Estimating Time of Arrival and Duration of the Rectangular Radio Signal with the Unknown Initial Phase

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Abstract. Quasi-likelihood, maximum likelihood and quasi-optimal algorithms for the estimation of the time of arrival and the duration of the rectangular radio signal with the unknown initial phase are synthesized. The asymptotic characteristics of their effectiveness are found. It is shown that the application of the suggested quasi-optimal approach makes the technical implementation of the measurer considerably easier, without appreciable losses in its performance.

1. Introduction

Reception of the signal with unknown time of arrival and duration is enough interesting problem for the researchers dealing with the many practical applications of communications theory, radiolocation, navigation and seismology. In paper [1], the problem of joint maximum likelihood (ML) estimation of the time of arrival and the duration of the rectangular radio pulse is considered. The estimates of the time of arrival and the duration of the free-form signal are studied in [2]. And in [3], there are examined the estimates of the time of arrival and the duration of the free-form signal with the unknown amplitude. However, in many cases the high-frequency signals (radio signals) are processed having an unknown initial phase due to the particular propagation features. Therefore, it is advisable to consider the algorithms of the joint estimation of the time of arrival and the duration of the radio signal with the unknown initial phase. In this paper, we have the algorithms for the estimation of the time of arrival and the duration of the rectangular radio pulse with the unknown initial phase synthesized by means of the ML method. For the synthesized algorithms the asymptotic characteristics of their performance are found, their accuracy increasing with a signal-to-noise ratio (SNR).

The model of a rectangular radio pulse with unknown initial phase, time of arrival and duration can be written as

$$s(t,\varphi_0,\lambda_0,\tau_0) = a\cos(\omega t - \varphi_0)\, I\left(\frac{t-\lambda_0}{\tau_0}\right), \qquad I(x) = \begin{cases} 1, & |x| \le 1/2, \\ 0, & |x| > 1/2. \end{cases} \tag{1}$$

Here a, φ_0, λ_0, τ_0 are signal amplitude, initial phase, time of arrival and duration. We presuppose that the signal phase φ_0 is located within the interval $[0,2\pi]$. The time of arrival λ_0 and the duration τ_0 can range from prior domain Λ, which is described by the inequalities $|\lambda_0| \leq \Lambda_0/2$, $T_1 \leq \tau_0 \leq T_2$, and the signal (1) is received against Gaussian white noise $n(t)$ with the one-sided spectral density N_0. In order to guarantee that the signal would not disappear before it appears, the following inequality is required: $\Lambda_0 \leq T_1$.

According to the process

$$\xi(t) = s(t, \varphi_0, \lambda_0, \tau_0) + n(t)$$

observable within the interval $[-T/2, T/2]$, the measurer should produce the joint estimates of the time of arrival and the duration.

If the initial phase of the useful signal (1) is a priori known, then the ML estimation algorithm can be applied. According to this algorithm, the estimates of the time of arrival and the duration coincide with the coordinates that belong to the position of the greatest maximum of the logarithm of the functional of likelihood ratio (FLR) [4]. However, when the time of arrival and the duration are unknown, together with the initial phase, the logarithm of FLR depends on the three variables [4]:

$$L(\varphi, \lambda, \tau) = \frac{2a}{N_0} \int_{\lambda-\tau/2}^{\lambda+\tau/2} \xi(t) \cos(\omega t - \varphi)\, dt - \frac{a^2 \tau}{2N_0}. \tag{2}$$

The last expression is obtained on condition that $\omega T_1 >> 1$.

By substituting some values of the initial phase into Eq. (2) instead of the unknown value φ, we can obtain a number of estimation algorithms for the time of arrival and the duration (both optimal and non-optimal). These values for the initial phase can be fixed, or otherwise they are determined by the realization of the observable data. The resulted estimation algorithms considered below are distinguished by both efficiency and simplicity their either hardware or software implementation.

2. Quasi-likelihood estimation algorithm

One of the ways to overcome the prior parametric uncertainty concerning the initial phase is to apply the quasi-likelihood (QL) estimation algorithm [5]. The QL receiver should generate the logarithm of FLR (2) for some expected value of the initial phase φ^* and for all the possible values of the time of arrival and the duration:

$$L^*(\lambda, \tau) = L(\varphi^*, \lambda, \tau). \tag{3}$$

Then, the QL estimates of the time of arrival and the duration are determined as positions of the absolute (greatest) maximum of the random field (3):

$$(\lambda^*, \tau^*) = \arg\sup L^*(\lambda, \tau), \tag{4}$$

Let us pass to new variables in Eq. (3):

$$\theta_1 = \lambda - \tau/2, \qquad \theta_2 = \lambda + \tau/2. \tag{5}$$

Here θ_1, θ_2 are the current values of the positions of rising $\theta_{01} = \lambda_0 - \tau_0/2$ and falling $\theta_{02} = \lambda_0 + \tau_0/2$ edges of the signal (1), respectively. Let us designate the domain of their possible values as Θ. It is clear that the linear transformations (5) are one-to-one, therefore, if we define the characteristics of the QL estimates positions of the signal edges θ_1^* and θ_2^*, then we could be able to find characteristics of QL estimates of the time of arrival and the duration

$$\lambda^* = \left(\theta_2^* + \theta_1^*\right)/2, \qquad \tau^* = \theta_2^* - \theta_1^*. \tag{6}$$

For unknown parameters θ_{01} and θ_{02} the logarithm of FLR (3) takes the form of

$$L^*(\theta_1, \theta_2) = L_1^*(\theta_1) + L_2^*(\theta_2), \tag{7}$$

where

$$L_1^*(\theta_1) = \frac{2a}{N_0} \int_{\theta_1}^{\theta} \xi(t) \cos\left(\omega t - \varphi^*\right) dt - \frac{a^2(\theta - \theta_1)}{2N_0}, \tag{8}$$

$$L_2^*(\theta_2) = \frac{2a}{N_0} \int_{\theta}^{\theta_2} \xi(t) \cos\left(\omega t - \varphi^*\right) dt - \frac{a^2(\theta_2 - \theta)}{2N_0}. \tag{9}$$

Here θ is a fixed point from the interval $\left[-(\mathrm{T}_1 - \Lambda_0)/2, (\mathrm{T}_1 - \Lambda_0)/2\right]$. The QL estimates θ_1^* and θ_2^* are determined as positions of the greatest maximum of the random field (7) with $(\theta_1, \theta_2) \in \Theta$. In order to simplify the measurer implementation and the analytical determination of the estimates characteristics, we widen the prior domain of the values $(\theta_1, \theta_2) \in \Theta$ into the square of the minimum area Θ_a. The sides of this square are parallel to the axes θ_1, θ_2 and the square includes the domain Θ. Thereafter the range of values belonging to Θ_a is presented by the inequalities

$\theta_{1\min} = -(\mathrm{T}_2 - \Lambda_0)/2 \le \theta_1 \le -(\mathrm{T}_1 - \Lambda_0)/2 = \theta_{1\max}$, $\theta_{2\min} = (\mathrm{T}_1 - \Lambda_0)/2 \le \theta_2 \le (\mathrm{T}_2 - \Lambda_0)/2 = \theta_{2\max}$.

According to Eqs. (8), (9), the random processes $L_1^*(\theta_1)$ and $L_2^*(\theta_2)$ are statistically independent as they are the integrals of white noise on the non-overlapping intervals. Hence, the maximum positions of the random field $L^*(\theta_1, \theta_2)$ by the variables θ_1 and θ_2 coincide with the maximum positions of the random processes (8) and (9), respectively. Consequently, the QL estimates of the moments of appearance and disappearance can be found as

$$\theta_i^* = \arg\sup L_i^*(\theta_i), \qquad i = 1,2, \qquad (\theta_1, \theta_2) \in \Theta_a. \tag{10}$$

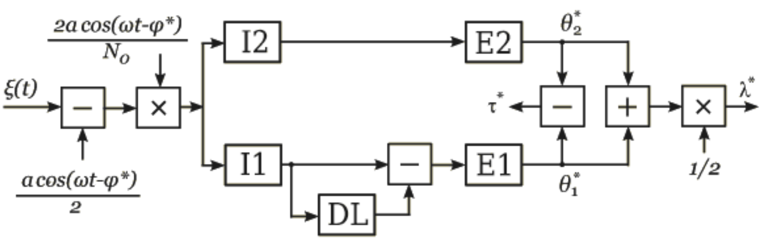

Fig.1 Block diagram of the quasi-likelihood measurer of the time of arrival and the duration

The block diagram of the QL measurer of the time of arrival and the duration is shown in Fig. 1. The designations here are: I1 is the integrator over the interval $[\theta_{1\min}, t]$, where $t \in [\theta_{1\min}, \theta]$, I2 is the integrator over the interval $[\theta, t]$, where $t \in [\theta, \theta_{2\max}]$, DL is the delay line for $t = \theta - \theta_{1\min}$, E1 and E2 are extremators searching the positions of the signal maxima within the time intervals $[\theta, \theta + \theta_{1\max} - \theta_{1\min}]$ and $[\theta_{2\min}, \theta_{2\max}]$, which, in their turn, are the QL estimates of the moments of appearance and disappearance, respectively. Out of them the estimates (6) are formed.

The statistical characteristics of the QL estimates of the moments of appearance and disappearance (10) are examined in [6]. It is shown that the estimates θ_1^* and θ_2^* are statistically independent and that they have the biases

$$B\left(\theta_i \middle| \theta_{0i}\right) = (-1)^i \, 2\tau_0 \left(R^2 - 1\right) \Big/ z_0^2 R^2 , \qquad i = 1,2 \tag{11}$$

and the equal variances

$$V\left(\theta_i \middle| \theta_{0i}\right) = 8\tau_0^2 \left[R^5 \left(2R^2 + 6R + 5\right) + 5R^2 + 6R + 2\right] \Big/ z_0^4 R^4 (R+1)^3 , \tag{12}$$

where $z_0^2 = a^2 \tau_0 / N_0$ is the SNR at the ML measurer output, $R = 2\cos\Delta\varphi - 1$, $\Delta\varphi = \varphi^* - \varphi_0$ is the initial phase mismatch of the QL measurer.

The bias and the variance of the QL estimates of the time of arrival and the duration (4) can be expressed by means of the bias and the variance of the QL estimates of the moments of appearance and disappearance as

$$B\left(\tau^* \middle| \lambda_0, \tau_0\right) = B\left(\theta_2^* \middle| \theta_{02}\right) - B\left(\theta_1^* \middle| \theta_{01}\right), \qquad B\left(\lambda^* \middle| \lambda_0, \tau_0\right) = \left[B\left(\theta_2^* \middle| \theta_{02}\right) + B\left(\theta_1^* \middle| \theta_{01}\right) \right] \Big/ 2 . \tag{13}$$

$$V\left(\tau^* \middle| \lambda_0, \tau_0\right) = V\left(\theta_2^* \middle| \theta_{02}\right) + V\left(\theta_1^* \middle| \theta_{01}\right), \qquad V\left(\lambda^* \middle| \lambda_0, \tau_0\right) = \left[V\left(\theta_2^* \middle| \theta_{02}\right) + V\left(\theta_1^* \middle| \theta_{01}\right) \right] \Big/ 4 . \tag{14}$$

According to Eqs. (5), (6), we can write down the expression for the correlation coefficient of estimates of the time of arrival and the duration as

$$K\left(\lambda^*, \tau^* \middle| \lambda_0, \tau_0\right) = \left[V\left(\theta_2^* \middle| \theta_{02}\right) - V\left(\theta_1^* \middle| \theta_{01}\right) \right] \Big/ \left[V\left(\theta_2^* \middle| \theta_{02}\right) + V\left(\theta_1^* \middle| \theta_{01}\right) \right] = 0 .$$

By substituting Eqs. (11), (12) into Eqs. (13), (14), we obtain the asymptotic formulas for the biases and the variances of the estimates of the time of arrival and the duration:

$$B\left(\tau^* \middle| \lambda_0, \tau_0\right) = 4\tau_0 P_B(R) / z_0^2 , \qquad B\left(\lambda^* \middle| \lambda_0, \tau_0\right) = 0 , \tag{15}$$

$$V\left(\tau^* \middle| \lambda_0, \tau_0\right) = 16\tau_0^2 P_V(R) / z_0^4 , \qquad V\left(\lambda^* \middle| \lambda_0, \tau_0\right) = 4\tau_0^2 P_V(R) / z_0^4 , \tag{16}$$

where

$$P_B(R) = \left[R^3 (R+2) - 2R - 1\right] \Big/ R^2 (R+1)^2 , \quad P_V(R) = \left[R^5 \left(2R^2 + 6R + 5\right) + 5R^2 + 6R + 1\right] \Big/ R^4 (R+1)^3 .$$

If $\Delta\varphi = 0$, then expressions (15), (16) coincide with the expressions for the biases $B_{0\lambda}$, $B_{0\tau}$ and the variances $V_{0\lambda}$, $V_{0\tau}$ of the ML estimates of the time of arrival and the duration of the radio signal with the a priori known initial phase obtained in [1], namely:

$$B_{0\tau} = B_{0\lambda} = 0, \qquad V_{0\tau} = 52\tau_0^2 / z_0^4 , \qquad V_{0\lambda} = 13\tau_0^2 / z_0^4 . \tag{17}$$

Influence of the prior ignorance of the initial phase upon the accuracy of the QL estimates (4) of the time of arrival and the duration can be characterized by means of the normalized bias $b_\tau = B\left(\tau^* \middle| \lambda_0, \tau_0\right) \Big/ \sqrt{V\left(\tau^* \middle| \lambda_0, \tau_0\right)}$ and variances $v = V\left(\tau^* \middle| \lambda_0, \tau_0\right) / V_{0\tau} = V\left(\lambda^* \middle| \lambda_0, \tau_0\right) / V_{0\lambda}$ of the QL estimates.

In Fig. 2 the dependence is presented of the normalized bias of the duration estimate b_τ on the mismatch of the initial phase $\Delta\varphi$. Fig. 3 demonstrates the coinciding dependencies of the normalized variances of the estimates of the time of arrival and the duration v on the mismatch of the initial phase $\Delta\varphi$. As can be seen from Figs. 2, 3, under the a priori known initial phase ($\Delta\varphi = 0$), the QL estimates of the signal time of arrival and duration have a zero bias and their variances coincide with the variances of the ML estimates. It is obvious that the non-zero initial phase mismatch leads to the appearance of the bias of the estimate of the duration and then – to the increase by ten times of the variances of the time parameters.

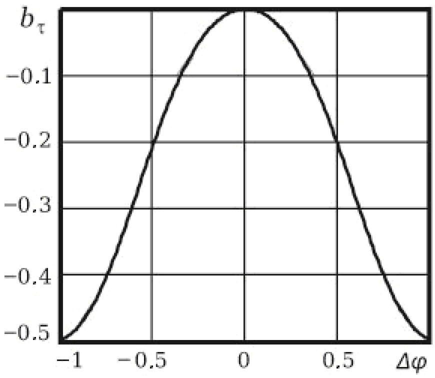

Fig.2 Normalized bias of the duration estimate.

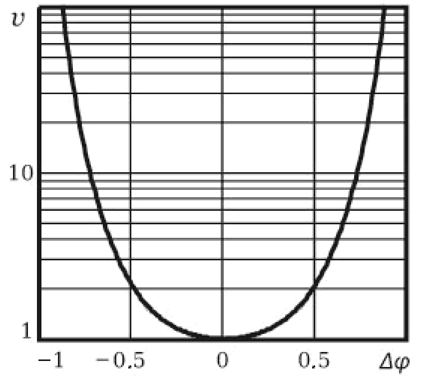

Fig.3 Normalized variance of the estimates of the arrival time and the duration.

3. Maximum likelihood estimation algorithm

In order to improve the accuracy of the estimation of the time of arrival and the duration, the ML algorithm can be applied. According to this algorithm, the unknown initial phase in Eq. (2) should be changed by its ML estimate φ_m. That is equivalent to the maximization of the logarithm of FLR (2) by the current value φ of the initial phase:

$$L(\lambda, \tau) = L(\varphi_m, \lambda, \tau) = \max_{\varphi} L(\varphi, \lambda, \tau). \tag{18}$$

The ML estimates of the time of arrival and the duration are determined as positions of the greatest maximum of decision statistics (18):

$$(\lambda_m, \tau_m) = \arg\sup L(\lambda, \tau). \tag{19}$$

The maximization of the logarithm of FLR (2) by the variable φ can be carried out analytically. As a result we get

$$L(\lambda, \tau) = \sqrt{X^2(\lambda, \tau) + Y^2(\lambda, \tau)} - a^2\tau/2N_0, \tag{20}$$

where $X(\lambda, \tau) = \frac{2a}{N_0} \int_{\lambda-\tau/2}^{\lambda+\tau/2} \xi(t)\cos(\omega t)\,dt$, $Y(\lambda, \tau) = \frac{2a}{N_0} \int_{\lambda-\tau/2}^{\lambda+\tau/2} \xi(t)\sin(\omega t)\,dt$, and the integrals from the terms, oscillating with the doubled frequency, are neglected.

In terms of Eq. (20), the optimal measurer structure can be constructed. But it should be noted that it is not possible to produce the decision statistics (20) as continuous function of the time of arrival and the duration. In this case, the measurer should generate the samples $L(\lambda_k, \tau_l) = L(k\Delta\lambda - \Lambda_0/2, \mathrm{T}_1 + l\Delta\tau)$ of the random field (20) for a discrete value set of the time of arrival and the duration. Here $\Delta\lambda$, $\Delta\tau$ are discretization steps on the parameters λ_0 and τ_0 accordingly, $k = \overline{1, n_1}$, $l = \overline{1, n_2}$, $n_1 = \{\Lambda_0/\Delta\lambda\}$, $n_2 = \{(\mathrm{T}_2 - \mathrm{T}_1)/\Delta\tau\}$ and $\{\cdot\}$ is an integral part. Moreover, the more accurately you need to estimate the time of arrival and the duration, the greater must be the values of n_1 and n_2, as well as the number of the channels necessary for the construction of the receiver. The necessity to form a two-dimensional random field causes problems in the hardware implementation of the estimation algorithm (20), since it is required to use a multi-channel reception.

In Fig. 4 the block diagram of a single channel of the ML measurer is represented, forming the logarithm of FLR (20) for the fixed values λ_k and τ_l. Here the integrators over the time interval $[\lambda_k - \tau_l/2, \lambda_k + \tau_l/2]$ are designated through "I".

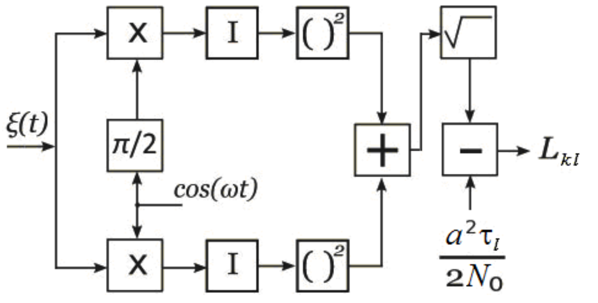

Fig.4 Block diagram of the quasi-likelihood measurer of the time of arrival and the duration.

In order to simplify the ML measurer hardware and software implementation and the determination of this measurer characteristics, it is possible to make use of the quasi-optimal (QO) estimates. For this purpose we pass to the new variables (5) in expression (2):

$$L(\varphi,\theta_1,\theta_2) = \frac{2a}{N_0}\int_{\theta_1}^{\theta_2} \xi(t)\cos(\omega t - \varphi)\,dt - \frac{a^2(\theta_2 - \theta_1)}{2N_0} \tag{21}$$

and present the logarithm of FLR (21) as the sum $L(\varphi,\theta_1,\theta_2) = L_1(\varphi,\theta_1) + L_2(\varphi,\theta_2)$ of the two summands

$$L_1(\varphi,\theta_1) = \frac{2a}{N_0}\int_{\theta_1}^{\theta} \xi(t)\cos(\omega t - \varphi)\,dt - \frac{a^2(\theta - \theta_1)}{2N_0}, \tag{22}$$

$$L_2(\varphi,\theta_2) = \frac{2a}{N_0}\int_{\theta}^{\theta_2} \xi(t)\cos(\omega t - \varphi)\,dt - \frac{a^2(\theta_2 - \theta)}{2N_0}. \tag{23}$$

Let us designate $L_{\varphi i}(\theta_i) = \max\limits_{\varphi} L_i(\varphi,\theta_i)$ and introduce the estimates

$$\theta^*_{mi} = \arg\sup L_{\varphi i}(\theta_i). \tag{24}$$

QO estimates (24), generally speaking, are not the ML ones but the application of the estimates (24) allows considerable simplification of the technical measurer implementation.

The maximization of functions (22), (23) by the variable φ is the carried out, similar to the one presented in [1,6], and thus we obtain

$$L_{\varphi 1}(\theta_1) = \max_{\varphi} L_1(\varphi,\theta_1) = \sqrt{X_1^2(\theta_1) + Y_1^2(\theta_1)} - a^2(\theta - \theta_1)/2N_0, \tag{25}$$

$$L_{\varphi 2}(\theta_2) = \max_{\varphi} L_2(\varphi,\theta_2) = \sqrt{X_2^2(\theta_2) + Y_2^2(\theta_2)} - a^2(\theta_2 - \theta)/2N_0, \tag{26}$$

where

$$X_1(\theta_1) = \frac{2a}{N_0}\int_{\theta_1}^{\theta} \xi(t)\cos(\omega t)\,dt, \qquad Y_1(\theta_1) = \frac{2a}{N_0}\int_{\theta_1}^{\theta} \xi(t)\sin(\omega t)\,dt,$$

$$X_2(\theta_2)=\frac{2a}{N_0}\int_{\theta}^{\theta_2}\xi(t)\cos(\omega t)\,dt\,,\qquad Y_2(\theta_2)=\frac{2a}{N_0}\int_{\theta}^{\theta_2}\xi(t)\sin(\omega t)\,dt\,.$$

With estimates (24) available, the QO estimates of the time of arrival and the duration can be obtained as

$$\lambda_m^*=\left(\theta_{m2}^*+\theta_{m1}^*\right)/2\,,\qquad \tau_m^*=\theta_{m2}^*-\theta_{m1}^*\,. \tag{27}$$

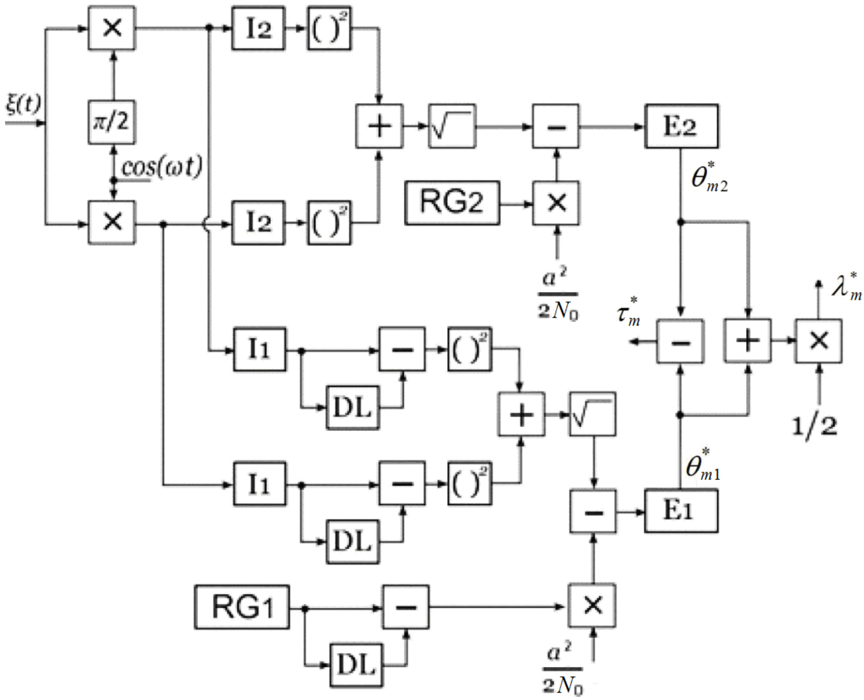

Fig.5 Block diagram of the quasi-optimal measurer of the time of arrival and the duration.

Fig. 5 shows the block diagram of the device generating the QO estimates (27) in terms of the expressions (25), (26). The designations in Fig. 5 are: I1 and I2 are the integrators over the time intervals $\left[-(\mathrm{T}_2-\Lambda_0)/2,\theta\right]$ and $\left[\theta,(\mathrm{T}_2-\Lambda_0)/2\right]$; RG1 and RG2 are ramp generators which are enabled in time $t=-(\mathrm{T}_2-\Lambda_0)/2$ and $t=\theta$, respectively; DL is the delay line for the time $\theta+(\mathrm{T}_2-\Lambda_0)/2$; E1 and E2 are the extremators fixing the positions of the absolute signal maxima within the time intervals $\left[\theta,\theta+(\mathrm{T}_2-\mathrm{T}_1)/2\right]$ and $\left[(\mathrm{T}_1-\Lambda_0)/2,(\mathrm{T}_2-\Lambda_0)/2\right]$, respectively. Thus, the application of the estimates (27) allows considerable simplification of the technical measurer implementation. Indeed, while for the realization of the ML estimation algorithm (19) we require the multichannel receiver constructed with the reference to appearance and disappearance moments, only the two-channel scheme is needed to determine the estimates in Eqs. (27).

The asymptotic characteristics for the estimates (24), which are valid under the large SNR, are found in [6]. The biases and the variances of the estimates of positions of the leading and the trailing edges of the radio pulse are defined by the expressions [6]

$$B_\varphi\left(\theta_{mi}^*\middle|\theta_{0i}\right)=0\,,\qquad V_\varphi\left(\theta_{mi}^*\middle|\theta_{0i}\right)=26\tau_0^2/z_0^4\,.$$

Taking into account Eqs. (27), we now find the expressions for the statistical characteristics of the QO estimates of the time of arrival and the duration, coinciding with Eqs. (17). Thus, the efficiency of the QO estimates (24) of the time of arrival and the duration of the signal with the a priori unknown initial phase asymptotically coincides with the efficiency of the ML estimates with the a priori known initial phase. Therefore, the dependences shown in Figs. 2, 3 characterize the accuracy gain provided by the ML estimates (19), or the QO estimates (27), when they are compared with the QL ones (6).

4. Summary

There are synthesized quasi-likelihood, maximum likelihood and quasi-optimal algorithms for the estimation of the time of arrival and the duration of the radio signal with the unknown initial phase. The efficiency characteristics of the synthesized algorithms are also found. The quasi-likelihood estimate is proved to have the simplest hardware or software implementation. However, in case of prior ignorance of the signal initial phase, this estimate suffers a considerable degradation in its accuracy. The maximum likelihood estimates of the time of arrival and the duration under the unknown initial phase can be realized by means of the multichannel receiver only. The application of the quasi-optimal algorithm yet allows making use of the two-channel receiver for the forming of the estimates without any losses in accuracy under the large signal-to-noise ratio.

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