DOI 10.1007/s11141-019-09942-5
*Radiophysics and Quantum Electronics, Vol. 61, No. 11, April, 2019*
*(Russian Original Vol. 61, No. 11, November, 2018)*

# DETECTION OF AN ULTRA-WIDEBAND QUASI RADIO SIGNAL WITH UNKNOWN DURATION AGAINST THE BACKGROUND OF WHITE NOISE

**Yu. É. Korchagin*** **and K. D. Titov**

UDC 621.321

*We develop the maximum likelihood algorithm for detecting an ultra-wideband quasi radio signal with an arbitrary shape and unknown amplitude, initial phase, and duration, which is observed against the background of additive Gaussian white noise. The structure and statistical characteristics of this algorithm are found. The influence of a priori ignorance of the duration of a quasi radio signal on its detection efficiency is studied. The operation efficiencies of the maximum-likelihood and quasioptimal detectors of the ultra-wideband quasi radio signal are compared. Using computer simulation, the efficiency of the synthesized algorithm is examined and the applicability ranges of the obtained asymptotic expressions for its characteristics are determined.*

## 1. INTRODUCTION

In radar, sonar, navigation, and seismology applications, radio-astronomy observations, etc., the radio-signal detection problem, which was considered in the literature many times on the assumption that a radio signal is narrowband [1–3], is topical. In the studies of recent years, the ultra-wideband signals are of keen interest and widely used in radiophysics [4–6]. Ultra-wideband quasi radio signals are referred to one of the types of such signals [4, 7]. Although their mathematical formulation (model) coincides with the radio-signal model, the condition of the relative narrowbandness is not fulfilled. The algorithm for detecting ultra-wideband quasi radio signals with unknown amplitude and initial phase is considered in [7]. However, in addition to the amplitude and the initial phase, the signal duration is also frequently unknown [3]. Moreover, the choice of the modulating function can significantly influence the detection efficiency. Therefore, it is expedient to study the algorithms for detecting an ultra-wideband quasi radio signal with unknown duration and an arbitrary-shaped modulating function. In this work, we consider the maximum-likelihood algorithm for detecting the ultra-wideband quasi radio signal with an arbitrary shape and unknown amplitude, initial phase, and duration.

As in [3, 7], the arbitrary-shaped ultra-wideband quasi radio signal is written in the form

$$s(t,a,\varphi,\tau)=\begin{cases}af(t)\cos(\omega t-\varphi), & 0\leqslant t\leqslant\tau,\\ 0, & t<0,\quad t>\tau.\end{cases}\tag{1}$$

Here, $a$, $\varphi$, $\omega$, and $\tau$ are the amplitude, initial phase, frequency, and duration of the ultra-wideband quasi radio signal, respectively, while $f(t)$ is the modulating function. If the frequency band $\Delta\omega$ and the frequency $\omega$ of signal (1) satisfy the condition

$$\Delta\omega\ll\omega,\tag{2}$$

* korchagin@phys.vsu.ru

Voronezh State University, Voronezh, Russia. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Radiofizika, Vol. 61, No. 11, pp. 958–973, November 2018. Original article submitted September 10, 2018; accepted November 30, 2018.

signal (1) is narrowband and $f(t)$ is its envelope function [1, 2]. If condition (2) is not fulfilled and only several periods or even some fractions of the period of the harmonic oscillation $a\cos(\omega t - \varphi)$ fall on the interval equal to the signal duration, then Eq. (1) describes the ultra-wideband quasi radio signal [7]. In the strict sense, the quantities $a$, $\varphi$, and $\omega$ are not the amplitude, initial phase, and frequency of the quasi radio signal, respectively, but are the parameters of the harmonic oscillation which is used for the formation of the signal. Nevertheless, by analogy with [7], for the sake of brevity, we call $a$, $\varphi$, and $\omega$ the amplitude, initial phase, and frequency of the ultra-wideband quasi radio signal (1), respectively.

Let us consider the problem of detecting signal (1) with unknown amplitude $a$, initial phase $\varphi$, and duration $\tau$ against the background of white Gaussian noise $n(t)$ with one-sided spectral density $N_0$. The additive mixture of signal (1) and noise $n(t)$, which is observed during the time interval $[0, T]$, is written as

$$\xi(t) = \gamma_0 s(t, a_0, \varphi_0, \tau_0) + n(t), \tag{3}$$

where $a_0$, $\varphi_0$, and $\tau_0$ are the true values of the unknown parameters and $\gamma_0$ is the discrete parameter, which is equal to zero in the signal absence ($\gamma_0 = 0$) and unity in the signal presence ($\gamma_0 = 1$). It is assumed that the signal duration $\tau$ can take the values from the *a priori* interval $[T_1, T_2]$. Using realization (3), the receiver should make the decision on the signal presence or absence. Then the detection problem is reduced to estimating the discrete parameter $\gamma_0$ on the basis of observed data (3).

## 2. MAXIMUM-LIKELIHOOD DETECTOR OF AN ULTRA-WIDEBAND QUASI RADIO SIGNAL WITH UNKNOWN DURATION

The algorithm for signal detection (estimation of the parameter $\gamma$) is synthesized using the maximum-likelihood method [1, 2]. For the unknown signal parameters, there exists an *a priori* parametric uncertainty with respect to the signal amplitude, initial phase, and duration. The quantities $a$, $\varphi$, and $\tau$ are assumed to be nonrandom [8; p. 378] and the *a priori* parametric uncertainty is overcome on the basis of the generalized likelihood-ratio criterion [8; p. 102]. In this case, the likelihood-ratio functional logarithm is a function of four unknown parameters [2] such that

$$L(\gamma, a, \varphi, \tau) = \frac{2\gamma}{N_0}\int_0^\tau \xi(t) s(t, a, \varphi)\,\mathrm{d}t - \frac{\gamma}{N_0}\int_0^\tau s^2(t, a, \varphi)\,\mathrm{d}t, \tag{4}$$

and the estimate of the discrete parameter $\gamma$ is determined by the expression

$$\gamma_\mathrm{m} : L(\gamma_\mathrm{m}) = \sup_\gamma \left[\sup_{a,\varphi,\tau} L(\gamma, a, \varphi, \tau)\right].$$

The first term in Eq. (4) and in what follows is the stochastic integral in the Itô sense. Taking into account that $L(\gamma = 0, a, \varphi, \tau) = 0$, we see that the maximum-likelihood detection algorithm involves comparison of the absolute (greatest) maximum of the likelihood-ratio functional logarithm (4) with a zero threshold, i.e.,

$$\gamma_\mathrm{m} = \begin{cases} 1, & L > 0, \\ 0, & L \leqslant 0, \end{cases} \tag{5}$$

$$L = \sup_\tau L(\tau), \qquad L(\tau) = \sup_{a,\varphi} L(a, \varphi, \tau) = L(a_\mathrm{m}, \varphi_\mathrm{m}, \tau),$$

$$a_\mathrm{m}, \varphi_\mathrm{m} : L(a_\mathrm{m}, \varphi_\mathrm{m}, \tau) = \sup_{a,\varphi} L(a, \varphi, \tau), \qquad L(a, \varphi, \tau) = L(\gamma = 1, a, \varphi, \tau). \tag{6}$$

By analogy with [1–3, 7], algorithm (5) can be replaced by the generalized detection algorithm based on comparing the absolute (greatest) maximum $L$ of the likelihood-ratio functional logarithm with some threshold $h$, which is not necessarily equal to zero. If the relationship $L > h$ or $L < h$ is fulfilled, then the

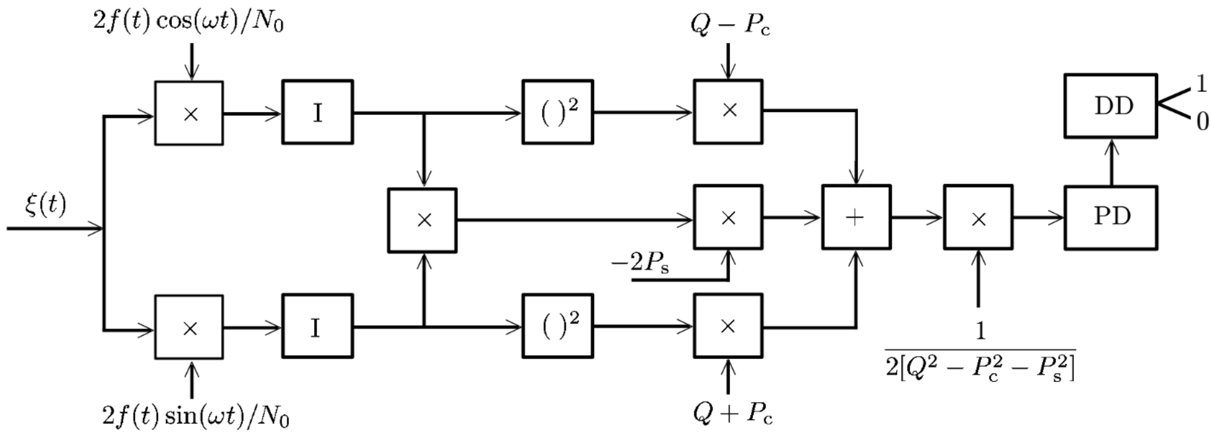


Fig. 1. Block diagram of the maximum-likelihood detection algorithm.

decisions on the signal presence ($\gamma_m = 1$) or absence ($\gamma_m = 0$), respectively, are made.

The function $L(\tau)$ is the likelihood-ratio functional logarithm in which the unknown amplitude and initial phase are replaced by their maximum-likelihood estimates $a_m$ and $\varphi_m$, respectively. This is equivalent to maximizing the likelihood-ratio functional logarithm $L(a, \varphi, \tau)$ in Eq. (6) with respect to the unknown parameters $a$ and $\varphi$. Analytically performing this maximization, we obtain

$$L(\tau) = \frac{[Q(\tau) - P_c(\tau)]X^2(\tau) + [Q(\tau) + P_c(\tau)]Y^2(\tau) - 2X(\tau)Y(\tau)P_s(\tau)}{2[Q^2(\tau) - P_c^2(\tau) - P_s^2(\tau)]}, \tag{7}$$

where

$$X(\tau) = \frac{2}{N_0}\int_0^\tau \xi(t)f(t)\cos(\omega t)\,dt, \qquad Y(\tau) = \frac{2}{N_0}\int_0^\tau \xi(t)f(t)\sin(\omega t)\,dt,$$

$$P_c(\tau) = \frac{1}{N_0}\int_0^\tau f^2(t)\cos(2\omega t)\,dt, \qquad P_s(\tau) = \frac{1}{N_0}\int_0^\tau f^2(t)\sin(2\omega t)\,dt,$$

$$Q(\tau) = \frac{1}{N_0}\int_0^\tau f^2(t)\,dt. \tag{8}$$

Equations (5)–(7) determine the receiver structure. It should form random process (7) for all possible duration values, find the value of its greatest maximum, and compare it with the threshold. Figure 1 shows a block diagram of the maximum-likelihood detection algorithm with the following notations: integrators in the time interval $[0, t]$, where $t \in [0, T_2]$ (I), the peak detector (PD), and the decision device (DD), which compares the output signal of the peak detector at the time $t = T_2$ with the threshold and makes decision on the signal presence or absence in the observed realization.

For the narrowband radio signal in Eq. (7), one can neglect the integrals of the functions oscillating with the double frequency, i.e., due to $P_s(\tau) \ll Q(\tau)$ and $P_c(\tau) \ll Q(\tau)$, write $P_s(\tau) \approx 0$ and $P_c(\tau) \approx 0$. Then Eq. (7) for the logarithm of the likelihood-ratio functional is significantly simplified and takes the form

$$L(\tau) = [X^2(\tau) + Y^2(\tau)]/[2Q(\tau)],$$

which agrees with the results presented in [9].

## 3. CHARACTERISTICS OF THE MAXIMUM-LIKELIHOOD DETECTOR

Let us analyze the maximum-likelihood detection algorithm given by Eqs. (5) and (7), i.e., find the false-alarm and signal-missing probabilities [1, 2, 10]. Let us use $L_1(\tau) = \{L(\tau)|\gamma_0 = 1\}$ and $L_0(\tau) = \{L(\tau)|\gamma_0 = 0\}$ to denote the likelihood-ratio functional logarithm (7) in the case of the signal presence and absence in the adopted realization, respectively.

Let us substitute the observed realization, which is given by Eq. (3), into Eq. (8) for $X(\tau)$ and $Y(\tau)$ and isolate the deterministic and random components

$$X(\tau) = \gamma_0 S_x(\tau) + N_x(\tau), \qquad Y(\tau) = \gamma_0 S_y(\tau) + N_y(\tau), \tag{9}$$

where

$$\begin{gathered} S_x(\tau) = a_0 \cos\varphi_0 \{Q(\min[\tau,\tau_0]) + P_{\rm c}(\min[\tau,\tau_0])\} + a_0 P_{\rm s}(\min[\tau,\tau_0]) \sin\varphi_0, \\ S_y(\tau) = a_0 \sin\varphi_0 \{Q(\min[\tau,\tau_0]) - P_{\rm c}(\min[\tau,\tau_0])\} + a_0 P_{\rm s}(\min[\tau,\tau_0]) \cos\varphi_0, \\ N_x(\tau) = \frac{2}{N_0}\int\limits_0^\tau n(t) f(t)\cos(\omega t)\,{\rm d}t, \qquad N_y(\tau) = \frac{2}{N_0}\int\limits_0^\tau n(t) f(t)\sin(\omega t)\,{\rm d}t. \end{gathered} \tag{10}$$

The random processes $N_x(\tau)$ and $N_y(\tau)$ are the linear transformations of the Gaussian white noise $n(t)$ and, therefore, are also Gaussian [11; p. 97, 8, p. 219]. They possess zero mathematical expectations and the following correlation functions (hereafter, the angular brackets denote the averaging over the realizations):

$$\begin{gathered} K_x(\tau_1,\tau_2) = \langle N_x(\tau_1) N_x(\tau_2)\rangle = Q(\min[\tau_1,\tau_2]) + P_{\rm c}(\min[\tau_1,\tau_2]), \\ K_y(\tau_1,\tau_2) = \langle N_y(\tau_1) N_y(\tau_2)\rangle = Q(\min[\tau_1,\tau_2]) - P_{\rm c}(\min[\tau_1,\tau_2]), \\ K_{xy}(\tau_1,\tau_2) = \langle N_x(\tau_1) N_y(\tau_2)\rangle = \langle N_y(\tau_1) N_x(\tau_2)\rangle = P_{\rm s}(\min[\tau_1,\tau_2]). \end{gathered}$$

To find the false-alarm probability, we study the decision statistic

$$L_0(\tau) = \frac{[Q(\tau) - P_{\rm c}(\tau)] N_x^2(\tau) + [Q(\tau) + P_{\rm c}(\tau)] N_y^2(\tau) - 2 N_x(\tau) N_y(\tau) P_{\rm s}(\tau)}{2[Q^2(\tau) - P_{\rm c}^2(\tau) - P_{\rm s}^2(\tau)]}. \tag{11}$$

It is a random process with the mathematical expectation

$$S_0(\tau) = \langle L_0(\tau)\rangle = 1 \tag{12}$$

and the correlation function

$$K_0(\tau_1,\tau_2) = \langle [L_0(\tau_1) - \langle L_0(\tau_1)\rangle][L_0(\tau_2) - \langle L_0(\tau_2)\rangle]\rangle = \frac{\Psi(\tau_1,\tau_2)}{\Psi(\max[\tau_1,\tau_2],\max[\tau_1,\tau_2])}, \tag{13}$$

where the following notation is used:

$$\Psi(\tau_1,\tau_2) = Q(\tau_1) Q(\tau_2) - P_{\rm c}(\tau_1) P_{\rm c}(\tau_2) - P_{\rm s}(\tau_1) P_{\rm s}(\tau_2).$$

Let us study the local properties of random process (11). To this end, we consider the behavior of correlation function (13) in the small neighborhood of an arbitrary point $\tau \in [T_1, T_2]$. Let us substitute $\tau_1 = \tau$ and $\tau_2 = \tau + \Delta$ into Eq. (12) and expand Eq. (13) into a Taylor series in terms of $\Delta$ in the neighborhood of $\tau$, rejecting all the terms in which the degree of $\Delta$ exceeds unity. As a result, we obtain

$$K_0(\tau,\tau+\Delta) \simeq 1 - \delta(\tau)\,|\Delta| + o(\Delta), \tag{14}$$

where

$$\delta(\tau) = \frac{1}{\Psi(\tau,\tau)} \left. \frac{\partial \Psi(\tau,x)}{\partial x} \right|_{x=\tau} = \frac{Q(\tau)Q'(\tau) - P_{\mathrm{c}}(\tau)P'_{\mathrm{c}}(\tau) - P_{\mathrm{s}}(\tau)P'_{\mathrm{s}}(\tau)}{Q^2(\tau) - P_{\mathrm{c}}^2(\tau) - P_{\mathrm{s}}^2(\tau)} \tag{15}$$

and the prime denotes the derivative with respect to $\tau$. According to Eqs. (12) and (14), the decision statistic $L_0(\tau)$ is a locally-stationary and locally-Markov random process. For such a process, the probability of that the boundary $h$ is not reached in the $\varepsilon$ neighborhood of the point $\tau$ was found in [1]:

$$F_\varepsilon(h,\tau) \simeq P\left\{L_0(x) < h, x \in \left[\tau - \frac{\varepsilon}{2}, \tau + \frac{\varepsilon}{2}\right]\right\} = \begin{cases} \exp\left[-\delta(\tau)\varepsilon h \exp(-h)\right], & h \geqslant 1, \\ 0, & h < 1. \end{cases} \tag{16}$$

The accuracy of approximation (16) improves with decreasing $\varepsilon$ and increasing threshold $h$. Let us divide the *a priori* interval $[T_1, T_2]$ of possible values of the duration into $N$ equal segments with the length $\varepsilon = (T_2 - T_1)/N$. The middle point of each interval is denoted as $t_i = T_1 + (i-1)\varepsilon/2$, where $i = 1, 2, \ldots N$. Then the probability of that the boundary $h$ is not reached by the decision statistic $L_0(\tau)$ in the $i$th interval is approximately equal to

$$F_{0i}(h) = P\{L_0(\tau) < h, \tau \in [t_i - \varepsilon/2, t_i + \varepsilon/2]\} = F_\varepsilon(h, t_i). \tag{17}$$

In this case, the quantity $\varepsilon$ should be sufficiently small to make approximation (14) valid. For sufficiently high thresholds $h$, overshoots of the realization $L_0(\tau)$ beyond the level $h$ in various elementary intervals $[t_i - \varepsilon/2, t_i + \varepsilon/2]$ can approximately be considered statistically independent [11]. Then the false-alarm probability can be expressed via the probability of that the threshold is not reached by the random process $L_0(\tau)$ in the interval $[T_1, T_2]$:

$$\alpha \simeq 1 - F_0(h) = 1 - P\{L_0(\tau) < h, \tau \in [T_1, T_2]\} = 1 - \prod_{i=1}^{N} F_{0i}(h, t_i). \tag{18}$$

Substituting Eq. (16) into Eq. (17) and then Eq. (17) into Eq. (18) and passing to the limit for $\varepsilon \to 0$ and $N \to \infty$, we obtain

$$\alpha \simeq \begin{cases} 1 - \exp\left[-h\exp(-h) \int\limits_{T_1}^{T_2} \delta(\tau)\,\mathrm{d}\tau\right], & h \geqslant 1, \\ 1, & h < 1. \end{cases} \tag{19}$$

After integration of function (15) in Eq. (19), we find the asymptotic expression for the false-alarm probability

$$\alpha = \begin{cases} 1 - \left[\dfrac{Q^2(T_1) - P_{\mathrm{c}}^2(T_1) - P_{\mathrm{s}}^2(T_1)}{Q^2(T_2) - P_{\mathrm{c}}^2(T_2) - P_{\mathrm{s}}^2(T_2)}\right]^{h\exp(-h)/2}, & h \geqslant 1, \\ 1, & h < 1. \end{cases} \tag{20}$$

To determine the signal-missing probability, we study the random process $L(\tau)$ in Eq. (7) for the signal presence. Let us consider the normalized (independent of the signal-to-noise ratio (SNR)) functions (8) and (10):

$$\begin{gathered} q(\tau) = Q(\tau)/z^2; \\ p_{\mathrm{c}}(\tau) = P_{\mathrm{c}}(\tau)/z^2, \qquad p_{\mathrm{s}}(\tau) = P_{\mathrm{s}}(\tau)/z^2, \\ s_x(\tau) = S_x(\tau)/z^2, \qquad s_y(\tau) = S_y(\tau)/z^2, \\ \eta_x(\tau) = N_x(\tau)/z, \qquad \eta_y(\tau) = N_y(\tau)/z, \end{gathered} \tag{21}$$

where

$$z^2 = 2a_0^2 T_2/N_0 \tag{22}$$

is the SNR at the output of the maximum-likelihood receiver for a rectangular pulse with the amplitude $a_0$ and duration $T_2$ without harmonic filling. Among the quantities in Eq. (21), the functions $p_{\rm c}(\tau)$, $p_{\rm s}(\tau)$, $s_x(\tau)$, and $s_y(\tau)$ are deterministic, while $\eta_x(\tau)$ and $\eta_y(\tau)$ are the random processes. Substituting Eqs. (9) and (10) into Eq. (7) for $\gamma_0 = 1$ and allowing for the notations given in Eq. (21), we obtain the following expression for the decision statistic in the signal presence:

$$L_1(\tau) = z^2 A(\tau) + zB_x(\tau)\eta_x(\tau) + zB_y(\tau)\eta_y(\tau) + C_x(\tau)\eta_x^2(\tau) + C_y(\tau)\eta_y^2(\tau) + C_{xy}(\tau)\eta_x(\tau)\eta_y(\tau), \tag{23}$$

where

$$A(\tau) = \frac{[q(\tau) - p_{\rm c}(\tau)]s_x^2(\tau) + [q(\tau) + p_{\rm c}(\tau)]s_y^2(\tau) - 2p_{\rm s}(\tau)s_x(\tau)s_y(\tau)}{2[q^2(\tau) - p_{\rm c}^2(\tau) - p_{\rm s}^2(\tau)]},$$
$$B_x(\tau) = \frac{[q(\tau) - p_{\rm c}(\tau)]s_x(\tau) - p_{\rm s}(\tau)s_y(\tau)}{q^2(\tau) - p_{\rm c}^2(\tau) - p_{\rm s}^2(\tau)},$$
$$B_y(\tau) = \frac{[q(\tau) + p_{\rm c}(\tau)]s_y(\tau) - p_{\rm s}(\tau)s_x(\tau)}{q^2(\tau) - p_{\rm c}^2(\tau) - p_{\rm s}^2(\tau)},$$
$$C_x(\tau) = \frac{q(\tau) - p_{\rm c}(\tau)}{2[q^2(\tau) - p_{\rm c}^2(\tau) - p_{\rm s}^2(\tau)]}, \qquad C_y(\tau) = \frac{q(\tau) + p_{\rm c}(\tau)}{2[q^2(\tau) - p_{\rm c}^2(\tau) - p_{\rm s}^2(\tau)]},$$
$$C_{xy}(\tau) = -\frac{p_{\rm s}(\tau)}{q^2(\tau) - p_{\rm c}^2(\tau) - p_{\rm s}^2(\tau)}.$$

The random processes $\eta_x(\tau)$ and $\eta_y(\tau)$ are Gaussian with the zero mathematical expectations and the correlation functions

$$K_{\eta x}(\tau_1, \tau_2) = K_x(\tau_1, \tau_2)/z^2 = \langle \eta_x(\tau_1)\eta_x(\tau_2)\rangle = q(\min[\tau_1, \tau_2]) + p_{\rm c}(\min[\tau_1, \tau_2]),$$
$$K_{\eta y}(\tau_1, \tau_2) = K_y(\tau_1, \tau_2)/z^2 = \langle \eta_y(\tau_1)\eta_y(\tau_2)\rangle = q(\min[\tau_1, \tau_2]) - p_{\rm c}(\min[\tau_1, \tau_2]),$$
$$K_{\eta xy}(\tau_1, \tau_2) = K_{xy}(\tau_1, \tau_2)/z^2 = \langle \eta_x(\tau_1)\eta_y(\tau_2)\rangle = \langle \eta_y(\tau_1)\eta_x(\tau_2)\rangle = p_{\rm s}(\min[\tau_1, \tau_2]).$$

The decision statistic $L_1(\tau)$ given by Eq. (23) is not Gaussian, since it contains the operations of multiplying and squaring the random processes $\eta_x(\tau)$ and $\eta_y(\tau)$. However, for sufficiently large SNRs ($z \gg 1$), the last three terms in Eq. (23) can be neglected compared with the preceding ones and approximately written as

$$L_1(\tau) \simeq z^2 A(\tau) + zB_x(\tau)\eta_x(\tau) + zB_y(\tau)\eta_y(\tau). \tag{24}$$

Since the random functions $\eta_x(\tau)$ and $\eta_y(\tau)$ enter Eq. (24) linearly, the process $L_1(\tau)$ is Gaussian. To obtain its full statistical description, it is sufficient to find the mathematical expectation and the correlation function. Performing the averaging, one obtains the mathematical expectation in the signal presence

$$S_1(\tau) = \langle L_1(\tau)\rangle = z^2 A(\tau) \tag{25}$$

and the correlation function

$$\begin{aligned} K(\tau_1, \tau_2) = \langle [L_1(\tau_1) - S_1(\tau_1)]\,[L_1(\tau_2) - S_1(\tau_2)]\rangle = A_1(\tau_1)A_1(\tau_2)\{q(\min[\tau_1, \tau_2]) + p_{\rm c}(\min[\tau_1, \tau_2])\} \\ + p_{\rm s}(\min[\tau_1, \tau_2])[A_1(\tau_1)A_2(\tau_2) + A_1(\tau_2)A_2(\tau_1)] \\ + A_2(\tau_1)A_2(\tau_2)\{q(\min[\tau_1, \tau_2]) - p_{\rm c}(\min[\tau_1, \tau_2])\}, \end{aligned} \tag{26}$$

where

$$A_1(\tau) = z\frac{[q(\tau) - p_{\rm c}(\tau)]s_x(\tau) - s_y(\tau)p_{\rm s}(\tau)}{q^2(\tau) - p_{\rm c}^2(\tau) - p_{\rm s}^2(\tau)}, \qquad A_2(\tau) = z\frac{[q(\tau) + p_{\rm c}(\tau)]s_y(\tau) - s_x(\tau)p_{\rm s}(\tau)}{q^2(\tau) - p_{\rm c}^2(\tau) - p_{\rm s}^2(\tau)}.$$

As is known, the location of the decision-statistic maximum shows the rms convergence to the true value of the duration $\tau_0$ with increasing SNR [1, 10]. Therefore, we study the likelihood-ratio functional logarithm (24) in the neighborhood of the point $\tau_0$. Expanding Eqs. (25) and (26) into the Taylor series in terms of the variable $\tau$ in the neighborhood of the point $\tau_0$, we obtain asymptotic expressions for the mathematical expectation

$$S_1(\tau) \approx \frac{\lambda_0}{2} + \frac{\tau - \tau_0}{2T_2}\begin{cases}\psi_1, & \tau \leqslant \tau_0,\\ -\psi_1, & \tau > \tau_0\end{cases} \tag{27}$$

and the correlation function

$$K_1(\tau_1, \tau_2) \approx \lambda_0 + \psi_1 \min(\tau_1 - \tau_0, \tau_2 - \tau_0)/T_2, \tag{28}$$

where $\lambda_0 = z^2[Q(\tau_0) + P_{\rm c}(\tau_0)\cos(2\varphi_0) + P_{\rm s}(\tau_0)\sin(2\varphi_0)]/2$ and $\psi_1 = z^2 f^2(\tau_0)\cos^2(\omega\tau_0 - \varphi_0)$.

Let us approximate the likelihood-ratio functional logarithm (24) by the Gaussian random process $\mu_1(\tau)$ with mathematical expectation (27) and correlation function (28). Such an approximation makes sense for all $\tau > \tau_{\rm d} = \tau_0 - T_2\lambda_0/\psi_1$ for which the random-process variance $\mu_1(\tau)$ is nonnegative, i.e., $K_1(\tau,\tau) \approx \lambda_0 + \psi_1(\tau - \tau_0)/T_2 \geqslant 0$. When using the approximation $\mu_1(\tau)$, it is assumed that the duration takes the values from the *a priori* interval $[T_{\rm d}, T_2]$, where $T_{\rm d} = \max(\tau_{\rm d}, T_1)$. Using Eqs. (27) and (28) and the Doob theorem [12], it can be shown that the decision statistic $\mu_1(\tau)$ is the Markov process with the drift and diffusion coefficients $k_{11}$ and $k_{21}$, respectively [12]:

$$k_{11} = \frac{1}{2T_2}\begin{cases}\psi_1, & T_d \leqslant \tau \leqslant \tau_0,\\ -\psi_1, & \tau_0 < \tau \leqslant T_2;\end{cases} \qquad k_{21} = \frac{\psi_1}{T_2}. \tag{29}$$

By definition, the signal-missing probability is equal to the probability $\beta$ that the boundaries $y = -\infty$ and $y = h$ are not reached by the Markov random process $\mu_1(\tau)$ in the interval $\tau \in [T_{\rm d}, T_2]$, i.e.,

$$\beta = F_1(h) = P\{\mu_1(\tau) < h, \tau \in [T_d, T_2]\}. \tag{30}$$

The desired probability (30) can be expressed via the probability density $W(y,\tau)$ of realizations of the random process $\mu_1(\tau)$ which have never reached the boundaries $y = -\infty$ and $y = h$ [12]:

$$F_1(h) = \int\limits_{-\infty}^{h} W(y, T_2)\,{\rm d}y. \tag{31}$$

The function $W(y,\tau)$ is the solution of the Fokker–Planck–Kolmogorov equation [12]

$$\frac{\partial W(y,\tau)}{\partial \tau} + \frac{\partial}{\partial y}[k_{11}(y,\tau)W(y,\tau)] - \frac{1}{2}\frac{\partial^2}{\partial y^2}[k_{21}(y,\tau)W(y,\tau)] = 0 \tag{32}$$

with coefficients (29) under the initial condition

$$W(y, T_{\rm d}) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(y-m)^2}{2\sigma^2}\right]$$

and the boundary conditions

$$W(-\infty,\tau) = W(h,\tau) = 0,$$

where $\sigma^2 = \lambda_0 + \psi_1(T_d - \tau_0)/T_2$ and $m = \lambda_0/2 + \psi_1(T_\mathrm{d} - \tau_0)/2T_2$.

Solving Eq. (32) by the method of reflection with the sign reversal [12], substituting the obtained solution into Eq. (31), and then Eq. (31) into Eq. (30), we obtain the expression for the signal-missing probability

$$\beta = \frac{1}{\sqrt{2\pi\lambda_0}} \int_0^\infty \exp\left[-\frac{(\xi+\lambda_0/2)^2 + h^2 - h\lambda_0}{2\lambda_0}\right] \left[\Phi\left(\frac{\sqrt{r}}{2} + \frac{\xi}{\sqrt{r}}\right) - \exp(-\xi)\Phi\left(\frac{\sqrt{r}}{2} - \frac{\xi}{\sqrt{r}}\right)\right] \left\{\Phi\left[h\sqrt{\frac{l}{\lambda_0(\lambda_0 - l)}} + \xi\sqrt{\frac{\lambda_0 - l}{\lambda_0 l}}\right] \exp\left(\frac{h\xi}{\lambda_0}\right) - \Phi\left[h\sqrt{\frac{l}{\lambda_0(\lambda_0 - l)}} - \xi\sqrt{\frac{\lambda_0 - l}{\lambda_0 l}}\right] \exp\left(-\frac{h\xi}{\lambda_0}\right)\right\} \mathrm{d}\xi, \quad (33)$$

where $l = \psi_1(\tau_0 - T_\mathrm{d})/T_2$ and $r = \psi_1(T_2 - \tau_0)/T_2$.

The false-alarm and signal-missing probabilities, which are given by Eqs. (20) and (33), respectively, are the generalizations of the similar expressions obtained in [3] for a narrowband radio signal. Indeed, if condition (2) is fulfilled for the received signal, which corresponds to detection of the narrowband radio signal, then $|P_\mathrm{c}(\tau)| \ll Q(\tau)$, $|P_\mathrm{s}(\tau)| \ll Q(\tau)$, and the false-alarm and the signal-missing probabilities, (20) and (33) coincide with similar probabilities obtained in [3] with allowance for the notations. Equation (33) was obtained by the method of the local-Markov approximation, which determines its presentation form which is similar to that used in [1; p. 73]. However, the calculation by Eq. (33) can be performed only numerically.

## 4. QUASIOPTIMAL DETECTION ALGORITHM

The maximum-likelihood algorithm for detecting an ultra-wideband quasi radio signal, which is shown in Fig. 1, has a more complicated structure than that of the algorithm for detecting a narrowband radio signal with unknown amplitude, initial phase, and duration, which was synthesized in [3] and developed using the classical quadrature scheme. Depending on the requirements to the efficiency and restrictions imposed on the complexity of the developed detector, it might be expedient to use a simpler quadrature algorithm for detecting the ultra-wideband quasi radio signal. This algorithm performs comparison with the absolute-maximum threshold of the decision statistic [3]

$$L_\mathrm{q}(\tau) = \frac{X^2(\tau) + Y^2(\tau)}{2Q(\tau)}, \qquad \tau \in [T_1, T_2], \quad (34)$$

$$\gamma_\mathrm{q} = \begin{cases} 1, & L_q > 0, \\ 0, & L_\mathrm{q} \leqslant 0, \end{cases} \qquad L_q = \sup_\tau L_\mathrm{q}(\tau). \quad (35)$$

Such a detector is called quasioptimal. Studying the quasioptimal algorithm for detecting an ultra-wideband quasi radio signal, one can estimate the efficiency of using the existing detectors of narrowband radio signals when an additive mixture of an arbitrary-shaped ultra-wideband quasi radio signal and Gaussian white noise is applied to the receiver input. In this case, the useful signal has unknown amplitude, initial phase, and duration. The quasioptimal detector can be realized on the basis of the block diagram shown in Fig. 2 in which the integrators (I) operate in the time interval $t \in [0, T_2]$.

Let us find the statistical characteristics of a quasioptimal detector based on Eq. (35). Substituting Eq. (9) into Eq. (34) for $\gamma_0 = 0$, one obtains the decision statistic of the quasioptimal detector in the signal

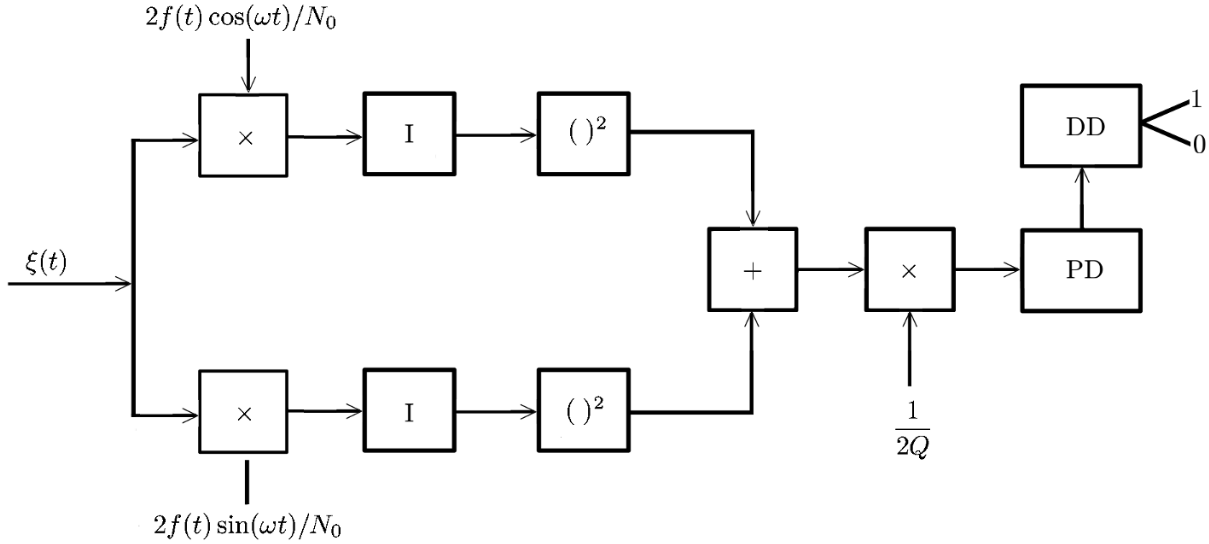


Fig. 2. Block diagram of the quasioptimal detection algorithm.

absence as

$$L_{0\mathrm{q}}(\tau) = \frac{N_x^2(\tau) + N_y^2(\tau)}{2Q(\tau)},$$

which coincides with Eq. (11) for $P_\mathrm{c}(\tau) = P_\mathrm{s}(\tau) = 0$. Then the false-alarm probability of the quasioptimal detection algorithm can be obtained from Eq. (20) in the form

$$\alpha_\mathrm{q} = \begin{cases} 1 - \left[\dfrac{Q(T_1)}{Q(T_2)}\right]^{h\exp(-h)}, & h \geqslant 1, \\ 1, & h < 1. \end{cases} \tag{36}$$

To find the probability of the signal missing by the quasioptimal detector, we substitute Eq. (9) into Eq. (34) for $\gamma_0 = 1$ and obtain the expression for the decision statistic of the quasioptimal detector in the signal presence:

$$L_{1\mathrm{q}}(\tau) = z^2 \frac{s_x^2(\tau) + s_y^2(\tau)}{2q(\tau)} + z\frac{s_x(\tau)\eta_x(\tau) + s_y(\tau)\eta_y(\tau)}{q(\tau)} + \frac{\eta_x^2(\tau) + \eta_y^2(\tau)}{2q(\tau)}. \tag{37}$$

For sufficiently large SNRs ($z \gg 1$), one can neglect the last term in Eq. (37) and approximately write

$$L_{1\mathrm{q}}(\tau) \approx z^2 \frac{s_x^2(\tau) + s_y^2(\tau)}{2q(\tau)} + z\frac{s_x(\tau)\eta_x(\tau) + s_y(\tau)\eta_y(\tau)}{q(\tau)}. \tag{38}$$

According to Eq. (38), the decision statistic $L_{1\mathrm{q}}(\tau)$ is a linear transformation of the random processes $\eta_x(\tau)$ and $\eta_y(\tau)$. The nonlinear operations of squaring in Eq. (38) are used only for deterministic quantities. This allows us to consider the random process $L_{1\mathrm{q}}(\tau)$ Gaussian. It has a mathematical expectation

$$S_{1\mathrm{q}}(\tau) = \langle L_{1\mathrm{q}}(\tau)\rangle = z^2 \frac{s_x^2(\tau) + s_y^2(\tau)}{2q(\tau)} \tag{39}$$

and a correlation function

$$\begin{aligned} K_\mathrm{q}(\tau_1, \tau_2) = \langle [L_{1\mathrm{q}}(\tau_1) - S_{1\mathrm{q}}(\tau_1)][L_{1\mathrm{q}}(\tau_2) - S_{1\mathrm{q}}(\tau_2)]\rangle = A_{1\mathrm{q}}(\tau_1)A_{1\mathrm{q}}(\tau_2)\{(q(\min[\tau_1, \tau_2]) \\ + p_\mathrm{c}(\min[\tau_1, \tau_2])\} + p_\mathrm{s}(\min[\tau_1, \tau_2])\,[A_{1\mathrm{q}}(\tau_1)A_{2\mathrm{q}}(\tau_2) + A_{1\mathrm{q}}(\tau_2)A_{2\mathrm{q}}(\tau_1)] \\ + A_{2\mathrm{q}}(\tau_1)A_{2\mathrm{q}}(\tau_2)\{q(\min[\tau_1, \tau_2]) - p_\mathrm{c}(\min[\tau_1, \tau_2])\}, \end{aligned} \tag{40}$$

where $A_{1q}(\tau) = zs_x(\tau)/q(\tau)$ and $A_{2q}(\tau) = zs_y(\tau)/q(\tau)$. Expanding Eqs. (39) and (40) into a Taylor series in terms of $\tau$ in the neighborhood of $\tau_0$, we obtain asymptotic expressions for the mathematical expectation

$$S_{1q}(\tau) \approx \frac{a_0}{2} + \frac{\tau - \tau_0}{2T_2} \begin{cases} a_1, & \tau \leqslant \tau_0, \\ -a_2, & \tau > \tau_0 \end{cases} \tag{41}$$

and the correlation function

$$K_q(\tau_1, \tau_2) \approx b_0 + b \min(\tau_1 - \tau_0, \tau_2 - \tau_0)/T_2. \tag{42}$$

Here,

$$a_0 = \frac{z^2}{2Q(\tau_0)} [Q^2(\tau_0) + P_c^2(\tau_0) + P_s^2(\tau_0) + 2P_c(\tau_0)Q(\tau_0)\cos(2\varphi_0) + 2P_s(\tau_0)Q(\tau_0)\sin(2\varphi_0)]; \tag{43}$$

$$a_1 = \frac{z^2 f^2(\tau_0)}{Q^2(\tau_0)} \left\{Q^2(\tau_0) - P_c^2(\tau_0) - P_s^2(\tau_0) \right. \\ \left. + 2Q(\tau_0)\left[P_c(\tau_0)\cos(2\pi\kappa) + Q(\tau_0)\cos(2\varphi_0 - 2\pi\kappa) + P_s(\tau_0)\sin(2\pi\kappa)\right]\right\}; \tag{44}$$

$$a_2 = \frac{z^2 f(\tau_0)}{Q^2(\tau_0)} \left[P_c^2(\tau_0) + P_s^2(\tau_0) - Q^2(\tau_0) + 2P_c(\tau_0)Q(\tau_0)\cos(2\varphi_0) + 2P_s(\tau_0)Q(\tau_0)\sin(2\varphi_0)\right]; \tag{45}$$

$$b_0 = \frac{z^4 f(\tau_0)}{2Q^2(\tau_0)} \left\{3Q(\tau_0)\left[P_c^2(\tau_0) + P_s^2(\tau_0)\right] + Q^3(\tau_0) \right. \\ \left. + [3Q(\tau_0) + P_c^2(\tau_0) + P_s^2(\tau_0)][2P_c(\tau_0)\cos(2\varphi_0) + 2P_s(\tau_0)\sin(2\varphi_0)]\right\}; \tag{46}$$

$$b = \frac{z^2 f^2(\tau_0)}{Q^3(\tau_0)} \left\{Q(\tau_0)[Q(\tau_0)\cos(\varphi_0 - \pi\kappa) + P_c(\tau_0)\cos(\varphi_0 + \pi\kappa) + P_s(\tau_0)\sin(\varphi_0 + \pi\kappa)]^2\right\} \\ - \left([P_c^2(\tau_0) + P_s^2(\tau_0) + Q^2(\tau_0)]\cos(2\varphi_0)[Q(\tau_0)\cos(2\pi\kappa) - P_c(\tau_0)] \right. \\ - [P_c^2(\tau_0) + P_s^2(\tau_0) + Q^2(\tau_0)]\sin(2\varphi_0)[P_s(\tau_0) - Q(\tau_0)\sin(2\pi\kappa)] \\ \left. -2Q(\tau_0)\left\{P_c^2(\tau_0) + P_s^2(\tau_0) - Q(\tau_0)[P_c(\tau_0)\cos(2\pi\kappa) + P_s(\tau_0)\sin(2\pi\kappa)]\right\}\right). \tag{47}$$

Here, we have introduced the quantity $\kappa = \omega\tau_0/(2\pi)$, which is numerically equal to the number of harmonic-carrier periods, which fall on the signal duration $\tau_0$. As in [7, 15], $\kappa$ is called the narrowbandness parameter. For $\kappa \to \infty$, signal (1) becomes narrowband.

The likelihood-ratio functional logarithm (38) is approximated by the Gaussian random process $\mu_q(\tau)$ with mathematical expectation (41) and correlation function (42). Such an approximation makes sense for all $\tau > \tau_q = \tau_0 - T_2 b_0/b$ for which the random-process variance $\mu_q(\tau)$ is nonnegative, i.e., $K_q(\tau,\tau) \approx b_0 + b(\tau - \tau_0)/T_2 \geqslant 0$. When using the approximation $\mu_q(\tau)$, it is assumed that duration takes the values from the *a priori* interval $[T_q, T_2]$, where $T_q = \max[\tau_q, T_1]$. Using Eqs. (41) and (42) and the Doob theorem (12), it can be shown that the decision statistic $\mu_q(\tau)$ is the Gaussian Markov process with the drift ($k_{1q}$) and diffusion ($k_{2q}$) coefficients [12]

$$k_{1q} = \frac{1}{2T_2} \begin{cases} a_1, & T_q \leqslant \tau \leqslant \tau_0, \\ -a_2, & \tau_0 < \tau \leqslant T_2; \end{cases} \qquad k_{2q} = \frac{b}{T_2}.$$

By analogy with probability (30), the signal-missing probability is equal to the probability of that the Markov random process $\mu_{\mathrm{q}}(\tau)$ does not reach the boundaries $y=-\infty$ and $y=h$ for $\tau \in [T_{\mathrm{q}}, T_2]$. Using the Markov properties of the process $\mu_{\mathrm{q}}(\tau)$, we find the expression for the probability of the signal missing by the quasioptimal detector in the form

$$\beta_q = \frac{1}{\sqrt{2\pi b_0}} \int\limits_0^{\infty} \left[ \Phi\left(\frac{1}{2}\sqrt{d_2 r} + \xi\sqrt{\frac{d_2}{r}}\right) - \exp(-d_2\xi)\Phi\left(\frac{1}{2}\sqrt{d_2 r} - \xi\sqrt{\frac{d_2}{r}}\right)\right]$$
$$\times \exp\left[-\frac{d_1\xi b_0 - l(h-m) + d_1^2 l^2/4}{2b_0}\right] \left\{ \exp\left[-\frac{(h-m_q-\xi)^2 - \sigma_q^2 d_1 \xi}{2b_0}\right]\right.$$
$$\times \Phi\left[\frac{(h-m)d_1 l + \sigma_q^2\xi + \sigma_q^2 l/2}{\sigma_q\sqrt{b_0 d_1 l}}\right] - \Phi\left[\frac{(h-m)d_1 l - \sigma_q^2\xi + \sigma_q^2 l/2}{\sigma_q\sqrt{b_0 d_1 l}}\right]$$
$$\left. \times \exp\left[-\frac{(h-m_q+\xi)^2 + \sigma_q^2 d_1\xi}{2b_0}\right]\right\} \mathrm{d}\xi, \quad (48)$$

where $l_{\mathrm{q}} = a_1(\tau_0 - T_{\mathrm{q}})/T_2$, $r_{\mathrm{q}} = a_2(T_2 - \tau_0)/T_2$, $m_{\mathrm{q}} = a_0/2 - l_{\mathrm{q}}/2$, $\sigma_{\mathrm{q}}^2 = b_0 - d_1 l_{\mathrm{q}}$, $d_1 = a_1/b$, $d_2 = a_2/b$, and the quantities $a_0$, $a_1$, $a_2$, $b_0$, and $b$ are defined in Eqs. (43)–(47), respectively. The signal-missing probability (48) coincides in form with Eq. (33) and differs from the latter only by the parameters, which results from using the method of the local-Markov approximation [1] when obtaining both expressions.

## 5. COMPARATIVE ANALYSIS OF THE CHARACTERISTICS OF DETECTORS OF A QUASI RADIO SIGNAL WITH UNKNOWN DURATION

In [7], the maximum-likelihood detector of an ultra-wideband quasi radio signal with unknown amplitude and initial phase, but known duration was synthesized. Although its block diagram resembles that of the maximum-likelihood detector, which is presented in Fig. 1, the integrators operate in the observation interval $[0, \tau_0]$ and the peak detector is absent. With allowance for notations (8) and (9), the false-alarm and signal-missing probabilities for such a detector have the form

$$\alpha_1 = \exp(-h), \quad (49)$$

$$\beta_1 = \exp\left(-\frac{z^2 V}{4}\right) \int\limits_0^h \exp(-L) I_0\left(z\sqrt{LV}\right) \mathrm{d}L, \quad (50)$$

where $V = Q(\tau_0) + P_c(\tau_0)\cos(2\varphi_0) + P_{\mathrm{s}}(\tau_0)\sin(2\varphi_0)$ and $I_0(x)$ is a modified Bessel function of the first kind of order zero.

Thus, we have obtained the detection characteristics (the false-alarm and signal-missing probabilities) for three detection algorithms, namely, the maximum-likelihood algorithm for detecting the ultra-wideband quasi radio signal with unknown duration (Eqs. (20) and (33)), the maximum-likelihood algorithm for detecting the ultra-wideband quasi radio signal with known duration (Eqs. (49) and (50)), and the quasioptimal algorithm for detecting an ultra-wideband quasi radio signal with unknown duration (Eqs. (36) and (48)). This allows one to perform comparative analysis of efficiency of the detectors of an ultra-wideband quasi radio signal with unknown duration with allowance for complexity of their hardware or software realizations.

As an example, let us consider the detection of an ultra-wideband quasi radio signal with the modulating function

$$f(t) = \exp(-\nu t/T_2), \quad (51)$$

where $\nu$ characterizes the signal decrease rate.

Figure 3 shows the dependences of the signal-missing probability for three various detection algorithms

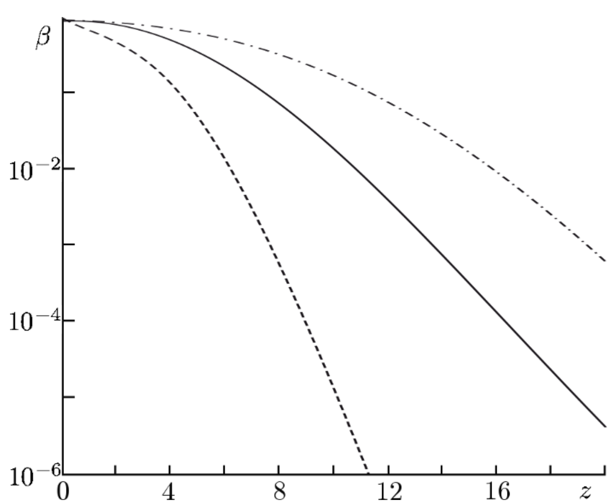


Fig. 3. Signal-missing probabilities as functions of the SNR for fixed false-alarm probabilities.

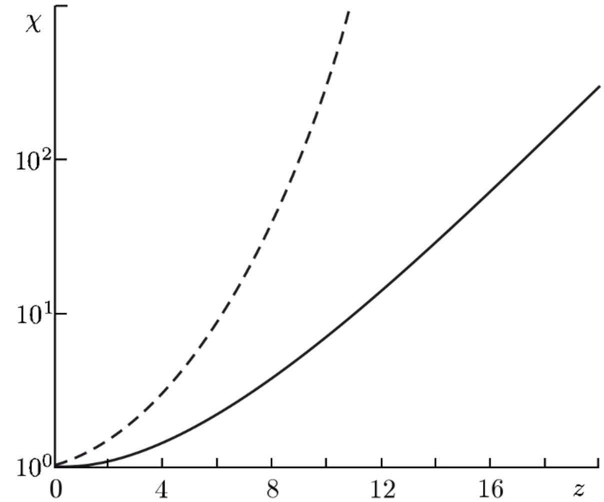


Fig. 4. Detection-efficiency loss as a function of the SNR.

on the SNR of Eq. (22) for the fixed level of the false-alarm probabilities $\alpha = \alpha_q = \alpha_1 = 0.1$. The solid, dashed, and dash-dotted curves characterize the efficiencies (33), (50), and (48) of the maximum-likelihood detector of the signal with unknown duration, the maximum-likelihood detector of the signal with known duration, and the quasioptimal detector of the signal with unknown duration, respectively. When calculating the curves in Fig. 3, it was assumed that the initial phase of the received signal is $\varphi_0 = 0$, the dynamic range of possible duration values is $k = T_2/T_1 = 10$, $\nu = 2$, $\kappa = 0.5$, and the true duration value was chosen in the middle of the *a priori* interval $\tau_0 = (T_1 + T_2)/2$. The dependences shown in Fig. 3 characterize the detection efficiency of various-complexity detectors for various *a priori* data on the signal duration. It is seen in Fig. 3 that the quasioptimal detector has the worst detection efficiency. The maximum-likelihood detection of the signal with unknown duration for small SNRs (22) has low efficiency because of the simultaneous *a priori* ignorance of the three signal parameters. It is confirmed by the several-order better characteristics of detection of the ultra-wideband quasi radio signal by the maximum-likelihood detector with unknown amplitude and initial phase, but known duration.

The influence of the *a priori* ignorance of duration on the detection efficiency is quantitatively characterized by the loss value

$$\chi_1(p) = \frac{\beta|\, \alpha = p}{\beta_1|\, \alpha_1 = p}, \tag{52}$$

which is the ratio of the probability of missing the signal with unknown duration, given by Eq. (33), to that of missing the signal with known duration, given by Eq. (50), for the fixed false-alarm probabilities.

The loss of the quasioptimal detection algorithm because of the input of the ultra-wideband quasi radio signal instead of the narrowband radio signal is quantitatively characterized by the quantity

$$\chi_2(p) = \frac{\beta_q\, |\alpha_q = p}{\beta|\, \alpha = p}, \tag{53}$$

which is the ratio of probabilities of missing the ultra-wideband quasi radio signal by the quasioptimal and maximum-likelihood detectors for the fixed false-alarm probabilities.

Figure 4 shows the dependences of losses (52) and (53) on SNR (22) for $\alpha = 0.1$, $\varphi_0 = 0$, $\nu = 2$, $k = 10$, and $\kappa = 0.5$. The solid curve characterizes loss (53) when choosing the nonoptimal detector of the ultra-wideband quasi radio signal, and the dashed curve, loss (52) in the case of *a priori* ignorance of the ultra-wideband quasi radio signal duration.

As is evident from Fig. 4, the *a priori* ignorance of the ultra-wideband quasi radio signal duration leads to an increase in the probabilities of errors by several orders of magnitude. The choice of the nonoptimal detector also leads to an increase in loss, but to a smaller degree because of the great number of unknown parameters and inefficient detection for low SNRs on the whole. With increasing SNR, the loss value increases as well as the efficiency of the maximum-likelihood detector for unknown duration. It should be noted that the detection efficiency is significantly influenced by the narrowbandness parameter $\kappa$. This is related to the fact that asymptotically, the probabilities of errors for large SNRs are independent of the form of the signal, but determined only by the value of the jump $f(\tau_0)$ of its trailing edge, which, in turn, is specified by the quantity $\kappa$.

## 6. STATISTICAL SIMULATION RESULTS

The above-obtained Eqs. (20) and (33) for the characteristics of the maximum-likelihood detector of the ultra-wideband quasi radio signal with unknown amplitude, phase, and duration are asymptotically accurate. Their accuracy increases with increasing *a priori* interval of possible values of duration, threshold, and SNR (22). To check the operability of the synthesized maximum-likelihood algorithm for detecting the ultra-wideband quasi radio signal and determine the applicability range of the asymptotic expressions (20) and (33) for the false-alarm and signal-missing probabilities, respectively, the statistical simulation of the maximum-likelihood detector of the ultra-wideband quasi radio signal with modulating function (51) was performed.

During the simulation, the discrete readouts of the decision statistic (7) are formed at the receiver output, on the basis of which the likelihood-ratio functional logarithm was approximated by the step function with the maximum relative rms error $\varepsilon = 0.1$. To simulate the detection algorithm, the readout, which is maximum among the decision-statistic readouts, was determined and compared with the threshold in the signal presence and absence. If the threshold was exceeded in the signal absence, the false alarm was recorded. By analogy, if the threshold was not exceeded in the signal presence, the signal missing was recorded. The relative frequencies of appearance of the corresponding errors were used as the signal-missing and false-alarm probability estimates. During the simulation, $10^6$ tests were realized for each SNR value.

The simulation results are shown in Fig. 5 as the dependences of the signal-missing probability (33) on the SNR (22) for various levels of the false-alarm probabilities (20). The lines and markers are used to denote the analytically-calculated and simulated dependences, respectively. The solid curve and the square markers correspond to the false-alarm level $\alpha = 10^{-1}$, the dashed curve and the triangular markers, to $\alpha = 10^{-2}$, and the dash-dotted curve and the circular markers, to $\alpha = 10^{-3}$. When calculating the curves in Fig. 5, it was assumed that $\varphi_0 = 0$, $\nu = 2$, $k = 4$, and $\kappa = 0.3$.

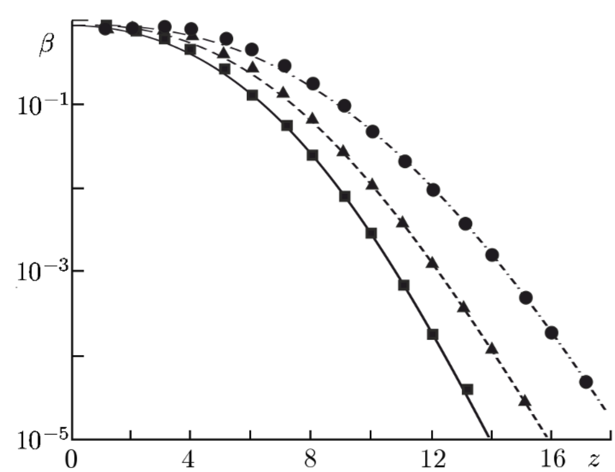

Fig. 5. Statistical simulation results.

As is evident from Fig. 5, the asymptotic expression for the signal-missing probability (33) satisfactorily describes the experimental dependences. Acceptable convergence of the experimental and theoretical dependences is already observed for $z > 4$.

## 7. CONCLUSIONS

Failure to fulfill the conditions of the relative narrowbandness of a radio signal leads to the necessity of using the maximum-likelihood detector whose structure significantly differs from that of the maximum-likelihood detector of the narrowband radio signal with unknown duration. The *a priori* ignorance of the signal duration can lead to a substantial deterioration in the detection quality, especially for small SNRs.

Although the synthesized quasioptimal detector has a rather low operation efficiency because of the receiver nonoptimality, its use can be justified by the structure simplicity in the case of relatively low requirements to the detection quality. The performed statistical simulation has confirmed operability of the synthesized detection algorithm and helped us to determine the applicability range of the asymptotic expressions for the probabilities of errors. The obtained results allow us to quantitatively characterize the influence of the *a priori* ignorance of duration and the choice of the structure of the detector of the ultra-wideband quasi radio signal on the detection efficiency.

**REFERENCES**


1. P. S. Akimov, P. A. Bakut, V. A. Bogdanovich, et al., *Signal Detection Theory* [in Russian], Radio i Svyaz', Moscow (1984).
2. V. I. Tikhonov, *Optimal Signal Reception* [in Russian], Radio i Svyaz', Moscow (1983).
3. A. P. Trifonov, Yu. É. Korchagin, and M. V. Trifonov, *Radiophys. Quantum Electron.*, **58**, No. 5, 361 (2015).
4. V. G. Radzievsky and P. A. Trifonov, *Processing of Ultra-Wideband Signals against a Background of Noise* [in Russian], Radiotekhnika, Moscow (2009).
5. H. Arslan, Z. N. Chen, and M. G. Benedetto, *Ultra Wideband Wireless Communication*, Wiley, Hoboken, N. J. (2006).
6. F. Nekoogar and F. Dowla, *Ultra-Wideband Radio Frequency Identification Systems*, Springer, New York (2011).
7. A. P. Trifonov and P. E. Rudnev, *Radiophys. Quantum Electron.*, **52**, No. 9, 671 (2009).
8. H. L. Van Trees, *Detection, Estimation, and Modulation Theory, Pt. 1*, Wiley, New York (1968).
9. Yu. É. Korchagin, *Radiotekhnika*, No. 9, 11 (2013).
10. V. I. Tikhonov, *Statistical Radioengineering* [in Russian], Radio i Svyaz', Moscow (1982).
11. V. I. Tikhonov and V. N. Kharisov, *Statistical Analysis and Synthesis of Radioengineering Devices and Systems* [in Russian], Radio i Svyaz', Moscow (1991).
12. V. I. Tikhonov and M. A. Mironov, *Markov Processes* [in Russian], Radio i Svyaz', Moscow (1977).
13. M. Abramowitz and I. A. Stegun, eds., *Handbook of Mathematical Functions*, Dover, New York (1965).
14. A. P. Trifonov, Yu. E. Korchagin, and K. D. Titov, in: *2017 IEEE Conf. on "Dynamics of Systems, Mechanisms and Machines," Omsk, Russia, November 14–16, 2017*, p. 17467664.
15. A. P. Trifonov, Yu. É. Korchagin, and K. D. Titov, *Radiotekhnika*, No. 6, 99 (2016).