Maximum likelihood detection of rectangle ultra-wideband quasi-radiosignal with unknown duration

Yury E. Korchagin, Konstantin D. Titov, Konstantin S. Kalashnikov
Department of Radiophysics,
Voronezh State University, VSU
Voronezh, Russia
korchagin@phys.vsu.ru

Abstract**—***The maximum likelihood algorithm detection of rectangle ultra-wideband quasi-radiosignal with unknown amplitude, initial phase and duration has been synthesized. The statistical characteristics of the efficiency synthesized detection algorithm – false-alarm and missing probabilities have been found.*

Keywords**—***signal detection, maximum likelihood, ultra-wideband quasi-radiosignal, false-alarm probability, missing probability.*

I. Introduction

There is an acute task of radiosignal detection in practical applications of radio- and hydrolocation, navigation, seismology, radiocommunication and others, which has been considered in literature [1-3]. It was supposed that the radiosignal is narrowband [1-3]. Ultra-wideband (UWB) signals find a wider application in the modern world [4], but algorithms of optimal processing of such signals have not been researched enough. The work [5] considers the task of detection of a radiosignal, which does not satisfy the condition of relative narrowbandness and is called UWB quasi-radiosignal (QRS). Amplitude and initial phase of the UWB QRS were considered in [5] unknown. However, often apart from amplitude and initial phase the duration of the received signal appears to be unknown. Quasi-likelihood algorithms detection of UWB QRS with unknown amplitude, initial phase and duration were researched in [6], where instead of duration, unknown in advance, its expected, predicted value was used, and in [7] has been researched adaptation by duration. This work considers maximum likelihood algorithm of rectangle UWB QRS detection with unknown amplitude, initial phase and duration.

II. Problem Statement

Let the signal be a subject to detection

$$s(t,a,\varphi,\tau)=\begin{cases} a\cos(\omega t-\varphi), & 0\le t\le\tau, \\ 0, & t<0,\quad t>\tau, \end{cases} \tag{1}$$

where a, φ, ω, τ – amplitude, initial phase, frequency and duration [5].

The reported study was supported by Russian Science Foundation (research project No. 17-71-10057).

If frequency band $\Delta\omega$ and frequency ω satisfy condition

$$\Delta\omega \ll \omega . \tag{2}$$

the signal (1) is a narrowband radiosignal [2,8]. If condition (2) is not performed, formula (1) describes UWB QRS [5]. Values a, φ, ω are parameters of harmonic oscillations, used for its forming. Nevertheless, similarly to [5] we will then name a, φ, ω amplitude, initial phase and frequency of UWB QRS (1). Let us consider that signal (1) is received at the background of white Gaussian noise $n(t)$ with unilateral spectral density N_0, and true values of amplitude a_0, initial phase φ_0 and duration τ_0 are prior unknown. We will submit additive mixture of signal (1) and noise $n(t)$, observed during time period $t\in[0,T]$ in a form

$$\xi(t)=\gamma_0 s(t,a_0,\varphi_0,\tau_0)+n(t), \tag{3}$$

where γ is a discrete parameter, taking the value $\gamma=0$ in the absence of signal and $\gamma=1$ in the presence, and γ_0 as its unknown true value. We will consider that the signal duration can take values from the prior interval $\tau\in[T_1,T_2]$. Having the accepted realization (3), the receiving device should make a solution about signal presence or absence. Then the task of detection comes down to the evaluation of the parameter γ on the basis of the observed data.

III. Synthesis of the Detection Algorithm

For the synthesis of the algorithm of UWB QRS (estimation of the parameter γ) we will use the maximum likelihood (ML) method [1,2,8]. Under unknown signal parameters there is prior parameter uncertainty in relation to amplitude, initial phase, and duration. In any case, the logarithm of the likelihood ratio functional (LRF) depends on four unknown parameters

$$L(\gamma,a,\varphi,\tau)=\frac{2\gamma}{N_0}\int_0^{\tau}\xi(t)s(t,a,\varphi)dt-\frac{\gamma}{N_0}\int_0^{\tau}s^2(t,a,\varphi)dt . \tag{4}$$

According to the ML algorithm [1,2], the discrete parameter γ is estimated by the expression

$$\gamma_m = \arg\sup_{\gamma}\left[\sup_{a,\varphi,\tau} L(\gamma,a,\varphi,\tau)\right]. \tag{5}$$

Given, that $L(\gamma=0,a,\varphi,\tau)=0$, we obtained that ML detection algorithm consists in comparing absolute maximum of the logarithm LRF (5) with zero threshold

$$\gamma_m = \begin{cases} 1, & L>0, \\ 0, & L\le 0, \end{cases} \tag{6}$$

$$\begin{gathered} L=\sup_{\tau} L(\tau), \quad L(\tau)=\sup_{a,\varphi} L(a,\varphi,\tau)=L(a_m,\varphi_m,\tau), \\ (a_m,\varphi_m)=\arg\sup_{a,\varphi} L(a,\varphi,\tau), \; L(a,\varphi,\tau)=L(\gamma=1,a,\varphi,\tau). \end{gathered} \tag{7}$$

Similarly to [1,2], instead of the algorithm (6), we can used a generalized detection algorithm based on comparing the absolute maximum L of the logarithm LRF with a certain threshold h, not necessarily zero. If the relation $L>h$ is satisfied, then a decision is made about the presence of signal ($\gamma_m=1$), if $L<h$ – about its absence ($\gamma_m=0$).

Function $L(\tau)$ is a logarithm LRF in which instead of unknown amplitude and initial phase in the expressions (7) is used by their ML estimations a_m and φ_m. This is the equivalent to maximizing the logarithm LRF (4) for the unknown parameters. Performing analytical maximization logarithm LRF (4) for the variables a and φ, we obtain

$$\begin{aligned} L(\tau)=&\frac{1}{2\left(Q(\tau)^2-P_c(\tau)^2-P_s(\tau)^2\right)}\Big(\left(Q(\tau)-P_c(\tau)\right)X(\tau)^2+ \\ &+\left(Q(\tau)+P_c(\tau)\right)Y(\tau)^2-2X(\tau)Y(\tau)P_s(\tau)\Big), \end{aligned} \tag{8}$$

where

$$\begin{gathered} X(\tau)=\frac{2}{N_0}\int_0^{\tau}\xi(t)\cos\omega t dt, \quad Y(\tau)=\frac{2}{N_0}\int_0^{\tau}\xi(t)\sin\omega t dt, \\ Q(\tau)=\frac{\tau}{N_0}, \quad P_c(\tau)=\frac{\sin 2\omega\tau}{2\omega N_0}, \quad P_s(\tau)=\frac{1-\cos 2\omega\tau}{2\omega N_0}. \end{gathered} \tag{9}$$

Formulas (6), (8) determine the structure of the receiver. A detector should form a random process (8) for all possible values of duration and find its maximum. The solution about absence or presence of a signal is made on the basis of the comparison of the maximum value (8) with threshold h.

Maximum likelihood detector of the UWB QRS (6) can be realized on the basis of the flow chart at the Fig.1, where integrators (I) work at the time interval $[0,t]$, $t\in[0,T_2]$, PD – peak detector, TD – thresholder, that makes the comparison of the maximum value L with threshold h and makes the solution about absence or presence of a signal.

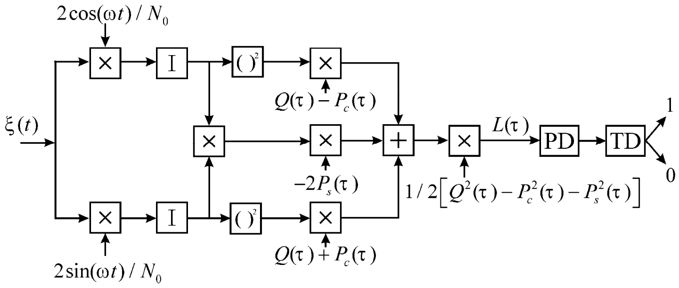

Fig.1. Block diagram of the ultra-wideband quasi-radiosignal detector

IV. CHARACTERISTICS OF THE DETECTION ALGORITHM

Let us make the analysis of the ML algorithm of detection (8), that find the probabilities of the false-alarm and the missing probability [1,2,9]. Let us consider $L_1(\tau)=\{L(\tau)|\gamma_0=1\}$ to be the logarithm LRF in the presence of signal in accepted realization, and $L_0(\tau)=\{L(\tau)|\gamma_0=0\}$ – in its absence.

Let us put observed realization (3) into the formulas (9) and select the deterministic and random components

$$X(\tau)=\gamma_0 a_0 S_x(\tau)+N_x(\tau), \; Y(\tau)=\gamma_0 a_0 S_y(\tau)+N_y(\tau), \tag{10}$$

where

$$\begin{gathered} S_x=\cos\varphi_0\left(Q(\min(\tau,\tau_0))+P_c(\min(\tau,\tau_0))\right)+P_s(\min(\tau,\tau_0))\sin\varphi_0, \\ S_y=\sin\varphi_0\left(Q(\min(\tau,\tau_0))-P_c(\min(\tau,\tau_0))\right)+P_s(\min(\tau,\tau_0))\cos\varphi_0, \\ N_x(\tau)=\frac{2}{N_0}\int_0^{\tau} n(t)\cos\omega t dt, \quad N_y(\tau)=\frac{2}{N_0}\int_0^{\tau} n(t)\sin\omega t dt \end{gathered}$$

— Gaussian random processes with zero mathematical expectations and correlation functions

$$\begin{gathered} K_x(\tau_1,\tau_2)=\langle N_x(\tau_1)N_x(\tau_2)\rangle=Q(\min(\tau_1,\tau_2))+P_c(\min(\tau_1,\tau_2)), \\ K_y(\tau_1,\tau_2)=\langle N_y(\tau_1)N_y(\tau_2)\rangle=Q(\min(\tau_1,\tau_2))-P_c(\min(\tau_1,\tau_2)), \\ K_{xy}(\tau_1,\tau_2)=\langle N_x(\tau_1)N_y(\tau_2)\rangle=\langle N_y(\tau_1)N_x(\tau_2)\rangle=P_s(\min(\tau_1,\tau_2)). \end{gathered}$$

To find the false-alarm probability let us research determinant statistics $L_0(\tau)$

$$\begin{aligned} L_0(\tau)=&\frac{1}{2\left(Q(\tau)^2-P_c(\tau)^2-P_s(\tau)^2\right)}\Big(\left(Q(\tau)-P_c(\tau)\right)N_x(\tau)^2+ \\ &+\left(Q(\tau)+P_c(\tau)\right)N_y(\tau)^2-2N_x(\tau)N_y(\tau)P_s(\tau)\Big). \end{aligned} \tag{11}$$

This is a random process with expected value

$$S_0(\tau)=\langle L_0(\tau)\rangle=1 \tag{12}$$

and correlation function

$$K_0(\tau_1,\tau_2)=\langle[L_0(\tau_1)-\langle L_0(\tau_1)\rangle][L_0(\tau_2)-\langle L_0(\tau_2)\rangle]\rangle= \\ =\frac{\Psi(\tau_1,\tau_2)}{\Psi(\max(\tau_1,\tau_2),\max(\tau_1,\tau_2))}, \tag{13}$$

where $\Psi(\tau_1,\tau_2)=Q(\tau_1)Q(\tau_2)-P_c(\tau_1)P_c(\tau_2)-P_s(\tau_1)P_s(\tau_2)$.

Let us investigate the local properties of a random process (11). To do this, we consider the behavior of the correlation function (13) in a small neighborhood of an arbitrary point $\tau\in[T_1,T_2]$. Substituting in (13) $\tau_1=\tau,\ \tau_2=\tau+\Delta$ and let us decompose correlation function in a Taylor series by Δ in the neighborhood of τ, discarding all terms above the first degree Δ

$$K_0(\tau,\tau+\Delta)=1-\delta(\tau)|\Delta|+o(\Delta), \tag{14}$$

where

$$\delta(\tau)=\frac{1}{\Psi(\tau,\tau)}\frac{\partial\Psi(\tau,x)}{\partial x}\bigg|_{x=\tau}= \\ =\frac{Q(\tau)Q'(\tau)-P_c(\tau)P_c'(\tau)-P_s(\tau)P_s'(\tau)}{Q^2(\tau)-P_c^2(\tau)-P_s^2(\tau)}, \tag{15}$$

and the prime denotes the derivative of τ. According to (12) and (14), the determinant statistics $L_0(\tau)$ is a locally stationary and locally Markovian random process. The probability of failure for the border h in such a process the ε neighborhood of τ found in [1]

$$F_\varepsilon(h,\tau)=P\left\{L_0(x)<h,x\in\left[\tau-\frac{\varepsilon}{2},\tau+\frac{\varepsilon}{2}\right]\right\}=\begin{cases}e^{-\delta(\tau)\varepsilon he^{-h}}, h\ge1,\\ 0,\ h<1.\end{cases} \tag{16}$$

We divide the a priori interval $[T_1,T_2]$ of possible values of duration into N equal segments by the value $\varepsilon=(T_2-T_1)/N$. We designate the middle of each interval, $t_i=T_1+(i-1)\varepsilon/2$, $i=\overline{1,N}$. Then the probability of failure to reach the boundary h by determinant statistics $L_0(\tau)$ on the i-th interval is

$$F_{0i}(h)=P\{L_0(\tau)<h,\tau\in[t_i-\varepsilon/2,t_i+\varepsilon/2]\}=F_\varepsilon(h,t_i). \tag{17}$$

The false-alarm probability expressed in terms of failure probability threshold for a random process $L_0(\tau)$ in the interval $[T_1,T_2]$

$$\alpha=1-F_0(h)=P\{L_0(\tau)<h,\tau\in[T_1,T_2]\}=\prod_{i=1}^{N}F_{0i}(h,t_i). \tag{18}$$

Putting (16) in (17), and then (17) в (18) and passing to the limit as $\varepsilon\to0$, $N\to\infty$, we obtain

$$\alpha=\begin{cases}1-e^{-he^{-h}\int_{T_1}^{T_2}\delta(\tau)d\tau},\ h\ge1,\\ 1,\ h<1.\end{cases}$$

After integrating the function (15), we find the asymptotic expression for the false-alarm probability

$$\alpha=\begin{cases}1-\left[\frac{Q^2(T_1)-P_c^2(T_1)-P_s^2(T_1)}{Q^2(T_2)-P_c^2(T_2)-P_s^2(T_2)}\right]^{\frac{he^{-h}}{2}},\ h\ge1,\\ 1,\ h<1.\end{cases} \tag{19}$$

Let us then find a formula for conditional the missing probability, for which we will research determinant statistics $L_1(\tau)$ in the presence of signal in accepted realization, coincides with (8). We later denote

$$z^2=2a_0^2T_2/N_0 \tag{20}$$

– signal-to-noise ratio (SNR) at the output of the ML receiver for a square signal with amplitude a_0 and duration T_2. Substituting (10) into (8), we obtain the expression for determinant statistics in the presence of a signal

$$L_1(\tau)=z^2\frac{(q-p_c)s_x^2+(q+p_c)s_y^2-2p_ss_xs_y}{4(q^2-p_c^2-p_s^2)}+ \\ +z\frac{(q-p_c)s_x\eta_x+(q+p_c)s_y\eta_y-p_s(s_y\eta_x+s_x\eta_y)}{\sqrt{2}(q^2-p_c^2-p_s^2)}+ \\ +\frac{(q-p_c)\eta_x^2+(q+p_c)\eta_y^2-2p_s\eta_x\eta_y}{2(q^2-p_c^2-p_s^2)}, \tag{21}$$

where

$$\eta_x=\eta_x(\tau)=N_x(\tau)\sqrt{N_0/T_2},\ \eta_y=\eta_y(\tau)=N_y(\tau)\sqrt{N_0/T_2} \tag{22}$$

– Gaussian random processes with zero mathematical expectations and correlation functions

$$K_{\eta x}(\tau_1,\tau_2)=\langle\eta_x(\tau_1)\eta_x(\tau_2)\rangle=q(\min(\tau_1,\tau_2))+p_c(\min(\tau_1,\tau_2)),$$
$$K_{\eta y}(\tau_1,\tau_2)=\langle\eta_y(\tau_1)\eta_y(\tau_2)\rangle=q(\min(\tau_1,\tau_2))-p_c(\min(\tau_1,\tau_2)),$$
$$K_{\eta xy}(\tau_1,\tau_2)=\langle\eta_x(\tau_1)\eta_y(\tau_2)\rangle=\langle\eta_y(\tau_1)\eta_x(\tau_2)\rangle=p_s(\min(\tau_1,\tau_2))$$

and normalized functions independent of SNR

$$q=q(\tau)=N_0Q(\tau)/T_2,$$
$$p_c=p_c(\tau)=N_0P_c(\tau)/T_2,\ p_s=p_s(\tau)=N_0P_s(\tau)/T_2,$$
$$s_x=s_x(\tau)=N_0S_x(\tau)/T_2,\ s_y=s_y(\tau)=N_0S_y(\tau)/T_2.$$

For sufficiently large SNR $z\gg 1$ of the last term in (21) can be neglected, and write approximately

$$L_1(\tau)\approx z^2\frac{(q-p_c)s_x^2+(q+p_c)s_y^2-2p_ss_xs_y}{4\left(q^2-p_c^2-p_s^2\right)}+ +z\frac{(q-p_c)s_x\eta_x+(q+p_c)s_y\eta_y-p_s\left(s_y\eta_x+s_x\eta_y\right)}{\sqrt{2}\left(q^2-p_c^2-p_s^2\right)}. \quad (23)$$

According to (23) a random process $L_1(\tau)$ is Gaussian. Thus, for its full statistical description it is enough to find expected value and correlation function. Making averaging out, we get mathematical expectation in the presence of signal

$$S_1=\langle L_1(\tau)\rangle=z^2\frac{(q-p_c)s_x^2+(q+p_c)s_y^2-2p_ss_xs_y}{4\left(q^2-p_c^2-p_s^2\right)} \quad (24)$$

and correlation function

$$\begin{aligned}K(\tau_1,\tau_2)&=\langle[L_1(\tau_1)-\langle L_1(\tau_1)\rangle][L_1(\tau_2)-\langle L_1(\tau_2)\rangle]\rangle=\\&=A_1(\tau_1)A_1(\tau_2)\left(q(\min(\tau_1,\tau_2))+p_c(\min(\tau_1,\tau_2))\right)+\\&+p_s(\min(\tau_1,\tau_2))\left(A_1(\tau_1)A_2(\tau_2)+A_1(\tau_2)A_2(\tau_1)\right)+\\&+A_2(\tau_1)A_2(\tau_2)\left(q(\min(\tau_1,\tau_2))-p_c(\min(\tau_1,\tau_2))\right),\end{aligned} \quad (25)$$

where

$$A_1(\tau)=z\frac{(q-p_c)s_x-p_ss_y}{\sqrt{2}\left(q^2-p_c^2-p_s^2\right)},\ A_2(\tau)=z\frac{(q+p_c)s_y-p_ss_x}{\sqrt{2}\left(q^2-p_c^2-p_s^2\right)}.$$

As it is known, with the growth of the SNR the maximum position of determinant statistics converges in mean square to the true meaning of an unknown parameter τ_0 [1,10]. We therefore explore logarithm LRF (23) near the true meaning of duration τ_0. Expanding forms (24) and (25) in a Taylor series by τ near τ_0 we get asymptotic forms for mathematical expectation

$$S_1\approx\frac{\lambda_1}{2}+\frac{\tau-\tau_0}{2T_2}\begin{cases}\psi_1, & \tau\le\tau_0,\\-\psi_1, & \tau>\tau_0,\end{cases} \quad (26)$$

and correlation function

$$K_1=K_1(\tau_1,\tau_2)\approx\lambda_1+\psi_1\min(\tau_1-\tau_0,\tau_2-\tau_0)/T_2, \quad (27)$$

where $\lambda_1=\frac{z^2}{2}\left(q(\tau_0)+p_c(\tau_0)\cos 2\varphi_0+p_s(\tau_0)\sin 2\varphi_0\right)$, $\psi_1=z^2\cos^2(\omega\tau_0-\varphi_0)$.

We will approximate the logarithm LRF (23) by Gaussian random process $\mu_1(\tau)$ with mathematical expectation (26) and correlation function (27) over the entire interval of a priori values of duration. Using formulas (26), (27) and Doob's theorem [9], it can be shown that the determinant statistics $\mu_1(\tau)$ is Gaussian Markovian process with drift coefficient k_{11} and diffusion coefficient k_{21} [10]

$$k_{11}=\psi_1/2T_2,\qquad k_{21}=\psi_1/T_2. \quad (28)$$

The missing probability is by the definition

$$\beta=F_1(h)=P\{\mu_1(\tau)<h,\ \tau\in[T_1,T_2]\} \quad (29)$$

– the failure probabilities to achieve borders $y=-\infty$ and $y=h$ by Markovian random process $\mu_1(\tau)$ at the interval $\tau\in[T_1,T_2]$. The searched probability (30) can be expressed through probability density $W(y,\tau)$ of the random process realizations $\mu_1(\tau)$, which have never achieved borders $y=-\infty$, $y=h$ [10]

$$F_1(h)=\int_{-\infty}^{h}W(y,T_2)\,dy. \quad (30)$$

Function $W(y,\tau)$ is the solution to the Fokker-Planck-Kolmogorov (FPK) equation with coefficients (28)

$$\frac{\partial W(y,\tau)}{\partial\tau}+\frac{\partial}{\partial y}\left[k_1W(y,\tau)\right]-\frac{1}{2}\frac{\partial^2}{\partial y^2}\left[k_2W(y,\tau)\right]=0, \quad (31)$$

under initial condition

$$W(y,\tau_0)=\frac{1}{\sqrt{2\pi\lambda_1}}e^{-\frac{(y-\lambda_1/2)^2}{2\lambda_1}}$$

and boundary conditions

$$W(-\infty,\tau)=W(h,\tau)=0\,.$$

Solving the FPK equation by the reflection method with a change in sign [7,10], putting found solution in (30), and then (29), we get a formula for the missing probability

$$\beta=\frac{e^{-\chi/4}}{2\cdot\sqrt{2\pi\left(\sigma^2+2\chi\right)}}\times\int\limits_0^\infty e^{-\frac{2\chi(h-S_1)^2+\xi\sigma^2(\xi+6\chi)}{4\chi\sigma^2}}\times$$
$$\times\left[\Phi\left(\frac{\chi-\xi}{\sqrt{2\chi}}\right)-e^{\xi}\Phi\left(\frac{\chi+\xi}{\sqrt{2\chi}}\right)\right]\left[e^{-\frac{\left(2\chi(h-S_1)+\sigma^2(\chi-\xi)\right)^2}{4\chi\sigma^2(\sigma^2+2\chi)}}\times\right.$$
$$\times\left(1+\Phi\left(\frac{2\chi(h-S_1)+\sigma^2(\chi-\xi)}{2\sqrt{\chi\sigma^2\left(\sigma^2+2\chi\right)}}\right)\right)-$$
$$\left.-e^{-\frac{\left(2\chi(h-S_1)+\sigma^2(\chi+\xi)\right)^2}{4\chi\sigma^2(\sigma^2+2\chi)}}\left(1+\Phi\left(\frac{2\chi(h-S_1)+\sigma^2(\chi+\xi)}{2\sqrt{\chi\sigma^2\left(\sigma^2+2\chi\right)}}\right)\right)\right]d\xi. \tag{32}$$

where $\chi=\frac{z_0^2(\delta-1)\cos^2(\varphi_0-4\pi\kappa)}{\delta+1}$, $\sigma^2=K_1$ and $\kappa=\omega\tau_0/2\pi$ – parameter of narrowbandness, which is equal to the number of periods of harmonic oscillations (1), settling at the signal duration τ_0.

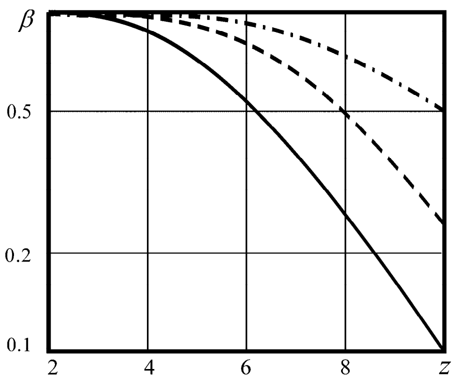

Fig. 2. The dependence of the missing probability on SNR

Fig.2 shows dependencies of the missing probability (32) from SNR (20) under different values of the false-alarm probability: solid-line curve corresponds to the false-alarm probability $\alpha=0.1$, dashed-line curve – $\alpha=0.01$, and dash-dotted line – $\alpha=0.001$. The threshold h was obtained from (19) in accordance with the Neyman-Pearson criterion [2]. In the calculation of lines at fig.2 it was supposed that initial phase of received signal $\varphi_0=0$, $T_2/T_1=4$, $\kappa=0.3$.

V. Conclusion

As the figure show, the missing probability decreases with increasing SNR. The a priori ignorance of the amplitude and initial phase of a rectangle UWB QRS and unknown duration complicates the detector structure. The results obtained us to calculate the values characterizing the efficiency detecting of a rectangular UWB QRS and calculate the required threshold by the Neyman-Pearson criterion. The expression for the false-alarm probability and the missing probability are approximate and their accuracy increases with increasing SNR.

References

[1] Carl W. Helstrom, Elements of Signal Detection and Estimation, Englewood Cliffs, Ney Jersey: PTR Prentice Hall, 1995.
[2] H. L. Van Trees, Detection, Estimation, and Modulation Theory, Part 1, New York: John Wiley & Sons, 2001.
[3] A.P. Trifonov, Y.É. Korchagin, M.V. Trifonov, “Detection of radio signals with unknown duration, amplitude and initial phase”, Radiophysics and Quantum Electronics, vol. 58, No. 5, pp. 361-372, October, 2015. DOI: 10.1007/s11141-015-9610-5.
[4] R. Aiello, A. Batra, Ultra Wideband Systems Technologies and Applications, U.K., London: Newnes Publication, 2006.
[5] A.P. Trifonov, P.E. Rudnev, “Detection of an ultra-wideband quasi radio signal against background random noise,” Radiophysics and Quantum Electronics, vol. 52, No. 9, pp. 671-681, September, 2009. DOI: 10.1007/s11141-010-9175-2.
[6] A.P. Trifonov, Y.É. Korchagin, K.D. Titov, “Quasi-likelihood detection of ultra-wideband quasi-radiosignal of a random shape with unknown duration”, Radiotechnics, No. 6, pp. 99-105, 2016.
[7] A.P. Trifonov, Y.É. Korchagin, K.D. Titov, “Quasi-likelihood detection of rectangle ultra-wideband quasi-radiosignal”, 2017 International Siberian Conference on Control and Communications (SIBCON). DOI: 10.1109/SIBCON.2017.7998454.
[8] M. Barkat, Signal Detection and Estimation, 2nd ed., Massachusetts: ARTECH HOUSE, INC., 2005.
[9] J.L. Doob, Stochastic processes, New York: John Wiley, 1953.
[10] Y.É. Korchagin, “Estimation duration of signal with unknown amplitude and phase”, Radiotechnics, No. 9, 2013.