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The Duration Estimate of the Missing Signal with the Unknown Amplitude

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Abstract

We synthesize the quasi-likelihood and maximum likelihood algorithms for estimating the duration of the free-form signal with the unknown amplitude. That the signal may be missing in the received realization of the observable data is taken into account. The characteristics of the introduced algorithms are found.

Keywords: Missing signal, Quasi-likelihood estimate, Maximum likelihood estimate, Signal duration, Signal amplitude, Estimate characteristics, Joint detection and estimation.

INTRODUCTION

The estimate of the duration of the signal observed against noise is important for many practical applications referring to communication and location theory, monitoring, seismology, etc. This problem is considered in a number of studies [1-4], where some optimal and quasi-optimal algorithms for the estimation of the free-form signal duration are introduced. The signal amplitude is often unknown, as well as the duration. The estimate of the duration of the free-form signal form with unknown amplitude is considered in paper [5], and the amplitude estimate of the signal with unknown duration – in paper [6]. However, the case of information transferred through an unstable communication channel, i.e. when the signal loss may occur is also of great importance. If the presence of the useful signal is not required, then we proceed to estimating the duration of the missing signal with unknown amplitude. In paper [7], the maximum likelihood and Bayesian algorithms are synthesized for the estimate of the duration of the missing free-form signal with the known amplitude. The algorithms for the estimate of the duration of the missing signal with unknown amplitude are considered below.

THE PROBLEM STATEMENT

Let the additive mix

$$\xi(t)=\gamma_0 s(t,a_0,\tau_0)+n(t) \tag{1}$$

of the useful signal

$$s(t,a_0,\tau_0)=\begin{cases} a_0 f(t), & 0\le t\le\tau_0 \\ 0, & t<0,\ t>\tau_0 \end{cases} \tag{2}$$

and Gaussian white noise $n(t)$ with the one-sided spectral density N_0 is observed over the interval $[0,T]$. Here a_0, τ_0 are the unknown signal amplitude and duration, correspondently, and $f(t)$ is a priori known continuous bounded function describing the signal form. We presuppose that the signal duration possesses the values from the prior interval

$$\tau\in[\mathrm{T}_1,\mathrm{T}_2], \tag{3}$$

while the signal amplitude is a spurious parameter and does not need to be estimated. Let the useful signal be present with the probability less than $p_1<1$ in the realization of the observable data (1). Then the discrete parameter γ_0 can take the value $\gamma_0=1$ (the signal is present) with the probability p_1 and the value $\gamma_0=0$ (the signal is not present) with the probability $p_0=1-p_1$. Based on the observable realization and the available prior information, it is necessary to form the estimate of the duration of the useful signal (2) taking the parameters a_0 and γ_0 as the spurious ones.

THE SYTHESIS OF THE ESTIMATION ALGORITHMS

To synthesize the algorithm for the duration estimation, we use a maximum likelihood (ML) method [1, 8, 9]. According to the specified conditions, the logarithm of the functional of the likelihood ratio (FLR) depends on the three unknown parameters:

$$L(a,\tau,\gamma)=\frac{2a\gamma}{N_0}\int_0^{\tau}\xi(t)f(t)\,dt-\frac{a^2\gamma^2}{N_0}\int_0^{\tau}f^2(t)\,dt\,. \quad (4)$$

If the useful signal (2) is present in the received realization (1) with the probability $p_1=1$, then the ML duration estimate is determined as the position of the absolute (greatest) maximum of the logarithm of FLR:

$$\tau_q=\arg\sup L(\tau), \quad (5)$$

where the designations are:

$$L(\tau)=\sup_a L(a,\tau), \quad L(a,\tau)=L(a,\tau,\gamma=1)\,. \quad (6)$$

The algorithm (5) does not account for the possible signal missing, therefore, under $p_1<1$, the estimate τ_q is not the maximum likelihood one. For all $p_1<1$ the estimate (5) is quasi-likelihood (QL) one [1, 9].

In case the useful signal may be absent, the ML approach is applied to find the position of the absolute maximum of the logarithm of FLR (4), while the current values of the spurious parameters a and γ are changed by their ML estimates:

$$\tau_m=\arg\sup_{\tau}\left\{\sup_{a,\gamma}L(a,\tau,\gamma)\right\}. \quad (7)$$

From Eq. (4), we see that $L(a,\tau,\gamma=0)=0$. Therefore, the algorithm (7) produces the following rule:

$$\tau_m=\begin{cases}\tau_q\,, & L(\tau_q)>0\,,\\ 0\,, & L(\tau_q)\le 0\,.\end{cases} \quad (8)$$

Similarly to [1, 2], we introduce the certain threshold h and the generalized ML duration estimate:

$$\tau_m=\begin{cases}\tau_q\,, & L(\tau_q)>h\,,\\ 0\,, & L(\tau_q)\le h\,.\end{cases} \quad (9)$$

It should be also noted that the QL estimate (5) is a special case of the generalized ML estimate (9), when $h=-\infty$.

According to Eq. (9), to find the ML duration estimate of the missing signal, there must be found the magnitude L_q and the position τ_q of the absolute maximum of the logarithm of FLR $L(\tau)$ (6). Determining the magnitude of the absolute maximum of the logarithm of FLR, we can write down

$$L_q=\sup_{\tau}L(\tau), \qquad L(\tau)=\sup_a L(a,\tau)=L(a_q(\tau),\tau),$$

$$a_q(\tau)=\arg\sup_a L(a,\tau)\,.$$

The maximization of the logarithm of FLR by the amplitude can be analytically implemented. For this purpose, we set the derivative of function $L(a,\tau)$ by the variable a equal to zero:

$$\frac{\partial L(a,\tau)}{\partial a}=\frac{2}{N_0}\int_0^{\tau}\xi(t)f(t)\,dt-\frac{2a}{N_0}\int_0^{\tau}f^2(t)\,dt=0$$

and solve the specified equation relative to the variable a:

$$a_q(\tau)=\int_0^{\tau}\xi(t)f(t)\,dt\Bigg/\int_0^{\tau}f^2(t)\,dt \quad (10)$$

By substituting the amplitude (10) into the expression (6), we get

$$L(\tau)=\left[\int_0^{\tau}\xi(t)f(t)\,dt\right]^2\Bigg/N_0\int_0^{\tau}f^2(t)\,dt\,. \quad (11)$$

We now see that the expression (11) is nonnegative for all τ from the prior interval (3) with the probability 1. Thus, $P[L_q\ge 0]=1$ and therefore the estimation algorithm (8) is the degenerate one. Then, after the maximization by the amplitude, it is necessary to apply the generalized ML estimation algorithm (9).

The expressions (5), (9) and (11) determine the meter structure. Its block diagram is presented in Fig. 1 where the designations are: I is the time interval $[0,t]$ integrator, where $t\in[\mathrm{T}_1,\mathrm{T}_2]$; E is the retriever searching position for the input signal maximum within time interval $[\mathrm{T}_1,\mathrm{T}_2]$ (extremator). By the dashed line, the scheme is selected, the one forming the QL estimate (5). To implement the ML algorithm (9), the logarithm of FLR as the function of current time is passed to the peak detector PD, its output signal being the magnitude of the maximum of the input signal L_q. The threshold device (TD) compares the maximum value of L_q with the threshold h at the time $t=\mathrm{T}_2$ and operates the resolver (RS) which generates the ML duration estimate. Either this estimate coincides with the QL one, if the threshold is exceeded, or it is equal to zero, if the threshold is not exceeded.

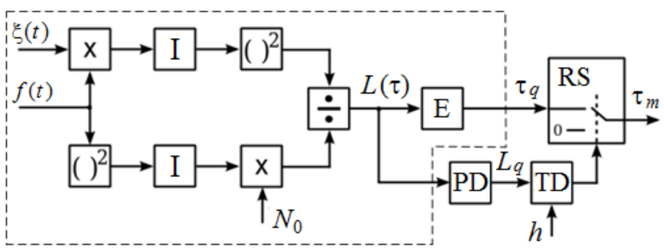

Figure 1: The block diagram of the duration meter

THE ANALYSIS OF THE ESTIMATION ALGORITHMS

Let us conduct the analysis of the synthesized algorithms for the duration estimation. We designate $L_j(\tau)=L(\tau|\gamma_0=j)$ as the decision statistics (11) with ($j=1$), or without ($j=0$) the useful signal in the received realization. By substituting the realization (1) in Eq. (11), we get

$$L_0(\tau)=\left[\int_0^\tau n(t)f(t)\,dt\right]^2 \Bigg/ N_0\int_0^\tau f^2(t)\,dt\,, \qquad (12)$$

$$L_1(\tau)=\left[a_0\int_0^{\min(\tau,\tau_0)} f^2(t)\,dt+\int_0^\tau n(t)f(t)\,dt\right]^2 \Bigg/ N_0\int_0^\tau f^2(t)\,dt\,. \qquad (13)$$

According to [1], the probability density of the ML duration estimate (9) can be presented in the form of

$$w(\tau)=p_0\left[\,w_0(\tau)+(1-\alpha)\delta(\tau)\,\right]+p_1\left[\,w_1(\tau)+\beta(\tau_0)\delta(\tau)\,\right]. \qquad (14)$$

Here the designations are:

$$w_j(\tau)=\int_h^\infty w_j(A,\tau)\,dA\,,$$

$w_j(A,\tau)$ is the joint probability density of the magnitude and the position of the absolute maximum of the random process $L_j(\tau)$, $j=0,1$;

$$\alpha=P[\sup L_0(\tau)>h]=\int_{T_1}^{T_2} w_0(\tau)\,d\tau=\int_{T_1}^{T_2}\int_h^\infty w_0(A,\tau)\,dA\,d\tau \qquad (15)$$

is the false alarm probability;

$$\begin{aligned}\beta(\tau_0)&=P[\sup L_1(\tau)<h]=\\&=1-\int_{T_1}^{T_2} w_1(\tau)\,d\tau=1-\int_{T_1}^{T_2}\int_h^\infty w_1(A,\tau)\,dA\,d\tau\end{aligned} \qquad (16)$$

is the missing probability. Under $h=-\infty$, the probability density (14) takes the form of

$$w(\tau)=p_0 w_0(\tau)+p_1 w_1(\tau),$$

where $w_1(\tau)$ is the probability density of the QL estimate (5), in the presence of the signal (2) in the received realization (1), and $w_0(\tau)$ is the probability density of the QL estimate (5) (pseudo-estimate) in the absence of the signal (2).

Now we consider the statistical properties of the decision statistics (11). Similarly to [5, 10], we introduce the auxiliary random process

$$M(\tau)=\int_0^\tau \xi(t)f(t)\,dt\,. \qquad (17)$$

This process is the Gaussian one with the mathematical expectation

$$S_M(\tau)=\langle M(\tau)\rangle=a_0\gamma_0\int_0^{\min(\tau,\tau_0)} f^2(t)\,dt=\frac{\gamma_0 N_0 q(\tau_0)}{2a_0}\min\left[1,\frac{q(\tau)}{q(\tau_0)}\right]$$

and the covariance function

$$\begin{aligned}K_M(\tau_1,\tau_2)&=\langle[M(\tau_1)-\langle M(\tau_1)\rangle][M(\tau_2)-\langle M(\tau_2)\rangle]\rangle=\\&=\frac{a_0N_0}{2}\int_0^{\min(\tau_1,\tau_2)} f^2(t)\,dt=\frac{N_0^2 q(\tau_0)}{4a_0^2}\min\left[\frac{q(\tau_1)}{q(\tau_0)},\frac{q(\tau_2)}{q(\tau_0)}\right].\end{aligned}$$

Here

$$q(\tau)=\frac{2a_0^2}{N_0}\int_0^\tau f^2(t)\,dt \qquad (18)$$

is the signal-to-noise ratio (SNR) at the ML receiver output for the signal with the duration τ.

In Eq. (17), we pass to the new variable $l=q(\tau)/q(\tau_0)$, $l\in[L_1,L_2]$, $L_1=q(T_1)/q(\tau_0)$, $L_2=q(T_2)/q(\tau_0)$. Then, for the random process (17), as the function of the variable l we can write down the following:

$$M(\tau)=M[\tau(l)]=\chi(l)=\frac{N_0 q(\tau_0)}{2a_0}\left[\gamma_0\min(1,l)+\frac{\omega(l)}{\sqrt{q(\tau_0)}}\right].$$

Here $\tau(l)$ is determined from the solution of equation $q(\tau)/q(\tau_0)=l$, and $\omega(l)$ is the standard Wiener process [11]. By applying the random process (17), the decision statistics (11) as the function of the variable l can be presented in the form of

$$L(l)=\frac{2a_0^2\chi^2(l)}{N_0^2 q(\tau_0)l}=\frac{z_0^2}{2l}\left[\min(1,l)+\frac{\omega(l)}{z_0}\right]^2,$$

where $z_0^2=q(\tau_0)$ is the SNR at the receiver output for the received signal.

Let us consider now the case when the signal is absent in the received realization. We express the probability density $w_0(A,\tau)$ of the magnitude and the position of the maximum of the random process $L_0(\tau)$ (12) through the probability density $w_{l0}(A,\tau)$ of the magnitude and the position of the maximum of the random process

$$L_0(l)=\omega^2(l)/2l \qquad (19)$$

as follows

$$w_0(A,\tau)=\frac{1}{q(\tau_0)}w_{l0}\left[A,\frac{q(\tau)}{q(\tau_0)}\right]\frac{dq(\tau)}{d\tau}. \qquad (20)$$

In turn, the probability density $w_{l0}(A,x)$ can be presented in the form of [1]

$$w_{l0}(A,x)=\left.\frac{\partial^2 F_{20l}(u,v,x)}{\partial u\partial x}\right|_{u=v=A}. \qquad (21)$$

Here

$$F_{20l}(u,v,x)=P\left[\underset{L_1\le l\le x}{L_0(l)<u},\ \underset{x<l\le L_2}{L_0(l)<v}\right]$$

is the two-dimensional distribution function of the magnitude of the maximum of the random process (19).

In Eq. (19), we carry out another change of variables: $m=\ln(l/L_1)$, $m\in[0,\tilde m]$, $\tilde m=\ln(L_2/L_1)$. Under it, the random process $\omega(l)/\sqrt{l}$ possesses the covariance function

$$\langle\omega(l_1)\omega(l_2)\rangle/\sqrt{l_1 l_2}=\exp(-|m_2-m_1|/2).$$

Therefore, the random process $\mathrm{X}(m)=\omega(l)/\sqrt{l}$, as the function of the variable *m*, is Gaussian Markov stationary random process. According to [11], it satisfies the stochastic differential equation

$$d\mathrm{X}(m)=-\mathrm{X}(m)\,dm/2+d\omega(m),$$

written down in the symmetrized form. We multiply the last equation by $\mathrm{X}(m)$ and, taking account that $dL_0(m)=\mathrm{X}(m)\,d\mathrm{X}(m)$, $\mathrm{X}(m)=\sqrt{2L_0(m)}$, we now get the stochastic differential equation for the decision statistics $L_0(m)$:

$$dL_0(m)=-L_0(m)\,dm+\sqrt{2L_0(m)}\,d\omega(m).$$

This equation coincides with the similar equation studied in [1], where the approximate expression is found for the two-dimensional distribution function of the maximum value of the random process $L_0(m)$ in the form of

$$F_{20m}(u,v,y)=P\left[\underset{0\le m\le y}{L_0(m)<u},\underset{y<m\le\tilde m}{L_0(m)<v}\right]\approx P_1(u,y)P_2(v,y). \qquad (22)$$

$$P_1(u,y)=P\left[\underset{0\le m\le y}{L_0(m)<u}\right]\approx\begin{cases}\exp\left[-y\sqrt{u/\pi}\exp(-u)\right], & u\ge 1/2,\\ 0, & u<1/2,\end{cases}$$

$$P_2(v,y)=P\left[\underset{y\le m\le\tilde m}{L_0(m)<v}\right]\approx\begin{cases}\exp\left[-(\tilde m-y)\sqrt{v/\pi}\exp(-v)\right], & v\ge 1/2,\\ 0, & v<1/2.\end{cases}$$

The probability density

$$w_{m0}(A,y)=\left.\frac{\partial^2 F_{20m}(u,v,y)}{\partial u\partial y}\right|_{u=v=A} \qquad (23)$$

is associated with the probability density (21) by the relation

$$w_{l0}(A,x)=w_{m0}[A,\ln(x/L_1)]L_1/x. \qquad (24)$$

By substituting the function (22) into the formula (23), and then Eq. (23) into Eq. (24) and Eq. (24) into Eq. (20), with subsequent integration from *h* up to ∞, we get

$$\begin{aligned}w_0(\tau)&=\int_h^\infty w_0(A,\tau)\,d\tau=\\&=\frac{\alpha}{\ln[q(\mathrm{T}_2)/q(\mathrm{T}_1)]}\frac{dq(\tau)}{d\tau}\begin{cases}1/q(\tau), & \mathrm{T}_1\le\tau\le\mathrm{T}_2,\\ 0, & \tau<\mathrm{T}_1,\ \tau>\mathrm{T}_2,\end{cases}\end{aligned} \qquad (25)$$

where

$$\alpha=\begin{cases}1-[q(\mathrm{T}_1)/q(\mathrm{T}_2)]^{\sqrt{h/\pi}\exp(-h)}, & h>1/2,\\ 1, & h\le 1/2\end{cases} \qquad (26)$$

is the false alarm probability (15).

Let us find now the probability density $w_1(A,\tau)$. Similarly to Eq. (20), we express it through the probability density $w_{l1}(A,l)$ of the magnitude and the position of the maximum of the random process

$$L_1(l)=\frac{z_0^2\min^2(1,l)}{2l}+\frac{z_0\min(1,l)}{l}\omega(l)+\frac{\omega^2(l)}{2l} \qquad (27)$$

as follows

$$w_1(A,\tau)=\frac{1}{q(\tau_0)}w_{l1}\left[A,\frac{q(\tau)}{q(\tau_0)}\right]\frac{dq(\tau)}{d\tau}. \qquad (28)$$

Under great SNR, we can neglect the last summand in Eq. (27) and write down the next expression approximately

$$L_1(l)\approx\frac{z_0^2\min^2(1,l)}{2l}+\frac{z_0\min(1,l)}{l}\omega(l). \qquad (29)$$

In Eq. (29), we carry out the change of variables $\lambda=q(\tau_0)l=z_0^2 l$. The value λ possesses the values from the interval $[\Lambda_1,\Lambda_2]$, where $\Lambda_1=q(\mathrm{T}_1)$, $\Lambda_2=q(\mathrm{T}_2)$, $\lambda_0=q(\tau_0)=z_0^2$. Then, we write down the decision statistics (29) as the function of the variable λ in the form of

$$\begin{aligned}L_1(l)&=L_1[l(\lambda)]=L_1(\lambda/z_0^2)=\mu(\lambda)=\\&=\min^2(\lambda,\lambda_0)/2\lambda+\min(\lambda,\lambda_0)\omega(\lambda)/\lambda.\end{aligned} \qquad (30)$$

This function is Gaussian random process with the mathematical expectation

$$S(\lambda)=\min^2(\lambda,\lambda_0)/2\lambda \tag{31}$$

and the covariance function

$$K(\lambda_1,\lambda_2)=\min(\lambda_1,\lambda_0)\min(\lambda_2,\lambda_0)\min(\lambda_1,\lambda_2)/\lambda_1\lambda_2 .$$

The correlation coefficient $R(\lambda_1,\lambda_2)=\min(\lambda_1,\lambda_2)/\sqrt{\lambda_1\lambda_2}$ of the decision statistics (30) satisfies the condition $R(x,y)=R(x,t)R(t,y)$, $x>t>y$ [8, 11]. Therefore, the random process (30) is the Markov one with drift and diffusion coefficients [8, 11]

$$k_1(\lambda)=\frac{1}{2}\begin{cases}1, & \lambda\le\lambda_0 ,\\ -\lambda_0^2/\lambda^2 , & \lambda>\lambda_0 ,\end{cases}\quad k_2(\lambda)=\begin{cases}1, & \lambda\le\lambda_0 ,\\ \lambda_0^2/\lambda^2 , & \lambda>\lambda_0 .\end{cases} \tag{32}$$

If SNR is big enough, then the position of the maximum of the decision statistics (30) is located in the neighborhood of the position of the maximum of its mathematical expectation [1]. The mathematical expectation (31) reaches the maximum value under $\lambda=\lambda_0$. We introduce the value $\varepsilon=(\lambda-\lambda_0)/\lambda_0$ which absolute value decreases with increasing SNR $\lambda_0=z_0^2$, and then rewrite the drift and diffusion coefficients (32) in the form of

$$k_1(\lambda)=\frac{1}{2}\begin{cases}1, & \lambda\le\lambda_0 ,\\ -(1+\varepsilon)^{-2} , & \lambda>\lambda_0 ,\end{cases}\quad k_2(\lambda)=\begin{cases}1, & \lambda\le\lambda_0 ,\\ (1+\varepsilon)^{-2} , & \lambda>\lambda_0 .\end{cases}$$

As $\varepsilon\to 0$ under $z_0\to\infty$, in the neighborhood of point $\lambda=\lambda_0$ the decision statistics (30) can be approximated by Gaussian Markov random process $\mu(\lambda)$ with drift and diffusion coefficients

$$k_1(\lambda)=\frac{1}{2}\begin{cases}1, & \lambda\le\lambda_0 ,\\ -1, & \lambda>\lambda_0 ,\end{cases}\qquad k_2(\lambda)=1 . \tag{33}$$

We use this approximation within all the interval of the possible values of the parameter $\lambda\in[\Lambda_1,\Lambda_2]$. Between the variables λ and τ there is one-to-one relation $\lambda=q(\tau)$. Therefore, the probability density $w_1(A,\tau)$ (28) of the magnitude and the position of the absolute maximum of the random process $L_1(\tau)$ (13) can be expressed through the probability density $w_{\lambda 1}(A,\lambda)$ of the magnitude and the position of the absolute maximum of the random process $\mu(\lambda)$ (30). Namely,

$$w_1(A,\tau)=w_{\lambda 1}[A,q(\tau)]\frac{dq(\tau)}{d\tau} . \tag{34}$$

Similarly to [1], we can write down

$$w_{\lambda 1}(A,x)=\left.\frac{\partial^2 F_{2\lambda}(u,v,x)}{\partial u\partial y}\right|_{u=v=A} , \tag{35}$$

where $F_{2\lambda}(u,v,x)$ refers to the two-dimensional distribution function of the absolute maxima of the random process $\mu(\lambda)$:

$$F_{2\lambda}(u,v,x)=P\left[\mu(\lambda)_{\Lambda_1\le\lambda\le x}<u,\ \mu(\lambda)_{x<\lambda\le\Lambda_2}<v\right].$$

It is noteworthy that under $\lambda=\Lambda_1$ the random variable $\mu(\Lambda_1)$ is described by the probability density

$$\left.w(y,\lambda)\right|_{\lambda=\Lambda_1}=\frac{1}{\sqrt{2\pi\Lambda_1}}\exp\left[-\frac{(y-\Lambda_1/2)^2}{2\Lambda_1}\right]. \tag{36}$$

The two-dimensional distribution function of the magnitude of the maximum of the Markov random process with drift and diffusion coefficients (33) for the initial condition (36) is found in [12]:

$$\begin{aligned}F_{2\lambda}(u,v,x)=&\int_0^\infty\!\!\int_0^\infty\left[\Phi\left(\frac{\sqrt{\Lambda_2-\lambda_0}}{2}+\frac{\xi_2}{\sqrt{\Lambda_2-\lambda_0}}\right)-\right.\\ &\left.-\exp(-\xi_2)\Phi\left(\frac{\sqrt{\Lambda_2-\lambda_0}}{2}-\frac{\xi_2}{\sqrt{\Lambda_2-\lambda_0}}\right)\right]\left[1-\exp\left(-\frac{2\xi_1\xi_2}{\lambda_0-x}\right)\right]\times\\ &\times\left\{\exp\left[-\frac{(v-\xi_1-x/2)^2}{2x}\right]\Phi\left[\frac{ux+\Lambda_1(\xi_1-v)}{\sqrt{x\Lambda_1(x-\Lambda_1)}}\right]-\right.\\ &\left.-\exp\left[u-\frac{(2u-v+\xi_1+x/2)^2}{2x}\right]\Phi\left[\frac{ux-\Lambda_1(\xi_1+2u-v)}{\sqrt{x\Lambda_1(x-\Lambda_1)}}\right]\right\}\times\\ &\times\exp\left[-\frac{(\xi_2-\xi_1+(\lambda_0-x)/2)^2}{2(\lambda_0-x)}\right]\frac{d\xi_1 d\xi_2}{2\pi\sqrt{x(\lambda_0-x)}} ,\qquad x<\lambda_0 ,\end{aligned}$$

(37)

$$\begin{aligned}F_{2\lambda}(u,v,x)=&\exp\left(\frac{u-v}{2}\right)\int_0^\infty\!\!\int_0^\infty\!\!\int_0^\infty\left[\Phi\left(\frac{\sqrt{\Lambda_2-x}}{2}+\frac{\xi_2}{\sqrt{\Lambda_2-x}}\right)-\right.\\ &\left.-\exp(-\xi_2)\Phi\left(\frac{\sqrt{\Lambda_2-x}}{2}-\frac{\xi_2}{\sqrt{\Lambda_2-x}}\right)\right]\left\{\exp\left[-\frac{(\xi_2-\xi_1)^2}{2(\lambda_0-\Lambda_1)}\right]-\right.\\ &\left.-\exp\left[-\frac{(\xi_2+\xi_1)^2}{2(\lambda_0-\Lambda_1)}\right]\right\}\exp\left(\frac{\xi+\xi_2-2\xi_1}{2}-\frac{x-\Lambda_1}{8}\right)\times\\ &\times\left\{\exp\left[-\frac{(\xi_1-\xi+u-v)^2}{2(x-\lambda_0)}\right]-\exp\left[-\frac{(\xi_1+\xi+u-v)^2}{2(x-\lambda_0)}\right]\right\}\times\\ &\times\exp\left[-\frac{(u-\xi_1+\Lambda_1/2)^2}{2\Lambda_1}\right]\frac{d\xi_1 d\xi_2}{2\pi\sqrt{2\pi\Lambda_1(x-\lambda_0)(\lambda_0-\Lambda_1)}} ,\quad x\ge\lambda_0 .\end{aligned}$$

Substituting Eqs. (37) into Eq. (35), and then Eq. (35) into Eq. (34), with subsequent integrating by variable *A* from *h* up to ∞, we find the probability density

$$w_{\lambda 1}(x) = \int_0^\infty \xi \exp\left[-\frac{(\xi + (\lambda_0 - x)/2)^2}{2(\lambda_0 - x)}\right] \times$$

$$\times \left[\Phi\left(\frac{\sqrt{\Lambda_2 - \lambda_0}}{2} + \frac{\xi}{\sqrt{\Lambda_2 - \lambda_0}}\right) - \exp(-x) \times\right.$$

$$\left.\times \Phi\left(\frac{\sqrt{\Lambda_2 - \lambda_0}}{2} - \frac{\xi}{\sqrt{\Lambda_2 - \lambda_0}}\right)\right] d\xi \times$$

$$\times \frac{1}{\pi\left[(x - \Lambda_1)(\lambda_0 - x)\right]^{3/2}} \int_0^\infty \xi_1 \Phi\left(\frac{\xi_1 - h + \Lambda_1/2}{\sqrt{\Lambda_1}}\right) \times$$

$$\times \exp\left[-\frac{(\xi_1 - (x - \Lambda_1)/2)^2}{2(x - \Lambda_1)}\right] d\xi_1, \quad x < \lambda_0,$$

$$w_{\lambda 1}(x) = \left[\sqrt{\frac{2}{\pi(\Lambda_2 - x)}} \exp\left(-\frac{\Lambda_2 - x}{8}\right) + \Phi\left(\frac{\sqrt{\Lambda_2 - x}}{2}\right)\right] \times$$

$$\times \frac{1}{2\pi(x - \lambda_0)^{3/2}\sqrt{\lambda_0 - \Lambda_1}} \int_0^\infty\int_0^\infty \xi_1 \Phi\left(\frac{\xi - h + \Lambda_1/2}{\sqrt{\Lambda_1}}\right) \times$$

$$\times \exp\left[-\frac{(\xi_1 - \xi + (\lambda_0 - \Lambda_1)/2)^2}{2(\lambda_0 - \Lambda_1)} - \frac{(\xi_1 + (x - \lambda_0)/2)^2}{2(x - \lambda_0)}\right] \times$$

$$\times \left[1 - \exp\left(-\frac{2\xi\xi_1}{\lambda_0 - \Lambda_1}\right)\right] d\xi d\xi_1, \quad x \geq \lambda_0.$$

This probability density is associated with the desired probability density $w_1(\tau)$ by the relation

$$w_1(\tau) = w_{\lambda 1}[q(\tau)][dq(\tau)/d\tau]. \tag{38}$$

According to [1], the missing probability (16) can be obtained from Eq. (37) as

$$\beta(\tau_0) = F_{21\lambda}(h, h, \Lambda_2) =$$

$$= \frac{1}{\sqrt{2\pi\lambda_0}} \int_0^\infty \exp\left[-\frac{(x + \lambda_0/2)^2 + h^2 - h\lambda_0}{2\lambda_0}\right] \times$$

$$\times \left[\exp\left(\frac{xh}{\lambda_0}\right) \Phi\left(h\sqrt{\frac{\lambda_0 - \Lambda_1}{\lambda_0 \Lambda_1}} + x\sqrt{\frac{\Lambda_1}{\lambda_0(\lambda_0 - \Lambda_1)}}\right) - \right.$$

$$\left. - \exp\left(-\frac{xh}{\lambda_0}\right) \Phi\left(h\sqrt{\frac{\lambda_0 - \Lambda_1}{\lambda_0 \Lambda_1}} - x\sqrt{\frac{\Lambda_1}{\lambda_0(\lambda_0 - \Lambda_1)}}\right)\right] \times \tag{39}$$

$$\left[\Phi\left(\frac{\sqrt{\Lambda_2 - \lambda_0}}{2} + \frac{x}{\sqrt{\Lambda_2 - \lambda_0}}\right) - \exp(-x) \times\right.$$

$$\left.\times \Phi\left(\frac{\sqrt{\Lambda_2 - \lambda_0}}{2} - \frac{x}{\sqrt{\Lambda_2 - \lambda_0}}\right)\right] dx.$$

By substituting the false alarm probability (26), the missing probability (39) and the probability densities (25), (38) into the expression (14), we get the probability density of the ML duration estimate.

The accuracy of the estimates (5), (9) can be also described by the conditional bias and variance that for the true duration value τ_0 are determined as [1]

$$b = p_0 b_0 + p_1[b_1 + \tau_0 \beta(\tau_0)], \; V = p_0 V_0 + p_1\left[V_1 + \tau_0^2 \beta(\tau_0)\right],$$

$$b_j = \int_{\mathrm{T}_1}^{\mathrm{T}_2} (\tau - \tau_0) w_j(\tau) d\tau, \quad V_j = \int_{\mathrm{T}_1}^{\mathrm{T}_2} (\tau - \tau_0)^2 w_j(\tau) d\tau.$$

As the QL algorithm (5) is the special case of the generalized ML algorithm (9) under $h = -\infty$, the characteristics of its performance can be obtained from the expressions (25) and (38) taking equal to $h = -\infty$ here. Then, for the probability densities of the QL duration estimate in the signal absence and presence we get

$$w_0(\tau) = \frac{1}{\ln[q(\mathrm{T}_2)/q(\mathrm{T}_1)]} \frac{dq(\tau)}{d\tau} \begin{cases} 1/q(\tau), & \mathrm{T}_1 \leq \tau \leq \mathrm{T}_2, \\ 0, & \tau < \mathrm{T}_1, \; \tau > \mathrm{T}_2, \end{cases}$$

$$w_1(\tau) = \frac{1}{2} \frac{dq(\tau)}{d\tau} \begin{cases} \Psi[\Delta q_1(\tau), \Delta q_2, \Delta q_3], & \tau \leq \tau_0, \\ \Psi[-\Delta q_1(\tau), \Delta q_3, \Delta q_2], & \tau > \tau_0. \end{cases}$$

Here it is designated: $\Delta q_1(\tau) = [q(\tau_0) - q(\tau)]/2$, $\Delta q_2 = [q(\tau_0) - q(\mathrm{T}_1)]/2$, $\Delta q_3 = [q(\mathrm{T}_2) - q(\tau_0)]/2$,

$$\Psi(y, y_1, y_2) = \frac{1}{2\sqrt{\pi}|y|^{3/2}} \left\{\frac{\exp[(y - y_1)/4]}{\sqrt{\pi(y_1 - y)}} + \Phi\left(\sqrt{\frac{y_1 - y}{2}}\right)\right\} \times$$

$$\times \int_0^\infty x \exp\left[-\frac{(x + y)^2}{4y}\right]\left[\Phi\left(\frac{y_2 + x}{\sqrt{2y_2}}\right) - \exp(-x)\Phi\left(\frac{y_2 - x}{\sqrt{2y_2}}\right)\right] dx.$$

PULSE WITH BEVEL TOP

We now specify the obtained expressions for the pulse with bevel top [13]. We write down the function describing the pulse shape as follows

$$f(t) = (1 + bt/\mathrm{T}_2) \Big/ \sqrt{1 + b + b^2/3} \tag{40}$$

where the b parameter determines the pulse top tilt. The multiplier $(1 + b + b^2/3)^{-1/2}$ is introduced to provide the independence of the energy value of the signal of the maximum duration from the pulse tilt. It allows us to compare the accuracy of the duration estimates of the signals with the different bevel top and identical energy. We calculate the function (18) with reference to the signal (40):

$$q(\tau) = \lambda = z_r^2 \eta \frac{1 + b\eta + b^2\eta^2/3}{1 + b + b^2/3}, \tag{41}$$

where $z_r^2 = 2a_0^2 \mathrm{T}_2 / N_0$ is the SNR for the rectangular pulse with the amplitude a_0 and the duration T_2; $\eta = \tau/T_2$ is the normalized duration, while $\eta \in [1/k, 1]$, where $k = T_2/T_1$ is the dynamic range of the unknown duration variation.

In Figs. 2, there are presented the dependences of the normalized variances V/τ_0^2 of the QL (5) and ML (9) duration estimates of the pulse (40) upon SNR z_r (41). By the dashed line the variance of the QL estimate is shown and by continuous lines – the variance of the ML estimate. The threshold h is calculated by Neumann-Pirson criterion in terms of the condition $\alpha(h) = p$, where α is determined by the expression (26). Curves 1 are plotted for $p = 10^{-1}$, curves 2 – for $p = 10^{-2}$, curves 3 – for $p = 10^{-3}$. For this calculation, it is presupposed that $k = 10$, $b = 9$ and the true duration value is found in the middle of the prior interval: $\tau_0 = (T_1 + T_2)/2$, $\eta_0 = (k+1)/2k$. In Fig 2a the dependences are constructed under $p_0 = 0.7$ and in Fig. 3 – under $p_0 = 0.3$.

As it can be seen from Figs. 2, the QL estimation algorithm (5), without taking into account that the possible signal missing, loses in accuracy to the ML algorithm (9), especially under the great SNR. The loss in accuracy of the QL estimate in relation to the ML estimate increases with the probability of signal absence. Nevertheless, as it follows from Fig. 2b, in the case of the low SNR and the probability p_0 of the signal missing, the QL estimate can provide a little gain in accuracy in comparison with the ML estimate.

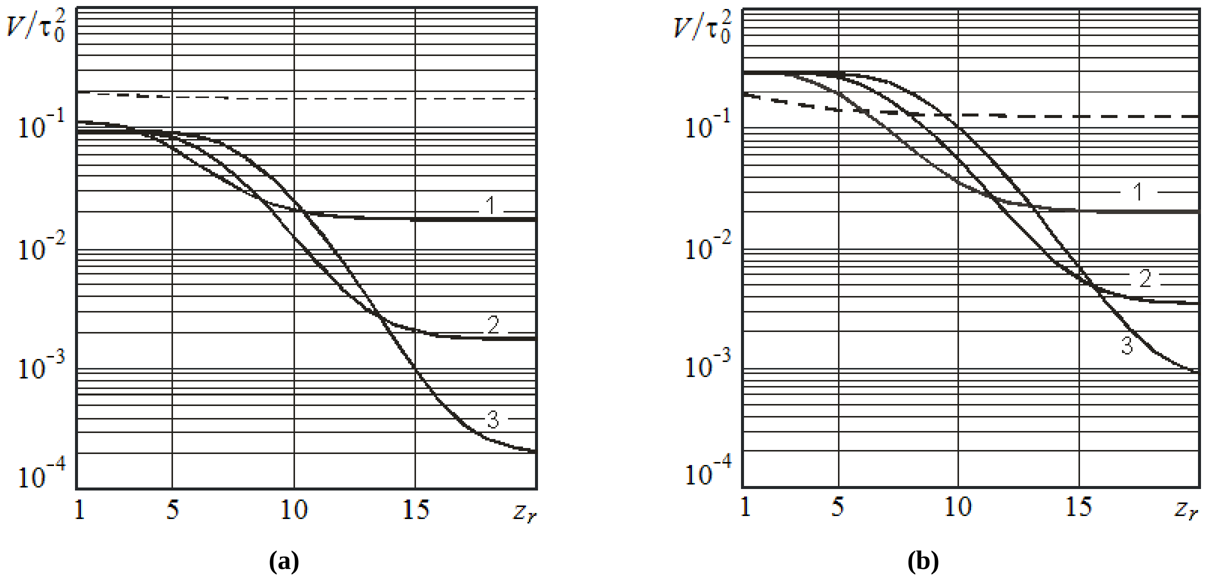

Figure 2: Normalized variances of the QL and ML pulse duration estimates.

CONCLUSION

On the basis of the conducted statistical analysis of the algorithms for the processing of the missing signal with unknown duration and amplitude, there can be evaluated an influence of a prior ignorance about signal presence or absence upon the accuracy of the duration estimate. The obtained results allow us to make the informed choice of the desired estimation algorithm depending on the requirements for the measurer implementation simplicity and the accuracy of estimate. By the example of the reception of the pulse with bevel top, it is shown that the accuracy of the quasi-likelihood duration estimate can essentially yield to the accuracy of the appropriate maximum likelihood estimate. Besides, the maximum likelihood estimation takes into account the possible signal missing in the received realization, which means that in fact it is a variant of the joint detection and estimation algorithm.

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