau_0, \tau_2 - \tau_0)/T_2, \tag{22}$$

where

$$\psi_1 = z^2 \Delta_a \left[\left(\cos\left(\Delta_\varphi\right) - \Delta_a/2 \right) + \cos(2\omega\tau_0) \times \right.$$
$$\times \left(\cos\left(2\varphi_0 + \Delta_\varphi\right) - \Delta_a \cos\left(2\varphi_0 + 2\Delta_\varphi\right)/2 \right) +$$
$$\left. + \left(\sin\left(2\varphi_0 + \Delta_\varphi\right) - \Delta_a \sin\left(2\varphi_0 + 2\Delta_\varphi\right)/2 \right) \sin(2\omega\tau_0) \right],$$
$$\psi_2 = \Delta_a^2 z^2 \cos^2\left(\omega\tau_0 - \varphi_0 - \Delta_\varphi\right),$$
$$\lambda_1 = z^2 \Delta_a \frac{\tau_0}{T_2} \left[\cos\left(\Delta_\varphi\right) - \frac{\Delta_a}{2} + \frac{\sin 2\omega\tau_0}{2\omega\tau_0} \times \right.$$
$$\times \left(\cos\left(2\varphi_0 + \Delta_\varphi\right) - \frac{\Delta_a}{2} \cos\left(2\varphi_0 + 2\Delta_\varphi\right) \right) +$$
$$\left. + \frac{1 - \cos 2\omega\tau_0}{2\omega\tau_0} \left(\sin\left(2\varphi_0 + \Delta_\varphi\right) - \frac{\Delta_a}{2} \sin\left(2\varphi_0 + 2\Delta_\varphi\right) \right) \right].$$

We will approximate the logarithm LRF $L_1(\tau)$ by Gaussian random process $\mu_1(\tau)$ with mathematical expectation (21) and correlation function (22). Such approximation makes sense for all $\tau > \tau_d = \tau_0 - T_2 \lambda_1/\psi_2$, under which dispersion of a random process $\mu_1(\tau)$ is nonnegative, that is $K_{q1}(\tau, \tau) \approx \lambda_1 + \psi_2(\tau - \tau_0)/T_2 \geq 0$. Using approximation $\mu_1(\tau)$, we will consider that duration takes the value from the prior interval $[T_d, T_2]$, where $T_d = \max(\tau_d, T_1)$. Using formulas (21), (22) and Doob's theorem [8,9], it can be shown that the determinant statistics $\mu_1(\tau)$ is Gaussian Markov process with drift coefficient k_{11} and diffusion coefficient k_{21} [9,10]

$$k_{11} = \frac{1}{2T_2} \begin{cases} \psi_1, & T_d \leq \tau \leq \tau_0, \\ -\psi_2, & \tau_0 < \tau \leq T_2, \end{cases} \quad k_{21} = \frac{\psi_2}{T_2}. \tag{23}$$

The missing probability a signal is by definition

$$\beta = F_1(h) = P\left\{ \mu_1(\tau) < h, \ \tau \in [T_d, T_2] \right\} \tag{24}$$

the probability's of failure to achieve borders $y = -\infty$ and $y = h$ by Markov random process $\mu_1(\tau)$ at the interval $\tau \in [T_d, T_2]$. The searched probability (24) can be expressed through probability density $W(y, \tau)$ of random process realizations $\mu_1(\tau)$, which have never achieved borders $y = -\infty$, $y = h$ [9]

$$F_1(h) = \int_{-\infty}^{h} W(y, T_2)\, dy. \tag{25}$$

Function $W(y, \tau)$ is the solution to the FPK equation (19) with coefficients (23) under initial condition

$$W(y, T_d) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y - m)^2}{2\sigma^2} \right)$$

and boundary conditions

$$W(-\infty, \tau) = W(h, \tau) = 0,$$

where $\sigma^2 = \lambda_1 + \psi_2(T_d - \tau_0)/T_2$, $m = \lambda_1/2 + \psi_1(T_d - \tau_0)/2T_2$.

Solving the FPK equation by the reflection method with a change in sign [8-10], putting found solution in (25), and then (24), we get a formula for missing probability a signal

$$\beta = \frac{\exp\left(-\psi_1^2\left(\tau_0 - T_d\right)/8\psi_2 T_2\right)}{\sqrt{2\pi\psi_2\left(\tau_0 - T_d\right)/T_2}} \int_0^\infty\int_0^\infty W\left(h-\xi, T_d\right)\times$$
$$\times\exp\left(\frac{\psi_1}{2\psi_2}\left(\xi-\xi_1\right)\right)\left\{\Phi\left(\frac{1}{2}\sqrt{\psi_2\left(1-\frac{\tau_0}{T_2}\right)}+\frac{\xi_1}{\sqrt{\psi_2\left(1-\frac{\tau_0}{T_2}\right)}}\right)-\right. \quad (26)$$
$$\left.-\exp\left(-\xi_1\right)\Phi\left(\frac{1}{2}\sqrt{\psi_2\left(1-\frac{\tau_0}{T_2}\right)}-\frac{\xi_1}{\sqrt{\psi_2\left(1-\frac{\tau_0}{T_2}\right)}}\right)\right\}\times$$
$$\times\left\{\exp\left(-\frac{T_2\left(\xi-\xi_1\right)^2}{2\psi_2\left(\tau_0-T_d\right)}\right)-\exp\left(-\frac{T_2\left(\xi+\xi_1\right)^2}{2\psi_2\left(\tau_0-T_d\right)}\right)\right\}d\xi d\xi_1.$$

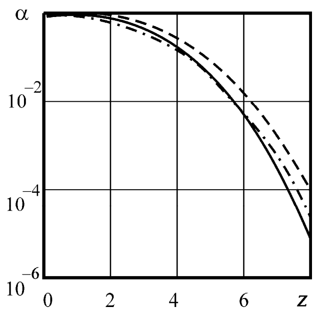

Fig. 2. The dependence of false-alarm probability on SNR

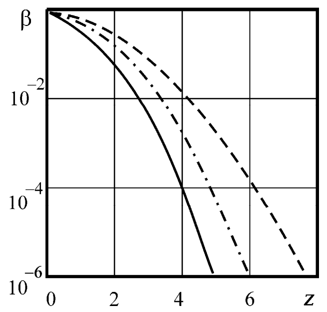

Fig. 3. The dependence of missing probability a signal on SNR

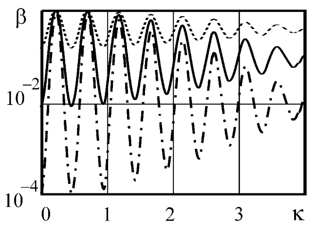

Fig. 4. The dependence of the missing probability a signal on parameter of narrowbandness

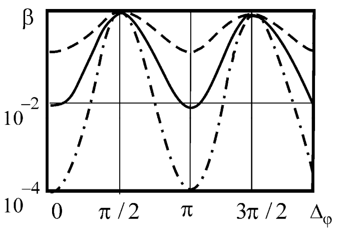

Fig. 5. The dependence of the missing probability a signal on detuning of initial phase of received signal

Fig.2 depicts dependencies of the false-alarm probability (20) from SNR (15) under different values of the parameter of narrowbandness $\kappa = \omega\tau_0/2\pi$, which is equal to the number of periods of harmonic oscillations (1), settling at the signal duration τ_0. Dashed-line curve corresponds to the parameter of narrowbandness $\kappa = 0.4$, solid-line curve $\kappa = 0.6$, and dash-dotted line – $\kappa = 0.8$. Fig.3 shows dependencies of the missing probability a signal (26) from SNR (15) under different values of the narrowbandness parameter: solid-line curve corresponds to the narrowbandness parameter $\kappa = 0.4$, dashed-line curve – $\kappa = 0.6$, and dash-dotted line – $\kappa = 0.8$. In the calculation of lines at fig.2 and 3 it was supposed that initial phase of received signal $\varphi_0 = 0$, threshold $h = 0$, $T_2/T_1 = 4$, and there are no detunings of amplitude $\Delta_a = 1$ and initial phase $\Delta_\varphi = 0$. Fig.4 and 5 depict dependencies of the missing probability a signal (26) under different values of the SNR (16) from the parameter of narrowbandness and detuning of initial phase, respectively. Dashed-line curve correspond to SNR $z = 3$, solid-line ones – $z = 5$, and dash-dotted lines – $z = 7$. In the calculation of lines of fig.4 it was supposed that initial phase of received signal $\varphi_0 = 0$, threshold $h = 0$, $T_2/T_1 = 4$, there are no detunings of amplitude $\Delta_a = 1$ and initial phase $\Delta_\varphi = 0$, and for lines at fig. 5 – $\varphi_0 = 0$, $h = 0$, $T_2/T_1 = 4$ and $\kappa = 0.5$.

V. Conclusion

As the figures show, the parameter of narrowbandness κ makes huge impact at the quality of detection of ultra-wideband quasi-radiosignal. With the growth of κ the impact on detection characteristics lowers, which is confirmed by a private case of ultra-wideband quasi-radiosignal – narrowband radiosignal ($\kappa >> 1$), characteristics of detection of which do not depend on number of periods of harmonic oscillations at the interval of its duration [1-3]. Ignorance of the parameter of narrowbandness, amplitude or initial phase (presence of detunings) lead to the growth of underflow errors. Thus, in practical applications it is appropriate to use such ultra-wideband quasi-radiosignals, for which the number of periods of harmonic oscillations at the interval of duration corresponds to the lowest error probabilities.

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