ISSN 1063-7842, Technical Physics, 2018, Vol. 63, No. 8, pp. 1199–1204. © Pleiades Publishing, Ltd., 2018.
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RADIOPHYSICS

Efficiency of the Detection of a Specific Wideband Signal under a Priori Parametric Uncertainty

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Received November 9, 2017

Abstract—A quasi-plausible algorithm is synthesized for detection of ultrawideband quasi-radio-signal with arbitrary shape and unknown amplitude, initial phase and duration. Statistical characteristics of the efficiency of the proposed algorithm (false-alarm and signal-omission probabilities) are determined. A decrease in the detection efficiency due to a priori ignorance of the signal parameters is characterized.

DOI: 10.1134/S1063784218080212

Ultrawideband (UWB) signals are widely employed in modern practical applications (radio- and hydrolocation, navigation, seismology, radio communication, etc.) [1–3]. Pulse-amplitude and pulse-time modulations that are used in most systems lead to the detection of UWB signals with unknown amplitude and arrival time. Optimal and quasi-optimal algorithms for the processing of the UWB signals with unknown amplitude and arrival time have been analyzed in [4, 5]. However, the duration of the UWB signal may also be unknown owing to specific features of the propagation process and possible inaccuracy of the model at the receiving side. The problem of the detection of signals with unknown duration is topical for both UWB signals and narrow-band radio signals that have been studied in [6–8]. When the duration, amplitude, and initial phase of a radio signal are unknown, the problem of the a priori parametric uncertainty must be solved to synthesize the detector. Quasi-plausible (QP) and maximum plausible (MP) algorithms for the detection of a narrow-band radio signal have been studied. Expected values are used instead of unknown parameters in the former case, and the MP estimations of the unknown parameters are used in the latter case [6–8]. However, the results obtained for the detection of narrow-band radio signals are inapplicable in the analysis of the UWB signals, since the condition for relatively narrow band is significant. Therefore, it is expedient to synthesize and analyze algorithms for the detection of UWB signals with unknown duration. Multiple UWB signals can be studied using various models. We consider UWB signals the structure of which is similar to the structure of narrow-band radio signals. However, note that the condition for relatively narrow band is not satisfied. Such UWB signals have been called quasi-radio-signals (QRSs) [1]. Then, narrow-band radio-signals can be considered as specific UWB QRSs. Such an approach accounts for topicality of the generalization of the results obtained for narrow-band signals to the UWB QRSs. Note also that a narrow band is rather qualitative than quantitative characteristic of signals in classical physics. The problem of the detection of UWB QRS makes it possible to formulate quantitative criteria of narrow band.

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A model of the signal that must be detected can be represented as

$$s(t, a, \varphi, \tau) = \begin{cases} af(t)\cos(\omega t - \varphi), & 0 \le t \le \tau, \\ 0, \quad t < 0, & t > \tau, \end{cases} \quad (1)$$

where a, φ, ω, and τ are the amplitude, initial phase, frequency, and duration of the signal, respectively, and $f(t)$ is the modulation function. When frequency band $\Delta\omega$ and central frequency ω satisfy the condition

$$\Delta\omega \ll \omega, \quad (2)$$

radio signal (1) is classified as the narrow-band signal [1, 7]. If condition (2) is not satisfied, formula (1) describes the UWB QRS [1, 4, 5].

The scheme of Fig. 1 illustrates generation of signal (1). Here, G is the generator of harmonic signal $a\cos(\omega t - \varphi)$ and K is the switch that is closed at time interval $[0, \tau]$. Parameters a, φ, and ω are the parameters of the harmonic oscillation that is used for generation of the UWB QRS. Nevertheless, we use the approach of [1, 4, 5] and interpret quantities a, φ, and ω as the amplitude, initial phase, and frequency of UWB QRS (1). Variations in modulation function $f(t)$ can be used to obtain frequency band $\Delta\omega$ that is close to frequency ω. Therefore, expression (1) makes it

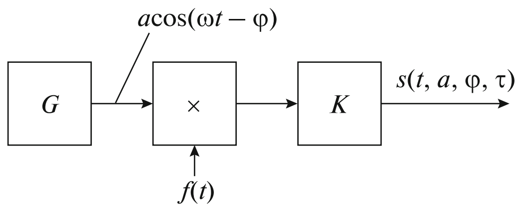

Fig. 1. Block diagram of the generation of the UWB QRS.

possible to describe both UWB QRS with a relatively wide frequency band and narrow-band signals for which condition (2) is satisfied [1, 4, 5].

We assume that signal (1) is received in the presence of white Gaussian noise $n(t)$ with single-sided spectral density N_0 and true amplitude a_0, initial phase φ_0, and duration τ_0 are a priori unknown parameters. An additive mixture of signal (1) and noise $n(t)$ that is observed over time interval $t \in [0, T]$ is represented as

$$\xi(t) = \gamma_0 s(t, a_0, \varphi_0, \tau_0) + n(t), \quad (3)$$

where γ_0 is the unknown true value of parameter γ that is $\gamma = 0$ in the absence of the signal and $\gamma = 1$ in the presence of the signal. We also assume that the signal duration belongs to a priori interval $\tau \in [T_1, T_2]$ ($0 < T_1 < T_2 \le T$. Based on the received signal (3), the detector must be able to solve the problem of the presence or absence of the signal. Hence, the problem of the detection is reduced to the estimation of discrete parameter γ using experimental data (3).

We use the MP method of [6, 7, 9] for the synthesis of the algorithm for the detection of the UWB QRS (estimation of parameter γ). When the parameters of the signal are unknown, we deal with the a priori parametric uncertainty with respect to amplitude, initial phase, and duration. In this case, the logarithm of likelihood ratio functional (LRF) depends on four unknown parameters:

$$L(\gamma, a, \varphi, \tau) = \frac{2\gamma}{N_0}\int_0^T \xi(t) s(t, a, \varphi, \tau) dt - \frac{\gamma}{N_0}\int_0^T s^2(t, a, \varphi, \tau) dt. \quad (4)$$

Several algorithms can be obtained when certain values are substituted for unknown parameters a, φ, and τ in expression (4). Such values can be fixed or determined from experimental data. In the QP algorithm for the detection of the UWB QRS of [10] at unknown amplitude, initial phase, and duration, an expected (predicted) value is used instead of a priori unknown duration and adaptation is employed with respect to unknown amplitude and initial phase. In this work, we consider the QP algorithm for the detection of the UWB QRS with adaptation with respect to duration. Following the approach of [10], we use expected values a^* and φ^* instead of unknown amplitude and initial phase in expression (4). The application of QP estimation instead of the unknown duration is equivalent to adaptation of the detection algorithm with respect to duration. Then, QP estimation $\hat{\gamma}$ of parameter γ is determined as the value at which the logarithm of LRF reaches the absolute maximum [9]. Thus, we follow the approach of [10] and represent the QP algorithm for signal detection (estimation of parameter γ) as

$$\hat{\gamma} = \begin{cases} 1, & L \ge h, \\ 0, & L < h. \end{cases} \quad (5)$$

Here, we have

$$L = \sup_{\tau} L(\tau), \quad L(\tau) = L(\gamma = 1, a = a^*, \varphi = \varphi^*, \tau). \quad (6)$$

Threshold h in formula (5) is chosen in accordance with optimization criterion [6, 7]. Expressions (4)–(6) determine the structure of the receiving unit. The detector must generate random process (6) for all possible values of the duration and search for the maximum. The decision on the presence or absence of the signal is based on the comparison of maximum (6) and threshold h. We substitute explicit representation of the UWB QRS (1) in expression (4) and represent the logarithm of LRF as

$$L(\tau) = a^*(X(\tau)\cos\varphi^* + Y(\tau)\sin\varphi^*) - a^{*2}[Q(\tau) + P_c(\tau)\cos 2\varphi^* + P_s(\tau)\sin 2\varphi^*]/2, \quad (7)$$

where we use the notation

$$X(\tau) = \frac{2}{N_0}\int_0^{\tau} \xi(t) f(t)\cos\omega t dt,$$
$$Y(\tau) = \frac{2}{N_0}\int_0^{\tau} \xi(t) f(t)\sin\omega t dt,$$
$$Q(\tau) = \frac{1}{N_0}\int_0^{\tau} f^2(t) dt, \quad (8)$$
$$P_c(\tau) = \frac{1}{N_0}\int_0^{\tau} f^2(t)\cos(2\omega t) dt,$$
$$P_s(\tau) = \frac{1}{N_0}\int_0^{\tau} f^2(t)\sin(2\omega t) dt.$$

The QP detector of the UWB QRS (expression (5)) can be implemented using the block diagram of Fig. 2 in which integrators I work at time interval $[0, t]$ ($t \in [0, T_2]$), F are the frequency doublers, PD is the peak detector, and TD is the threshold device that compares maximum L and threshold h and makes decision regarding the presence of the signal. For a narrow-band signal, we have $|P_c(\tau)| \ll Q(\tau)$ and $|P_s(\tau)| \ll Q(\tau)$ and logarithm of LRF (7) can be approximated as

$$L(\tau) \approx L_n(\tau)$$
$$= a^*(X(\tau)\cos\varphi^* + Y(\tau)\sin\varphi^*) - a^{*2}Q(\tau)/2.$$

The dashed line in Fig. 2 shows the block-diagram of the QP of the detector of the narrow-band radio signal. It is seen that the block diagram becomes more complicated when a wideband signal must be detected. In particular, we must employ frequency doublers and multipliers.

We analyze QP algorithm for the detection (5) to determine false-alarm and signal-omission probabilities [6, 7, 11]. Evidently, the absence of information on the amplitude and initial phase affects the detection efficiency. Thus, we introduce quantities that characterize the mismatch of the QP detector with respect to amplitude $\Delta_a = a^*/a_0$ and initial phase $\Delta_\varphi = \varphi^* - \varphi_0$. Then, the expected amplitude and initial phase can be represented using true values and mismatch as $a^* = a_0\Delta_a$ and $\varphi^* = \varphi_0 + \Delta_\varphi$. Substituting expected values a^* and φ^* in expression (7), we represent the logarithm of LRF as

$$L(\tau) = a_0\Delta_a(X(\tau)\cos(\varphi_0 + \Delta_\varphi) + Y(\tau)\sin(\varphi_0 + \Delta_\varphi)) - \frac{(a_0\Delta_a)^2}{2}(Q(\tau) + P_c(\tau)\cos(2\varphi_0 + 2\Delta_\varphi) + P_s(\tau)\sin(2\varphi_0 + 2\Delta_\varphi)). \quad (9)$$

Random processes $X(\tau)$ and $Y(\tau)$ are the Gaussian processes, since they represent linear transformations (8) of Gaussian process (3). Therefore, random process (9) is the Gaussian process that can statistically be described using mean value and correlation function. Let $L_1(\tau) = \{L(\tau)|\gamma_0 = 1\}$ be the logarithm of LRF (9) in the presence of the signal and $L_0(\tau) = \{L(\tau)|\gamma_0 = 0\}$ be the logarithm of LRF in the absence of the signal. Averaging yields expected mean values in the presence of the signal

$$S_1(\tau) = \langle L_1(\tau)\rangle = a_0^2\Delta_a[Q(\min(\tau,\tau_0))\cos(\Delta_\varphi) + P_c(\min(\tau,\tau_0))\cos(2\varphi_0 + \Delta_\varphi) + P_s(\min(\tau,\tau_0))\sin(2\varphi_0 + \Delta_\varphi)] - \frac{a_0^2\Delta_a^2}{2}[Q(\tau) + P_c(\tau)\cos(2\varphi_0 + 2\Delta_\varphi) + P_s(\tau)\sin(2\varphi_0 + 2\Delta_\varphi)] \quad (10)$$

and in the absence of the signal

$$S_0(\tau) = \langle L_0(\tau)\rangle = -\frac{a_0^2\Delta_a^2}{2}[Q(\tau) + P_c(\tau)\cos(2\varphi_0 + 2\Delta_\varphi) + P_s(\tau)\sin(2\varphi_0 + 2\Delta_\varphi)], \quad (11)$$

and the correlation function

$$K(\tau_1,\tau_2) = \langle[L_1(\tau_1) - \langle L_1(\tau_1)\rangle][L_1(\tau_2) - \langle L_1(\tau_2)\rangle]\rangle = \langle[L_0(\tau_1) - \langle L_0(\tau_1)\rangle][L_0(\tau_2) - \langle L_0(\tau_2)\rangle]\rangle = a_0^2\Delta_a^2[Q(\min(\tau_1,\tau_2)) + P_c(\min(\tau_1,\tau_2)) \times\cos(2\varphi_0 + 2\Delta_\varphi) + P_s(\min(\tau_1,\tau_2))\sin(2\varphi_0 + 2\Delta_\varphi)]. \quad (12)$$

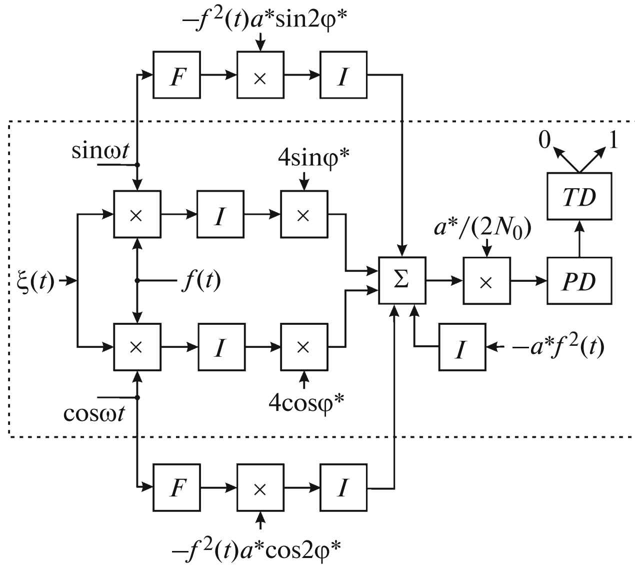

Fig. 2. Block diagram of the QP detector of the UWB QRS.

Then, we assume that the output signal-to-noise ratio (SNR) is relatively high for the received signal. To determine the false-alarm probability, we study decision statistics $L_0(\tau)$ in the vicinity of the maximum. When the SNR increases, the position of the maximum of the decision statistics exhibits the mean-square convergence to the position of the maximum of the expected mean value [11]. The derivative of mean value (10) in the absence of the signal

$$\frac{\partial S_0(\tau)}{\partial\tau} = -a_0^2\Delta_a^2 f^2(\tau)\cos^2(\omega\tau - \varphi_0 - \Delta_\varphi)/N_0$$

is negative at all possible durations. Hence the position of the maximum of expected mean value $S_0(\tau)$ of the decision statistics at interval $[T_1, T_2]$ coincides with the left-hand boundary of the a priori interval of possible durations T_1. Using the expansion of expressions (11) and (12) in the Taylor series in terms of τ in the vicinity of T_1, we obtain asymptotic expressions for expected mean value and correlation function in the absence of the signal:

$$S_0(\tau) \approx -\lambda_0/2 - (\tau - T_1)\psi_0/2T_2, \quad (13)$$

$$K_{q0}(\tau_1,\tau_2) \approx \lambda_0 + \psi_0\min(\tau_1 - T_1, \tau_2 - T_1)/T_2, \quad (14)$$

where

$$\psi_0 = \Delta_a^2 z^2 f^2(T_1)\cos^2(\omega T_1 - \varphi_0 - \Delta_\varphi),$$
$$\lambda_0 = (a_0\Delta_a)^2(Q(T_1) + P_c(T_1)\cos(2\varphi_0 + 2\Delta_\varphi) + P_s(T_1)\sin(2\varphi_0 + 2\Delta_\varphi)), \quad z^2 = 2a_0^2T_2/N_0 \quad (15)$$

is the output SNR of the MP detector of the UWB QRS with amplitude a_0, duration T_2, and rectangular-shaped modulation function.

We approximate logarithm of LRF $L_0(\tau)$ at relatively high SNRs using Gaussian random process $\mu_0(\tau)$ with expected mean value (13) and correlation function (14) at the entire a priori interval of the durations. Expressions (13) and (14) and the Doob theorem [12, 13] can be used to show that decision statistics $\mu_0(\tau)$ is the Gaussian Markovian process with drift coefficient k_{10} and diffusion coefficient k_{20} given by [12, 13]

$$k_{10} = -\psi_0/2T_2, \quad k_{20} = \psi_0/T_2. \tag{16}$$

By definition, the false-alarm probability is represented as $\alpha = 1 - F_0(h)$, where

$$F_0(h) = P\{\mu_0(\tau) < h, \tau \in [T_1, T_2]\} \tag{17}$$

is the probability with which Markovian random process $\mu_0(\tau)$ does not reach boundaries $y = -\infty$ and $y = h$ at interval $\tau \in [T_1, T_2]$. Desired probability (17) can be represented in terms of probability density $W(y, \tau)$ of realizations in which random process $\mu_0(\tau)$ does not reach boundaries $y = -\infty$ and $y = h$ [13]:

$$F_0(h) = \int_{-\infty}^{h} W(y, T_2)dy. \tag{18}$$

Probability density $W(y, \tau)$ is a solution to the Fokker–Planck–Kolmogorov (FPK) equation [12, 13]

$$\frac{\partial W(y,\tau)}{\partial \tau} + \frac{\partial}{\partial y}[k_1 W(y,\tau)] - \frac{1}{2}\frac{\partial^2}{\partial y^2}[k_2 W(y,\tau)] = 0 \tag{19}$$

with coefficients (16) $k_1 = k_{10}$ and $k_2 = k_{20}$ under the initial condition

$$W(y, T_1) = \frac{1}{\sqrt{2\pi\lambda_0}}\exp\left(-\frac{(y + \lambda_0/2)^2}{2\lambda_0}\right)$$

and the boundary conditions

$$W(-\infty, \tau) = W(h, \tau) = 0.$$

We solve FPK equation (19) using the method of reflection with sign changing [13] and substitute the solution in expression (18) and then expression (18) in expression (17) to derive the following formula for the false-alarm probability:

$$\begin{aligned}\alpha = \alpha(h, z) = 1 - \frac{1}{\sqrt{2\pi\lambda_0}}\int_0^{\infty}\exp\left(-\frac{(h - \xi + \lambda_0/2)^2}{2\lambda_0}\right) \\ \times\left[\Phi\left(\frac{1}{2}\sqrt{\frac{\psi_0(k-1)}{k}} + \xi\sqrt{\frac{k}{\psi_0(k-1)}}\right)\right. \\ \left. - \exp(-\xi)\Phi\left(\frac{1}{2}\sqrt{\frac{\psi_0(k-1)}{k}} - \xi\sqrt{\frac{k}{\psi_0(k-1)}}\right)\right]d\xi,\end{aligned} \tag{20}$$

where $k = T_2/T_1$ is the dynamic range of possible durations and

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp(-t^2/2)dt$$

is the probability integral.

Then, we search for an approximate expression for the conditional signal-omission probability that is valid at relatively high SNRs. It is known from [6, 7] that an increase in the SNR leads to the mean-square convergence of the maximum of the logarithm of LRF (7) to the position of the maximum of expected mean value $\tau_s = \arg\sup S_1(\tau)$. Thus, we analyze decision statistics $L_1(\tau)$ in the vicinity of τ_s. Below, we restrict consideration to combinations of expected and true amplitudes and initial phases at which the position of the maximum of expected mean value (10) coincides with the true value of the unknown duration, so that $\tau_s = \tau_0$. We expand functions (10) and (12) in the Taylor series in terms of τ in the vicinity of τ_0 and consider only the terms of the first order of smallness. Thus, we obtain asymptotic expressions for the expected mean value and correlation function in the presence of the signal:

$$S_1(\tau) \approx \frac{\lambda_1}{2} + \frac{\tau - \tau_0}{2T_2}\begin{cases}\psi_1, & \tau \le \tau_0,\\ -\psi_2, & \tau > \tau_0,\end{cases} \tag{21}$$

$$K_{q1}(\tau_1, \tau_2) \approx \lambda_1 + \psi_2 \min(\tau_1 - \tau_0, \tau_2 - \tau_0)/T_2, \tag{22}$$

where

$$\lambda_1 = S_1(\tau_0), \quad \psi_1 = z^2\Delta_a f^2(\tau_0)\cos(\omega\tau_0 - \Delta_\varphi - \varphi_0) \times[2\cos(\varphi_0 - \omega\tau_0) - \Delta_a\cos(\omega\tau_0 - \Delta_\varphi - \varphi_0)],$$

$$\psi_2 = \Delta_a^2 z^2 f^2(\tau_0)\cos^2(\omega\tau_0 - \Delta_\varphi - \varphi_0).$$

We approximate logarithm of LRF $L_1(\tau)$ using Gaussian random process $\mu_1(\tau)$ with expected mean value (21) and correlation function (22). Such an approximation is meaningful at any $\tau > \tau_d = \tau_0 - T_2\lambda_1/\psi_2$ for which the variance of random process $\mu_1(\tau)$ is nonnegative (i.e., $K_{q1}(\tau, \tau) \approx \lambda_1 + \psi_2(\tau - \tau_0)/T_2 \ge 0$). When approximation $\mu_1(\tau)$ is used, we assume that the duration belongs to a priori interval $[T_d, T_2]$, where $T_d = \max(\tau_d, T_1)$. Expressions (21) and (22) and the Doob theorem [12, 13] can be used to show that decision statistics $\mu_1(\tau)$ is the Gaussian Markovian process with drift coefficient k_{11} and diffusion coefficient k_{21} given by [12, 13]

$$k_{11} = \frac{1}{2T_2}\begin{cases}\psi_1, & T_d \le \tau \le \tau_0,\\ -\psi_2, & \tau_0 < \tau \le T_2,\end{cases} \quad k_{21} = \frac{\psi_2}{T_2}. \tag{23}$$

By definition, the signal-omission probability is written as

$$\beta = F_1(h) = P\{\mu_1(\tau) < h, \tau \in [T_d, T_2]\}. \tag{24}$$

Such an expression is interpreted as the probability with which Markovian random process $\mu_1(\tau)$ does not reach boundaries $y = -\infty$ and $y = h$ at interval $\tau \in [T_d, T_2]$. Desired probability (24) can be represented in terms of probability density $W(y, \tau)$ of realizations in which random process $\mu_1(\tau)$ does not reach boundaries $y = -\infty$ and $y = h$ [13]:

$$F_1(h) = \int_{-\infty}^{h} W(y, T_2)dy. \tag{25}$$

Function $W(y, \tau)$ is the solution to FPK equation (19) with coefficients (23) under the initial condition

$$W(y, T_d) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(y-m)^2}{2\sigma^2}\right)$$

and the boundary conditions

$$W(-\infty, \tau) = W(h, \tau) = 0,$$

where

$$\sigma^2 = \lambda_1 + \psi_2(T_d - \tau_0)/T_2,$$
$$m = \lambda_1/2 + \psi_1(T_d - \tau_0)/2T_2.$$

We solve FPK equation (19) using the method of reflection with sign changing [13], substitute the solution in formula (25), and substitute resulting expression (25) in expression (24) to obtain the following formula for the signal-omission probability:

$$\beta(h, z) = \frac{\exp[-\psi_1^2(k_d(1+k) - 2k)/16\psi_2 k k_d]}{\sqrt{\pi\psi_2(k_d(1+k) - 2k)/kk_d}}$$
$$\times\int_0^\infty\int_0^\infty W(h-\xi, T_d)\exp\left(\frac{\psi_2}{2\psi_2}(\xi - \xi_1)\right)$$
$$\times\left\{\Phi\left(\frac{1}{2}\sqrt{\frac{\psi_2(k-1)}{2k}} + \xi_1\sqrt{\frac{2k}{\psi_2(k-1)}}\right)\right.$$
$$\left.-\exp(-\xi_1)\Phi\left(\frac{1}{2}\sqrt{\frac{\psi_2(k-1)}{2k}} - \xi_1\sqrt{\frac{2k}{\psi_2(k-1)}}\right)\right\} \tag{26}$$
$$\times\left\{\exp\left(-\frac{(\xi-\xi_1)^2 kk_d}{\psi_2(k_d(1+k) - 2k)}\right)\right.$$
$$\left.-\exp\left(-\frac{(\xi+\xi_1)^2 kk_d}{\psi_2(k_d(1+k) - 2k)}\right)\right\}d\xi d\xi_1,$$

where $k_d = \dfrac{T_2}{T_d}$.

By way of example, we analyze the QP algorithm for the detection of the UWB QRS the modulation function of which is given by $f(t) = \exp(-\nu t/T_2)$, where parameter ν is the decay rate of the modulation function. We consider quantity $\kappa = \omega\tau_0/2\pi$ that is numerically equal to the number of periods of the harmonic carrier corresponding to signal duration τ_0. Following the approach of [4, 5], we interpret parameter κ as the parameter characterizing the narrow-band character of the process. Signal (1) is the narrow-band signal when $\kappa \to \infty$. Figure 3 shows the dependences of signal-omission probabilities (26) on SNR (15) at several levels of false-alarm probability (20): the solid,

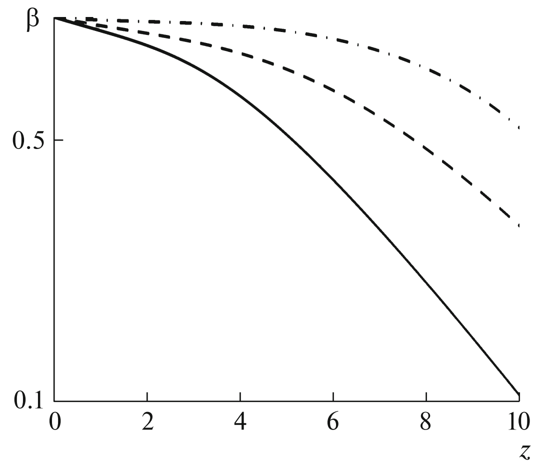

Fig. 3. Plots of signal-omission probability vs. SNR at several levels of the false-alarm probability.

dashed, and dashed-and-dotted curves correspond to $\alpha = 0.1$, 0.01, and 0.001, respectively. The curves are calculated on the assumption that the initial phase is $\varphi_0 = 0$, $\nu = 2$, $k = 4$, $\kappa = 0.3$, and the mismatches of the amplitude and phase are absent ($\Delta_a = 1$ and $\Delta_\varphi = 0$). The curves of Fig. 3 characterize the detection efficiency in the absence of the amplitude and phase mismatches. In this case, the QP algorithm for the detection coincides with the MP algorithm at a priori known amplitude and initial phase.

The effect of a priori absence of information on the amplitude and initial phase on the detection efficiency can be quantitatively characterized using the following expression:

$$\chi = \frac{\beta(h', z, \Delta_a, \Delta_\varphi)}{\beta(h', z, \Delta_a = 1,\ \Delta_\varphi = 0)}. \tag{27}$$

Such a quantity can be interpreted as the ratio of the signal-omission probabilities for signals with unknown amplitude, initial phase, and duration in the presence and absence of amplitude and phase mismatches. In expression (27), h' is the threshold found from the solution to equation $\alpha(h', z) = p$. Figures 4 and 5 present loss of efficiency (27) for the detection of the UWB QRS with unknown duration versus amplitude (Δ_a) and phase (Δ_φ) mismatches, respectively, for several SNRs and $p = 0.1$. The solid, dashed, and dashed-and-dotted lines in Figs. 4 and 5 correspond to SNRs $z = 3$, 5, and 7, respectively. The curves are calculated for the parameters $\varphi_0 = 0$, $\nu = 2$, $k = 4$, and $\kappa = 0.3$.

Thus, the absence of information on the amplitude, initial phase, and duration of the UWB QRS leads to a several-fold increase in the error probability. Note that the detection efficiency substantially depends on parameter κ that characterizes the spectral width of the signal. Such a result is due to the fact that

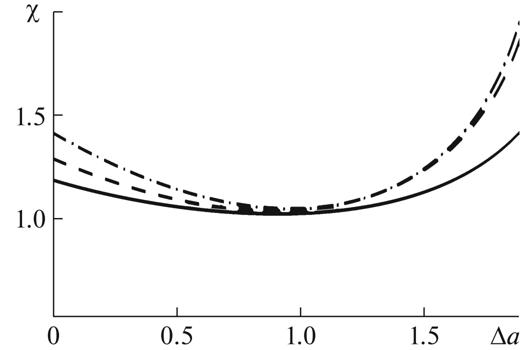

Fig. 4. Plots of the loss of efficiency for the detection of the UWB QRS vs. amplitude mismatch.

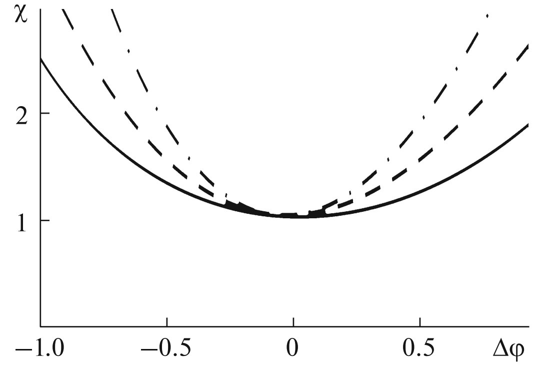

Fig. 5. Plots of the loss of efficiency for the detection of the UWB QRS vs. mismatch of the initial phase.

the error probabilities are asymptotically independent of the signal shape at relatively high SNRs and determined by trailing edge $f(t_0)$, which depends on parameter κ. The results of this work make it possible to quantitatively characterize the effect of a priori absence of information on parameters of the UWB QRS on the detection efficiency and calculate the allowed mismatches of the amplitude and initial phase of the UWB QRS using permissible false-alarm and signal-omission probabilities.

ACKNOWLEDGMENTS

We are grateful to our coworker A.P. Trifonov who passed away when this paper was in press.

REFERENCES

1. V. G. Radzievskii and P. A. Trifonov, *Processing of Noisy Ultrawideband Signals* (Radiotekhnika, Moscow, 2009).
2. H. Arslan, Z. N. Chen, and M. G. Benedetto, *Ultra Wideband Wireless Communication* (Wiley, New Jersey, 2006).
3. F. Nekoogar and F. Dowla, *Ultra-Wideband Radio Frequency Identification Systems* (Springer, New York, 2011).
4. A. P. Trifonov and P. E. Rudnev, Radiophys. Quantum Electron. **52**, 671 (2009).
5. A. P. Trifonov and P. E. Rudnev, Radiophys. Quantum Electron. **54**, 415 (2011).
6. P. S. Akimov, P. A. Bakut, and V. A. Bogdanovich, *Theory of Signal Detection* (Radio i Svyaz', Moscow, 1984).
7. V. I. Tikhonov, *Optimum Signal Reception* (Radio i Svyaz', Moscow, 1983).
8. A. P. Trifonov, Yu. E. Korchagin, and M. V. Trifonov, Radiophys. Quantum Electron. **58**, 361 (2015).
9. E. I. Kulikov and A. P. Trifonov, *Estimation of Parameters of Noisy Signals* (Sovetskoe Radio, Moscow, 1978).
10. A. P. Trifonov, Yu. E. Korchagin, and K. D. Titov, Radiotekhnika, No. 6, 99 (2016).
11. A. P. Trifonov and Yu. S. Shinakov, *Joint Discrimination of Noisy Signals and Estimation of Their Parameters* (Radio i Svyaz', Moscow, 1986).
12. J. L. Doob, *Stochastic Processes* (Wiley, New York, 1953).
13. V. I. Tikhonov and M. A. Mironov, *Markov Processes* (Radio i Svyaz', Moscow, 1977).

Translated by A. Chikishev